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ELASTIC PION-DEUTERON SCATTERING NEAR THE 3-3 RESONANCE

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A B S T R A C T

Elastic pion-deuteron scattering at medium energies is analyzed on the basis of the Brueckner model, particular attention being paid to the kinematic approximations that have to be made. In contrast to an earlier Glauber-type calculation, the energy dependence of the large angle cross-section is reasonably well reproduced.

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1. INTRODUCTION

It can be argued that the low energy pion-deuteron system represents the simplest three-body scattering problem in nuclear physics. Until recently however, this has only been studied in bubble chambers with poor statistics ¹⁾, which has not posed much of a challenge to the theorist. In the last couple of years the quality of the data has improved significantly ^{2),3)}, and this trend is likely to continue with the advent of the new high intensity accelerators.

The pion-deuteron elastic scattering amplitude, in the energy range of the first pion-nucleon resonance, depends critically on the pion-nucleon spin-flip amplitude and charge-exchange amplitude as well as the deuteron D state probability. Thus any complete dynamical study must take into account many different scattering channels. There exists a recent calculation by Myrher and Koltun ⁴⁾ using the Fadeev equations, but in order to keep the computation relatively simple, they had to neglect the complications mentioned above. On the other hand, these can easily be built into a Glauber calculation ⁵⁾, but its validity at the large angles and low energies we are interested in is extremely doubtful. There are two basic assumptions in the Glauber scheme. Firstly the pion-deuteron amplitude is calculated as if the nucleons in the nucleus were fixed scatterers, and subsequently the result is averaged over the deuteron wave function. This may be justified ⁶⁾ using the closure approximation providing that the pion is allowed to transfer only a small part of its energy to the nucleons. This would be the case for a very light, low energy, incident particle, for which the nucleon is relatively massive. Alternatively, in the Glauber limit of high energy scattering, the dominance of the pion-nucleon diffraction peak achieves the same effect for small angle scattering. With this first assumption the problem is reduced to the solution of a series of rather complicated one-body problems. The second assumption of the Glauber model is that all scatterings are small angle high energy ones, which can be described reasonably well by the eikonal approximation. In particular, the eikonal form of the pion propagator is used between the pion-nucleon scatterings. This is unrealistic when each pion-nucleon amplitude is dominated by a single partial wave. Gibbs ⁷⁾ attempted to improve upon the eikonal propagation, but his ansatz for the replacement is still poor at large angles. In the present paper we shall follow the method of Brueckner ^{8),9)}, which keeps the first, averaging, assumption, but improves upon the calculation of the fixed scatterer amplitude.

2. BRUECKNER MODEL CALCULATION

If two potentials do not overlap, then the total on-shell scattering amplitude for the compound system is a function only of the individual on-shell amplitudes ^{10),11)}. As a simple example of this, if there is only s wave scattering from each "nucleon", the pion-deuteron amplitude for fixed scatterers is

$$F_{\pi d}(\vec{q}, \vec{r}) = \frac{\alpha_p e^{i\vec{q} \cdot \vec{r}/2} + \alpha_n e^{-i\vec{q} \cdot \vec{r}/2} + 2\alpha_p \alpha_n e^{i\vec{k} \cdot \vec{r}} \cos(\vec{k} \cdot \vec{r})/r}{1 - \alpha_p \alpha_n e^{2i\vec{k} \cdot \vec{r}}/r^2} \quad (1)$$

where $\vec{q}$ and $\vec{k}$ are the momentum transfer and average momentum respectively

$$\begin{aligned} \vec{q} &= (\vec{k} - \vec{k}') \\ \vec{k} &= \frac{1}{2}(\vec{k} + \vec{k}') \end{aligned} \quad (2)$$

α_p is the pion-proton s wave amplitude and $\frac{1}{2}\vec{r}$ is the co-ordinate of the proton. Equation (1) is derived by merely matching boundary conditions on the surface of the two scattering centres, and therefore does not depend upon the nature of the pion-nucleon potential, or even upon its existence. Single and double scattering can be explicitly seen in Eq. (1), and the expansion of the denominator leads to higher order multiple scatterings.

To obtain the real pion-deuteron amplitude, this fixed scatterer amplitude must be averaged over the distribution of the two nucleons.

$$F_{\pi d}^{n'n}(\vec{q}) = \int \mathcal{G}_{n'}^*(\vec{r}) F_{\pi d}(\vec{q}, \vec{r}) \mathcal{G}_n(\vec{r}) d^3 r \quad (3)$$

where $\mathcal{G}_M(\vec{r})$ is the deuteron wave function of polarization M. For high energy, small angle scattering ($kR_d \gg 1$, $k \gg q$) this leads essentially to the same result as Glauber theory, including the modification proposed by Gottfried ¹²⁾. For an S wave deuteron

$$F_{\pi d}(q) \approx (\alpha_p + \alpha_n) \langle e^{i\vec{q} \cdot \vec{r}/2} \rangle + i \frac{\alpha_p \alpha_n}{R} \left\langle \frac{e^{i\frac{q^2 r}{8R}}}{r^2} \right\rangle \quad (4)$$

Near the backward direction, however, the formula is quite different from Glauber's, the double scattering getting important contributions from the real part of the propagator as well as the imaginary

$$F_{\pi d}(q=2k) \approx (\alpha_p + \alpha_n) \left\langle \frac{\sin kr}{Rr} \right\rangle + 2\alpha_p \alpha_n \left\langle \frac{e^{i\frac{kr}{r}}}{r} \right\rangle \quad (5)$$

For simplicity in this paper we shall confine our attention to single and double scattering, in which case the fixed scatterer amplitude of Eq. (1) may be obtained in a rather different way, which can be easily generalized to include the higher pion-nucleon partial waves. For arbitrary pion-nucleon amplitudes f , the expansion up to second order in f is given by 9), 13)

$$\begin{aligned} F_{\pi d}(\vec{q}, \vec{r}) = & f_p(\vec{k}', \vec{k}) e^{i\vec{q} \cdot \vec{r}/2} + f_n(\vec{k}', \vec{k}) e^{-i\vec{q} \cdot \vec{r}/2} \\ & + \frac{1}{2\pi^2} \int \frac{d^3 q'}{(\vec{K} - \vec{q}')^2 - k^2 - i\epsilon} \left\{ f_p(\vec{K} - \vec{q}'/2, \vec{K} - \vec{q}') f_n(\vec{K} - \vec{q}', \vec{K} + \vec{q}'/2) e^{-i\vec{q}' \cdot \vec{r}} \right. \\ & \left. + f_n(\vec{K} - \vec{q}'/2, \vec{K} - \vec{q}') f_p(\vec{K} - \vec{q}', \vec{K} + \vec{q}'/2) e^{i\vec{q}' \cdot \vec{r}} \right\} \quad (6) \end{aligned}$$

To go any further we need the result, proved explicitly in this framework by Beg¹⁰⁾, that if the two potentials do not overlap, then the double scattering integral only depends upon the on-shell pion-nucleon amplitudes. Thus to obtain the simple s wave formula of Eq. (1) we replace the f by α , and take them out of the integral at their on-shell values. The remaining integral is then evaluated using calculus of residues, and leads immediately to the required result. Note that although the final answer depends only upon the on-shell quantities, we do not neglect the off-shell propagation between the scatterings [the principal part of the integral in Eq. (6)]. Because of the no-overlap hypothesis, this also gives a contribution which is just a function of the physical amplitudes.

The procedure is very similar for the inclusion of higher partial waves: the angular dependence of the pion-nucleon scattering amplitude must be inserted into the integral. After performing the angular integrations, we merely pick up the residue of the propagator pole. The solution for the double scattering amplitude for the case of an S wave deuteron, but including all pion-nucleon partial waves is given by Beg¹⁰⁾.

The low energy pion-nucleon amplitude is dominated by the s and p waves, with very little d wave scattering below 300 MeV. Neglecting these waves, the pion-nucleon amplitude may be written as

$$f = \alpha + \beta \hat{k} \cdot \hat{k}' + i \gamma \vec{\sigma} \cdot (\hat{k} \wedge \hat{k}') \quad (7)$$

where the spin-flip amplitude γ ensures that f is an operator in the nucleon spin. These amplitudes may be constructed in the centre-of-mass frame directly from the pion-nucleon phase shifts, which are well determined from 80 to 310 MeV¹⁴⁾. Deferring for the moment the problem of the choice of reference frames, the functional form in Eq. (7) may be inserted into the integral of Eq. (6), which can then be evaluated as just described. To carry out the averaging over the deuteron co-ordinates as prescribed in Eq. (3), we first decompose the deuteron wave function $\mathcal{D}_M$ into S and D waves [see for example Ref. 15]. Then the angular integrations may be performed analytically for each element of the amplitude $F_{M'M}(q)$, leaving only the integration over the radial co-ordinate to be done numerically. The result of this angular averaging leads to extremely lengthy expressions, especially for the deuteron D wave contributions. The explicit formulae are therefore relegated to Appendix A. The only extra simplification we have made is to neglect the double spin-flip contribution from an initial deuteron D state to a final D state. This should be very small.

The possibility of double charge exchange can be taken into account by making the algebraic substitution¹⁶⁾ in the double scattering

$$\delta_p \varepsilon_n \rightarrow \delta_p \varepsilon_n - \frac{1}{2} \delta^{c.e.} \varepsilon^{c.e.} \quad (8)$$

where δ and ε refer to any of α , β , or γ , and the c.e. superscript denotes the corresponding charge exchange amplitude.

3. KINEMATIC PROBLEMS

The most serious difficulties in applying the formulae we have just derived are connected with the recoil corrections to the theory. The Brueckner model is based on the assumption of two very heavy scatterers, and this is only roughly true for the nucleons. Already in the single-scattering term this gives rise to problems of interpretation. Near the backward direction, for a given incident energy the maximum momentum transfer increases as the target mass gets larger. Because $F_{\pi d}(q) \propto f_{\pi p}(q)$, we will need the pion-nucleon amplitude at an unphysical point. This can be avoided if one postulates that for targets of finite mass the formalism should be set up in the Breit or Brick-wall frame. As a consequence of this the laboratory energy of the pion-nucleon amplitude becomes higher than that for pion-deuteron, and for single scattering the two momentum transfer regions overlap completely. From the kinematics of the Breit frame illustrated in Fig. 1, it can be seen that there is no energy transfer to the nucleus, which is very much in the spirit of the Brueckner model.

We must now calculate the Breit frame amplitudes in terms of the pion-nucleon phase shifts, and this we do in two stages. First we obtain the centre-of-mass amplitudes, and from these the invariant amplitudes A and B¹⁷⁾. Secondly, the invariant amplitudes are reduced in the Breit frame into a spin-non-flip and a spin-flip part

$$f_{\pi N} = f + i \vec{\sigma} \cdot (\hat{q} \wedge \hat{k}) g \quad (9)$$

where

$$f = N \left[A \sqrt{1 + q^2/4m^2} + B \left\{ \frac{s - m^2 - \mu^2 - q^2/2}{2m \sqrt{1 + q^2/4m^2}} \right\} \right] \quad (10a)$$

$$g = N B K q / 2m \quad (10b)$$

and the Jacobian factor

$$N = \frac{m}{4\pi\sqrt{s}} \left[\frac{\partial(\cos\theta^*)}{\partial(\cos\theta_{\text{BREIT}})} \right]^{1/2} \quad (11)$$

In the above m is the nucleon mass, μ the pion mass and s the square of the pion-nucleon centre-of-mass energy.

This frame transformation distorts considerably the angular variation of the amplitudes, and an example of this is shown in Fig. 2 for the spin-non-flip, isospin $\frac{3}{2}$ case. If the change were unimportant, then the amplitude would be essentially a straight line, which is clearly not the case. The origin of the distortion is to be found not only in the Lorentz transformation of the amplitudes, but also in the fact that the amplitudes are required at increasing centre-of-mass energies for increasing momentum transfer.

How should we choose the pion-nucleon scattering amplitudes such as to minimise the effect of neglecting this faster angular variation? For single scattering there is no real problem since we can there use the full pion-nucleon amplitude without truncating the partial waves. This is not easy for the double scattering (see Appendix A).

If we introduce the deuteron averaging inside the integral of Eq. (6), the expectation value of $e^{iq'\cdot r}$ is just a deuteron form factor. The most important deuteron form factors are sharply peaked towards small values of the momentum transfer. Consequently we should determine the parameters of the s and p waves such as to give the correct behaviour at the maximum of the form factors, i.e. $q'=0$. Thus for pion-deuteron scattering through an angle θ , for single scattering we should take the pion-nucleon amplitude at an angle θ , but for double scattering rather in the vicinity of half this angle. The actual fitting of the amplitude for the spin-non-flip part may be accomplished by matching the value of the amplitude and its first derivative at the required angle (see Fig. 2). For the spin-flip amplitude only one parameter is required.

We have so far neglected the internal motion of the nucleons in the deuteron. This may be rectified somewhat by smearing the input pion-nucleon amplitudes over the momentum spectrum in the deuteron. The principal effect is due to the longitudinal motion of the target nucleons, taken to lowest order. Second order terms are very ambiguous, being intimately connected with binding energy problems. Expanding the pion kinetic energy T around the value for a stationary nucleon T_0 and keeping only the leading correction, the smeared amplitudes are simple functionals of the unsmeared ones

$$f^{\text{SMEARED}}(T) = f(T_0) + \frac{k^2 \langle p^2 \rangle}{6m^2} \left. \frac{\partial^2 f(T)}{\partial T^2} \right|_{T_0} \quad (12)$$

where k is the Breit momentum and $\langle p^2 \rangle$ the average value of the square of the Fermi momentum. Reasonable deuteron wave functions¹⁸⁾ give $\langle p^2 \rangle^{\frac{1}{2}} \approx 110$ MeV/c. In practise the Fermi momentum does not play a vital rôle in this approximation, giving at most an 8% correction. At the present stage of sophistication it is probably not worth improving upon.

Equation (12) is derived assuming a spherically symmetric deuteron. Because of the deuteron D state, there are terms linear in the Fermi momentum which can induce magnetic scattering (terms like F_{10}), in complete analogy with electron scattering. They are, however, proportional to the deuteron D state probability¹⁹⁾ and will be neglected in the present work.

4. RESULTS AND DISCUSSION

The deuteron wave function used in this calculation is that derived by Humberston¹⁸⁾ from a modified Hamada-Johnston potential. This has a D state probability of 7%. It is of course essential to work with a wave function containing a hard core, or otherwise the integral over the p wave double-scattering expression [Eq. (A)] would diverge. Although this divergence may be eliminated by considering the higher order scatterings, the Brueckner model becomes meaningless when there is a significant overlap of the two nucleons. In the region of such overlap, the propagation of the pion is completely falsified by taking the free Green's function, the more so for the higher pion-nucleon partial waves. There is a

calculation ²⁰⁾ which suggests that though the range of the pion-nucleon force is larger than the hard-core separation, it is less than the average internucleon distance in the deuteron, so that the model might be a reasonable approximation from this point of view.

The pion-nucleon phase shifts were taken from the analysis of Carter et al. ¹⁴⁾. It can be seen from Fig. 2 that near the forward direction the kinematic transformation induces a large positive effective p wave amplitude, but a negative s wave. These of course cancel to give an amplitude of moderate dimensions, but it makes it very difficult to investigate the consequences of truncating the multiple scattering series after the first two terms.

In Figs. 3 and 4 are plotted the angular distribution for pion-deuteron elastic scattering at 256 MeV and the energy distribution for a fixed laboratory scattering angle of 160° , the experimental data being taken from Ref. 3). The theoretical curves show the result of single scattering as well as the full calculation. In the single-scattering approximation, in the absence of pion-nucleon spin-flip terms there would be a deep minimum in the vicinity of 90° due to the vanishing there of the p wave amplitude. At large angles the deuteron charge form factor gets large contributions from the quadrupole form factor (the Fourier transform of the overlap of the S and D state wave functions), so that the cross-section here is influenced tremendously by the D state probability. At 256 MeV, switching off the D state reduces the cross-section by a factor of two, but the effect at lower energies is less marked, since a fixed angle then corresponds to a smaller momentum transfer. These sensitivities are maintained with the inclusion of double scattering. In our model the double scattering acts constructively near the forward direction at 256 MeV, and since the single scattering agrees moderately well with the experimental points there, our calculated cross-section is consistently higher than the data. Around 90° the most important amplitude still seems to be F^{10} (single deuteron spin flip), but the interference between single and double scattering here depends largely upon the S-D contribution to the double scattering. The neglect of the D state would give a large enhancement. The importance of the double charge exchange can easily be gauged for large angles from Eq. (5). Near a supposed elastic s wave resonance, $\alpha_p = \alpha_n = ik$, the on-shell part of the double scattering completely extinguishes the single scattering, leaving only the result of the off-shell propagation. This cancellation, which is also true for the p wave case, is not total when the double scattering is reduced by the charge exchange [Eq. (8)].

Although the predicted angular distribution is quite reasonable, especially in view of its sensitivity to many small terms, it does not look much of an improvement upon the Glauber calculation of Ref. 3). This is probably because the latter fits well by accident! On the other hand the present calculation reproduces the fixed angle energy dependence much better (Fig. 4). In Fig. 5 are shown the results of the best of the bubble chamber measurements at 180 MeV ²⁾. The statistics are much poorer than those of the counter experiment of Ref. 3). The shape of the cross-section past 60° is not well reproduced.

In what way could the present application of the model be improved? The Brueckner model has been rederived by a summation of a class of non-relativistic Feynman diagrams ²¹⁾ and this work suggests some possible corrections to the model. In the present calculation it would clearly be more consistent to take the higher partial waves into account explicitly in the double scattering integral, rather than using our half-angle prescription. Then the triple scattering could be examined without having to worry whether the amplitude should be evaluated say at one third of the angle! The effects of a finite range of the pion-nucleon force must also be investigated, but this requires more knowledge of the dynamics of the pion-nucleon system.

These modifications may only be palliatives, and the model of neglecting the deuteron dynamics too crude at the level of accuracy we are working. In this case one must start using the Fadeev scheme seriously, with realistic nucleon-nucleon forces, with accurate deuteron wave functions, and include spin and isospin dependence. Even this may not be a complete answer, since it is an unsolved problem of how to include the effects of true pion absorption without double counting.

A P P E N D I X

The pion-deuteron scattering amplitude $F_{M'M}$ is a 3×3 matrix in the polarisation of the deuteron, satisfying the following constraints:

$$F_{11} = F_{-1-1} : F_{1-1} = F_{-11} : F_{10} = -F_{01} = -F_{0-1} = +F_{-10} \quad (A.1)$$

The spin-averaged cross-section is

$$\frac{d\sigma}{d\Omega} = \frac{1}{3} \sum_{n'n} |F_{n'n}|^2 \quad (A.2)$$

Our calculations have been carried out with the quantisation axis lying in the direction of the momentum transfer q , and the x axis parallel to the average momentum K . Wherever possible the results were checked with those of other authors. The single scattering amplitude may be found in Refs. 9) and 14), and the double scattering for a purely S wave deuteron also in Ref. 9). In addition, the leading terms at high energy and small angles must agree with the Glauber calculation of Ref. 14).

The expressions are given as expectation with respect to deuteron S and D radial wave functions of spherical Bessel functions j_m and propagators f , g and h , defined by

$$\begin{aligned} h(x) &\equiv R e^{ix}/x \\ f(x) &\equiv R^2 h'(x)/x \\ g(x) &\equiv x f'(x) \end{aligned} \quad (A.3)$$

The argument of the Bessel function is $\frac{1}{2}qr$ for single scattering, and Kr for double scattering, whereas for the propagators it is kr . In single scattering, the pion-nucleon amplitude appearing in the equations is the arithmetic mean of the proton and neutron values, but for double scattering, neglecting charge exchange, terms like $2\alpha\beta$ should be interpreted as $\alpha_p\beta_n + \alpha_n\beta_p$.

$$\begin{aligned}
 F_{11} = & 2(\alpha + \beta \cos \theta) \{ \langle S | j_0 | S \rangle - \sqrt{2} \langle S | j_2 | D \rangle \\
 & + \langle D | j_0 + \frac{1}{2} j_2 | D \rangle \} + 2\alpha^2 \{ \langle S | h j_0 | S \rangle + \frac{1}{\sqrt{2}} \langle S | h j_2 | D \rangle \\
 & + \langle D | h (j_0 - \frac{1}{4} j_2) | D \rangle \} - 4\alpha\beta \frac{K}{R^2} \{ \langle S | r f j_1 | S \rangle \\
 & + \sqrt{2} \langle S | r f (-\frac{1}{5} j_1 + \frac{3}{10} j_3) | D \rangle + \langle D | r f (\frac{11}{10} j_1 - \frac{3}{20} j_3) | D \rangle \} \\
 & - \frac{2\beta^2}{R^2} \{ \cos \theta [\langle S | f j_0 | S \rangle + \frac{1}{\sqrt{2}} \langle S | f j_2 | D \rangle + \langle D | f (j_0 - \frac{1}{4} j_2) | D \rangle] \\
 & + \frac{1}{3} \frac{K^2}{R^2} [\langle S | g (j_0 - 2j_2) | S \rangle + \sqrt{2} \langle S | g (-\frac{1}{5} j_0 + \frac{11}{14} j_2 - \frac{18}{35} j_4) | D \rangle \\
 & + \langle D | g (\frac{11}{10} j_0 - \frac{67}{28} j_2 + \frac{9}{35} j_4) | D \rangle] - \frac{9^2}{12 R^2} [\langle S | g (j_0 + j_2) | S \rangle \\
 & + \sqrt{2} \langle S | g (\frac{2}{5} j_0 + \frac{11}{14} j_2 + \frac{27}{70} j_4) | D \rangle + \langle D | g (\frac{4}{5} j_0 + \frac{17}{28} j_2 - \frac{27}{140} j_4) | D \rangle] \} \\
 & - \frac{\gamma^2}{R^4} \{ 2K^2 [\langle S | f j_0 | S \rangle + \frac{1}{3} \langle S | g (j_0 + j_2) | S \rangle - 2\sqrt{2} \langle S | f j_2 | D \rangle \\
 & + \sqrt{2} \langle S | g (-\frac{4}{15} j_0 - \frac{2}{21} j_2 + \frac{6}{35} j_4) | D \rangle \\
 & - \frac{9^2}{\sqrt{2}} \langle S | g (-\frac{4}{5} j_0 + \frac{4}{7} j_2 - \frac{9}{70} j_4) | D \rangle \} \quad (A.4a)
 \end{aligned}$$

$$\begin{aligned}
 F_{00} = & 2(\alpha + \beta \cos \theta) \{ \langle S | j_0 | S \rangle + 2\sqrt{2} \langle S | j_2 | D \rangle \\
 & + \langle D | j_0 - j_2 | D \rangle \} + 2\alpha^2 \{ \langle S | h j_0 | S \rangle - \sqrt{2} \langle S | h j_2 | D \rangle \\
 & + \langle D | h (j_0 + \frac{1}{2} j_2) | D \rangle \} - 4\frac{\alpha\beta K}{R^2} \{ \langle S | r f j_1 | S \rangle \\
 & - \sqrt{2} \langle S | r f (-\frac{2}{5} j_1 + \frac{3}{5} j_3) | D \rangle + \langle D | r f (\frac{4}{5} j_1 + \frac{3}{10} j_3) | D \rangle \}
 \end{aligned}$$

$$\begin{aligned}
 & - \frac{2\beta^2}{K^2} \left\{ \cos \theta \left[\langle S | f j_0 | S \rangle - \sqrt{2} \langle S | f j_2 | D \rangle + \langle D | f (j_0 + \frac{1}{2} j_2) | D \rangle \right] \right. \\
 & + \frac{1}{3} \frac{K^2}{K^2} \left[\langle S | g (j_0 - 2 j_2) | S \rangle - 2\sqrt{2} \langle S | g \left(-\frac{1}{5} j_0 + \frac{11}{14} j_2 - \frac{18}{35} j_4 \right) | D \rangle \right. \\
 & + \left. \langle D | g \left(\frac{4}{5} j_0 - \frac{17}{14} j_2 - \frac{18}{35} j_4 \right) | D \rangle \right] - \frac{9^2}{12 K^2} \left[\langle S | g (j_0 + j_2) | S \rangle \right. \\
 & - 2\sqrt{2} \langle S | g \left(\frac{2}{5} j_0 + \frac{11}{14} j_2 + \frac{27}{70} j_4 \right) | D \rangle + \left. \langle D | g \left(\frac{7}{5} j_0 + \frac{25}{14} j_2 + \frac{27}{70} j_4 \right) | D \rangle \right] \Big\} \\
 & + \frac{\gamma^2}{K^4} \left\{ 9^2 \left[\langle S | f j_0 | S \rangle + \frac{1}{3} \langle S | g (j_0 - \frac{1}{2} j_2) | S \rangle \right. \right. \\
 & - \sqrt{2} \langle S | f j_2 | D \rangle + \frac{1}{\sqrt{2}} \langle S | g \left(\frac{4}{15} j_0 - \frac{10}{21} j_2 + \frac{9}{35} j_4 \right) | D \rangle \Big] \\
 & \left. \left. - 2\sqrt{2} K^2 \langle S | g \left(-\frac{4}{5} j_0 - \frac{8}{7} j_2 - \frac{12}{35} j_4 \right) | D \rangle \right] \right\}
 \end{aligned}$$

(A.4b)

$$\begin{aligned}
 F_{1-1} &= 2 \alpha^2 \left\{ - \frac{3\sqrt{2}}{2} \langle S | h j_2 | D \rangle + \frac{3}{4} \langle D | h j_2 | D \rangle \right. \\
 & + \frac{\beta^2}{K^2} \left\{ 2 \cos \theta \left[\frac{3\sqrt{2}}{2} \langle S | f j_2 | D \rangle - \frac{3}{4} \langle D | f j_2 | D \rangle \right] \right. \\
 & - \frac{2 K^2}{K^2} \left[\frac{3\sqrt{2}}{2} \langle S | g \left(\frac{2}{15} j_0 - \frac{11}{21} j_2 + \frac{12}{35} j_4 \right) | D \rangle \right. \\
 & - \frac{3}{4} \langle D | g \left(\frac{2}{15} j_0 - \frac{11}{21} j_2 + \frac{12}{35} j_4 \right) | D \rangle \Big] + \frac{39^2}{14 K^2} \left[\frac{1}{\sqrt{2}} \langle S | g (-j_2 - j_4) | D \rangle \right. \\
 & - \frac{1}{4} \langle D | g (-j_2 - j_4) | D \rangle \Big] \Big\} - \frac{4 \alpha \beta K}{K^2} \left\{ \sqrt{2} \langle S | \sqrt{f} \left(\frac{3}{5} j_1 - \frac{9}{10} j_3 \right) | D \rangle \right. \\
 & \left. - \langle D | \sqrt{f} \left(\frac{3}{10} j_1 - \frac{9}{20} j_3 \right) | D \rangle \right\} + \frac{\gamma^2}{K^4} \left\{ 2 K^2 \left[\langle S | f j_0 | S \rangle \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{1}{3} \langle S | g (j_0 + j_2) | S \rangle + 2\sqrt{2} \langle S | f j_2 | D \rangle \\
 & + \sqrt{2} \langle S | g \left(\frac{8}{15} j_0 + \frac{22}{21} j_2 + \frac{18}{35} j_4 \right) | D \rangle \\
 & + \frac{1}{2} g^2 \left[\langle S | g j_2 | S \rangle + \sqrt{2} \langle S | g \left(-\frac{2}{7} j_2 + \frac{3}{14} j_4 \right) | D \rangle \right] \quad (A.4c)
 \end{aligned}$$

$$\begin{aligned}
 F_{10} = & \sqrt{2} \gamma \sin \theta \left\{ \langle S | j_0 | S \rangle + \frac{1}{\sqrt{2}} \langle S | j_2 | D \rangle - \frac{1}{2} \langle D | j_0 - j_2 | D \rangle \right\} \\
 & - \sqrt{2} \frac{\alpha \gamma g}{R^2} \left\{ \langle S | r f j_1 | S \rangle - \frac{1}{\sqrt{2}} \langle S | r f j_1 | D \rangle + \langle D | r f j_1 | D \rangle \right\} \\
 & - 2\sqrt{2} \frac{\beta \gamma \sin \theta}{R^2} \left\{ \langle S | f j_0 | S \rangle + \frac{1}{3} \langle S | g (j_0 - \frac{1}{2} j_2) | S \rangle \right. \\
 & + \frac{1}{\sqrt{2}} \langle S | f j_2 | D \rangle - \frac{1}{2} \langle D | f (j_0 - j_2) | D \rangle \\
 & \left. - \frac{1}{3\sqrt{2}} \langle S | g (j_0 - \frac{1}{2} j_2) | D \rangle - \frac{1}{3} \langle D | g (j_0 - \frac{1}{2} j_2) | D \rangle \right\} \quad (A.4d)
 \end{aligned}$$

R E F E R E N C E S

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FIGURE CAPTIONS

Figure 1 :

The kinematics in the Breit frame.

Figure 2 :

The imaginary part of the π^+p amplitude in the Breit frame for an incident energy of 256 MeV. The input amplitude for the double scattering through an angle θ_B is obtained from the tangent to the graph at an angle $\frac{1}{2}\theta_B$.

Figure 3 :

Differential cross-section for elastic π^+d scattering at 256 MeV versus laboratory scattering angle, the data being taken from Ref. 3). The Brueckner model calculation includes only single and double scattering.

Figure 4 :

Energy dependence of elastic π^+d scattering at fixed laboratory scattering angle of 160° , the data being taken from Ref. 3). The Brueckner model calculation includes only single and double scattering.

Figure 5 :

Differential cross-section for elastic π^+d scattering at 180 MeV versus laboratory scattering angle, the data being taken from Ref. 2) (circles), and Ref. 3) (cross). The Brueckner model calculation includes only single and double scattering.

FIG 1

FIG 2

FIG 3

FIG 4

FIG 5