The strong equivalence principle
from a gravitational gauge structure

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Abstract

Gravitational self-interactions are assumed to be determined by the covariant derivative
acting on the Riemann-Christoffel field strength. Once imposed on a metric theory, this
Yang-Mills gauge constraint extends the equality of gravitational mass and inertial mass
to compact bodies with non-negligible gravitational binding energy. Applied to generalized
Brans-Dicke theories, it singles out one tensor theory and one scalar theory for gravity but
also suggests a way to implement a minimal violation of the strong equivalence principle.

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**Introduction**

The weak equivalence principle (WEP) rests upon the empirical equality of the inertial and gravitational masses: all test bodies fall with the same acceleration if tidal forces due to inhomogeneities of the gravitational field are negligible. It implies that somebody in a free-falling elevator would experience no apparent weight. From this “happiest thought”, Einstein inferred that all physical laws of special relativity (electromagnetism included) are valid in a sufficiently small free-falling laboratory to eventually establish his successful general theory of relativity.

The strong equivalence principle (SEP) naturally extends this universality of physical results to local gravitational experiments [1]. In particular, it states that the free-fall of a compact body is also independent of its gravitational binding energy. Eötvös-like experiments with laboratory-size bodies verify only the WEP, not the SEP. Yet, the Lunar Laser Ranging experiment beautifully corroborates this statement for celestial bodies: the Moon orbit around the Earth does not appear to be polarized toward the Sun. In this Letter, we would like to raise this heuristic hypothesis about non-linear effects of gravity at the level of a fundamental gauge principle.

**Self-interaction Gauge Structure**

**A. The case for strong interactions**

Non-abelian gauge fields carry themselves the quantum numbers with which they interact. Indeed, in the classical equations of motion

\[ \partial_{\mu} F^{\mu\nu} = -g j^{\nu} - ig [A_{\mu}, F^{\mu\nu}] \]  

(1)

the first term on the rhs is the (gauge-invariant but not conserved) matter current, while the second one represents the non-linear self-coupling effect of the Yang-Mills fields [2]. Consequently, these gauge fields act themselves as a source and their self-interaction is fixed by their coupling to matter. In particular, the effective colour charges of the octet gluons (the gauge fields \( A \) associated with strong interactions) and triplet quarks (the fermionic matter fields \( F \)) are equal up to well-defined \( SU(3) \) group theory factors. LEP precision measurements around the electroweak scale provide the corresponding quadratic Casimir operators [3].

\[
C_A = 2.89 \pm 0.01 \text{ (stat.)} \pm 0.21 \text{ (syst.)} \quad (2a)
\]

\[
C_F = 1.30 \pm 0.01 \text{ (stat.)} \pm 0.09 \text{ (syst.)} \quad (2b)
\]

which are in very good agreement with the non-abelian gauge structure of QCD \((C_A = 3, C_F = 4/3)\) and rule out any abelian vector gluon model \((C_A = 0, C_F = 1/2)\). They amount to a universal running of the triple-gluon coupling defined by Eq.(1). This running as a function of energy points then at another genuine property of the strong interactions, namely quark and gluon confinement into hadronic bound states.
For pure gauge fields, i.e. without matter, Eq.(1) simply becomes the Yang-Mills conditions

$$D_\mu F^{\mu\nu} = 0.$$  \hspace{1cm} (3)

The covariant derivative introduced in Eq.(3) defines the field strength

$$[D_\mu, D_\nu] \equiv iF_{\mu\nu}$$  \hspace{1cm} (4)

in a way similar to the definition of the curvature tensor for gravity. This well-known feature will be exploited here to implement the SEP in a covariant way, namely without reference to some effective compactness, sometimes called the “sensitivity”.

B. The case for gravitational interactions

If the gravitational field interacts with the matter fields through the general covariance which turns the ordinary derivative $\partial_\mu$ into the covariant derivative $D_\mu$, any test body moves following geodesics, regardless of its mass or internal structure. It results from this kinematics in curved space-time that the WEP is fulfilled. However, the gravitational field may also interact with itself, like non-abelian fields do. Indeed, the affine connection $\Gamma^\lambda_{\mu\nu}$ corresponds to the gauge field $A_\mu$ and the Riemann-Christoffel tensor $R^\sigma_{\lambda\mu\nu}$ corresponds to the non-abelian field strength $F^{\mu\nu}$ with

$$[D_\mu, D_\nu]^\sigma_\lambda \equiv -R^\sigma_{\lambda\mu\nu}. $$  \hspace{1cm} (5)

Compact bodies which differ by their binding energy could therefore fall with different accelerations if the gravitational self-interaction was not universal. But the present null results on an anomalous polarization of the Moon orbit around the Earth (the Nordtvedt effect \[4\]) plead in favour of a single triple-graviton coupling known with a precision of $10^{-3}$. Inspired by the strong interaction gauge theory, we impose then the Yang-Mills conditions (3) on the curvature tensor :

$$D_\sigma R^\sigma_{\lambda\mu\nu} = 0.$$  \hspace{1cm} (6)

These tensorial equations naturally embody the gravitational self-interaction: gravitons may gravitate the way gluons glue. If such is the case, the SEP would be an intrinsic property of the gravitational dynamics, entirely determined by the geometry of space-time.

Let us express Eq.(6) for an asymptotically flat isotropic metric expanded in the weak stationary field $V(r) = -GM/r$:

$$g_{00} = 1 + 2\alpha \left( \frac{V}{c^2} \right) + 2\beta \left( \frac{V}{c^2} \right)^2 + \mathcal{O} \left( \frac{1}{c^6} \right)$$  

$$g_{ij} = -\delta_{ij} \left\{ 1 - 2\gamma \left( \frac{V}{c^2} \right) + \frac{3}{2} \delta \left( \frac{V}{c^2} \right)^2 + \mathcal{O} \left( \frac{1}{c^6} \right) \right\}. $$  \hspace{1cm} (7)

The $\alpha$, $\beta$, $\gamma$ and $\delta$ dimensionless coefficients, normalized to one for the Schwarzschild solution of general relativity, have to be determined experimentally \[5\]. From such a parametrization, we
easily obtain the following relations

\[ D_\sigma R^\sigma_{\ 00n} = - \left( \frac{1}{2c^4} \right) (4\beta - \alpha\gamma - 3\alpha^2) \partial_n (\partial_l V \partial^l V) \]  

(8a)

\[ D_\sigma R^\sigma_{\ tmn} = + \left( \frac{1}{c^4} \right) (6\delta - 6\gamma^2 - \alpha\gamma + \alpha^2) (\partial_t \partial_m V \partial_n V - \partial_t \partial_n V \partial_m V) \]  

(8b)

if the $O(1/c^6)$ terms are neglected. So, the third order differential equations (6) for the metric $g_{\mu\nu}$ manifestly contain unphysical solutions. For example, the exact solution with $g_{00} = 1$ (i.e. $\alpha = \beta = 0, \delta = \gamma^2$) possesses no gravitational redshifts [6] and violates thereby the WEP. Thus, at this level Eq.(6) should not be regarded as the fundamental gravitational field equations derived from some variational principle [7], but rather as further tensorial constraints on any metric theory incorporating by definition the WEP.

The SEP in Metric Theories

In metric theories, the WEP corresponds to the zeroth-order condition $\alpha \equiv 1$. The higher-order constraints imposed by Eq.(6) reduce then respectively to

\[ \eta \equiv 4\beta - \gamma - 3 = 0 \]  

(9a)

\[ \eta' \equiv 6\delta - 6\gamma^2 - \gamma + 1 = 0. \]  

(9b)

So, the Yang-Mills conditions (3) applied to the curvature field strength go beyond Einstein’s heuristic hypothesis and imply that the non-linear parameters ($\beta$ and $\delta$) are independently determined by the intrinsic space curvature ($\gamma$). The recent conjunction experiment with Cassini spacecraft [5] provides the range $\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$. If the conditions (9) prove physical, we may already infer that $|\beta - 1| < 10^{-5}$ and $|\delta - 1| < 10^{-4}$ in the vicinity of the Sun. These theoretical constraints are much stronger than the existing observational bounds from the solar system [9]. At present, the secular advance of Mercury’s perihelion proportional to the combination $(-\beta + 2\gamma + 2)/3$ yields a weaker constraint on the Eddington parameter $\beta$, $|\beta - 1| < 3 \times 10^{-3}$. On the other hand, the crucial parameter $\delta$ which appears only at the second order in the light deflection angle (consistently expressed in terms of the physical impact parameter $b$):

\[ \Delta = \left( \frac{4GM}{c^2 b} \right) \left\{ \frac{(1 + \gamma)}{2} + \frac{(8 + 8\gamma - 4\beta + 3\delta)}{16} \right\} \left( \frac{\pi GM}{c^2 b} \right) \]  

(10)

is far from being constrained nowadays. Based on optical interferometry between two microspacecrafts, the LATOR experiment [10] aims at measuring $\delta$ with a precision of $10^{-3}$. But this experiment would simultaneously reach the impressive level of accuracy of 1 to $10^8$ for the parameter $\gamma$.

We are of course pleased to recover, as expected, the well-known Nordtvedt condition (9a) derived from a phenomenological relation [11] between the gravitational mass ($m_{gr}$) and the inertial mass ($m_{in}$):

\[ \frac{m_{gr}}{m_{in}} \approx 1 + \eta \left( \frac{\Omega mc^2}{\Omega mc^2} \right) \]  

(11)
for compact bodies with non-negligible gravitational binding energy $\Omega$. Although the fraction of gravitational self-energy is only $4.5 \times 10^{-10}$ for our planet, the Lunar Laser Range experiment confirms indeed that Earth and Moon fall toward the Sun at equal rates with a precision of about $2 \times 10^{-13}$. So, the gravitational binding energy equally contributes to the inertial mass and to the gravitational mass with a precision given by \[ |\eta^{obs.}| = (4.4 \pm 4.5) \times 10^{-4}. \] (12)

However the general covariance of Eq.(6) unavoidably requires a second condition (9b) to be also fulfilled!

To illustrate this important point, let us first briefly revive the Einstein-Grossmann “Entwurf” \[12\]. This “outline” is based on a spatially flat metric, $g_{ij} = -\delta_{ij}$, and predicts an advance of Mercury’s perihelion of about 18” per century \[13\], i.e. 5/12 of the observed value. In the weak field approximation, the corresponding Eddington parameters are $\gamma = \delta = 0$ and $\beta = 3/4$, respectively. As a consequence, the theory obeys the Nordtvedt constraint (9a) but not (9b). This can easily be understood from the fact that the kinetic term in the associated action functional involves ordinary derivatives of the metric field. So, general covariance is lost and the action is only invariant with respect to arbitrary linear transformations.

On the other hand, both constraints (9a) and (9b) are satisfied by the Einstein final theory \[14\] with all Eddington parameters equal to unity ($\beta \equiv 1, \gamma \equiv 1, \delta \equiv 1$), but also by the Nordström scalar theory \[15\]. The geometric reformulation \[16\] of this first consistent relativistic theory of gravitation leads indeed to a special conformally flat metric, $g_{\mu\nu} = (1 + V/c^2)^2 \eta_{\mu\nu}$, i.e. a finite set of Eddington parameters ($\beta = 1/2, \gamma = -1, \delta = 2/3$). These particular values imply a retrogression of Mercury’s perihelion of 1/6 of the observed value as well as the vanishing of the deflection angle $\Delta$ expressed in Eq.(10).

The Bianchi identities, $D_\sigma R^\sigma_{\lambda\mu\nu} + D_\nu R^\nu_{\sigma\lambda\mu} + D_\mu R^\mu_{\nu\sigma\lambda} = 0$, allow us to extend the analysis of the Yang-Mills conditions beyond the weak field approximation. Indeed, these identities imply that the basic Eq.(6) is equivalent to

$$D_\nu R_{\lambda\mu} - D_\mu R_{\lambda\nu} = 0. \tag{13}$$

A contraction of Eq.(13) yields the necessary condition $D_\nu R^\nu_\mu = \partial_\mu R$. A direct comparison with the contracted Bianchi identities, $2 D_\nu R^\nu_\mu = \partial_\mu R$, implies therefore a constant scalar curvature $R$. But our hypothesis of an asymptotically flat metric eventually requires $R$ to be vanishing. Using then the standard decomposition of the Riemann tensor $R^\sigma_{\lambda\mu\nu}$ into the Weyl tensor $W^\sigma_{\lambda\mu\nu}$ and the Ricci tensor $R_{\lambda\mu}$, one easily infers that Eq.(13) is, in its turn, equivalent to

$$D_\nu W^\sigma_{\lambda\mu\nu} = 0 \tag{14a}$$

$$R = 0. \tag{14b}$$
So, we immediately conclude from Eqs.(13) and (14) that both Einstein tensor theory \( R_{\lambda\mu} = 0 \) and Nordström-Einstein-Fokker scalar theory \( W^{\gamma}_{\lambda\mu\nu} = 0, R = 0 \) are in fact exact solutions of Eq.(6).

At the \( 1/c^4 \) order in the weak field approximation, the tensorial Eqs.(14a) and (14b) respectively constrain two linear combinations of the \( \eta \) and \( \eta' \) parameters:

\[
\begin{align*}
(2\eta - \eta') &= 0 \\
(\eta + \eta') &= 0.
\end{align*}
\]

So, \( \eta' \) has definitely to be on an equal footing with the Nordtvedt parameter \( \eta \) defined in Eq.(11).

This nicely confirms our intuition that Eq.(6) provides a covariant formalism for the SEP in the general framework of metric theories. Such a formalism seems to single out one pure tensor and one pure scalar theory of gravity. It is therefore worth checking if they both survive within a restricted class of metric theories which naturally emerge as low-energy approximations of superstring or Kaluza-Klein theories.

### The SEP in Tensor-Scalar Theories

Let us consider the following tensor-scalar (TS) action

\[
S_{TS}(\omega) = -\left(\frac{c^4}{16\pi}\right) \int d^4 x \sqrt{-g} \left\{ \phi R - \frac{\omega(\phi)}{\phi} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right\} + S_{\text{Matter}}(g_{\mu\nu}, \psi) \tag{16}
\]

involving in principle an arbitrary function \( \omega(\phi) \) of the scalar field \( \phi \).

#### A. Constant-\( \omega \)

If \( \omega(\phi) \) is a constant parameter \( \omega_0 \), we recover the original Brans-Dicke (BD) theory [17]. In this theory, Eq.(11) simply amounts to a spatial variation of the Newton constant:

\[
G_{\text{lab}}(r) \approx G_\infty \left\{ 1 - \eta_{BD} \frac{V(r)}{c^2} \right\} \tag{17}
\]

where \( G_\infty \) is the value of \( G \) measured far from the gravity source. Such a variation of the gravitational coupling obviously violates the universality of free-fall for compact bodies, i.e. the SEP. The inner structure of a compact body is indeed sensitive to changes of the gravitational “constant”.

In the weak field approximation, we easily obtain

\[
\begin{align*}
\eta_{BD} &= 1 - \gamma \\
\eta'_{BD} &= 2\gamma(\gamma - 1)
\end{align*}
\]

with \( \gamma = (\omega_0 + 1)/(\omega_0 + 2) \). Consequently, Eqs.(9) are only fulfilled by general relativity \( (\omega_0 = \infty) \). This result can easily be extended beyond the weak field approximation. Indeed, variations of the
BD action with respect to the gravitational fields imply that the scalar \( \phi \) plays the role of a source for the Riemann-Christoffel tensor:

\[
D_\sigma (\phi R^\sigma_{\lambda \mu \nu}) = -(\omega_0 + 1)(R_{\lambda \mu \nu} \phi - R_{\lambda \nu} \phi_\mu).
\]

(19)

The basic Yang-Mills condition (6) requires a constant scalar field and the tensor-scalar action (16) reduces to the Hilbert one for \( G\phi = 1 \). Notice that the rhs of Eq.(19) vanishes for \( \omega_0 = -1 \), a value predicted by duality in the graviton-dilaton low-energy effective superstring action [18]. Such is not the case for the low-energy limit of a \( (4 + \varepsilon) \) dimensional Kaluza-Klein theory characterized by \( \omega_0 + 1 = 1/\varepsilon \).

**B. Variable-\( \omega \)**

We have seen that the SEP requires \( \gamma_{BD} = +1 \). This is not a surprise since the Brans-Dicke theory obeys \( \beta_{BD} = 1 \), i.e. rules out the Nordström-Einstein-Fokker theory \( (\beta = 1/2) \) from the start. But if \( \omega(\phi) \) is an arbitrary function of the scalar field, \( \beta \) is now proportional to its first derivative \( \omega'(\phi) \).

Remarkably, we still have an exact relation between the parameters \( \beta, \gamma \) and \( \delta \):

\[
\delta_{TS} = \frac{4}{3}(\beta_{TS} - 1) + \frac{1}{6}(8\gamma_{TS}^2 - \gamma_{TS} - 1)
\]

(20)

such that one linear combination of \( \eta \) and \( \eta' \) can be directly expressed in terms of the space curvature parameter:

\[
2\eta_{TS} - \eta'_{TS} = 2(1 - \gamma_{TS}^2)
\]

(21)

even if \( \omega(\phi) \) is not known! Consequently, a necessary condition to preserve the SEP in any tensor-scalar theory is \( \gamma_{TS} = \pm 1 \). This generic result on the SEP is in agreement with [19]. Combined with the constraint (9a) and the relation (20), it singles out both Einstein and Nordström-Einstein-Fokker metrics at the full \( 1/c^4 \) order in the weak field approximation. This is clearly sufficient to settle the latter. Yet, our covariant formalism (14) allows us to go again beyond such an approximation.

The linear combination of \( \eta \) and \( \eta' \) in Eq.(21) is precisely associated with the tensor \( D_\sigma W^\sigma_{\lambda \mu \nu} \) in the weak field approximation (see Eq.(15a)). Following then Eq.(14), we still have the condition of vanishing scalar curvature at our disposal:

\[
R = \left\{ \omega - 3\frac{\omega' \phi}{(2\omega + 3)} \right\} \frac{\phi' \phi_\alpha}{\phi^2} = 0.
\]

(22)

This second condition completely fixes the arbitrary functional \( \omega(\phi) \) appearing in the tensor-scalar action (16):

\[
\omega(\phi) = -\left(\frac{3}{2}\right) \frac{G\phi}{(G\phi - 1)}.
\]

(23)

A simple scalar field redefinition:

\[
G\phi = 1 - \frac{\kappa \phi^2}{6}
\]

(24)
with $\kappa = 8\pi G/c^4$ leads then immediately to the improved Einstein-massless scalar theory advocated by Deser [20]. The resulting action

$$S_{R=0} = -\left(\frac{1}{2\kappa}\right) \int d^4x \sqrt{-g} R + \int d^4x \sqrt{-g} \left\{ \left(\frac{\kappa^2}{12}\right) R + \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi \right\}$$

(25)

corresponds indeed to general relativity coupled to a matter field $\varphi$ whose energy-momentum tensor is traceless, as required by classical scale invariance [21]. But we have shown that it may as well be considered as the generalized Brans-Dicke tensor-scalar theory with a vanishing scalar curvature $R$. From this latter point of view, the other condition (14a) is also satisfied if either the scalar (helicity-0) or the tensor (helicity-2) degree of freedom is frozen ($\varphi \to 0$ or $g_{\mu\nu} \to \eta_{\mu\nu}$, respectively).

In the first case, the action simply reduces to the Hilbert one and we are in the presence of the Einstein tensor theory. In the second case, a massless scalar freely propagates in the Minkowski space-time and we end up with the original Nordström scalar theory [15].

We conclude that only the Einstein and Nordström-Einstein-Fokker metric theories do comply with the SEP as defined by Eq.(6). From an observational viewpoint, it is obvious that perihelion advance and light deflection measurements ($\Delta \neq 0$) exclude the latter in favour of the former. However, it would be more satisfactory if one could already discriminate them at the theoretical level.

**Toward a minimal violation of the SEP**

The sub-class of tensor-scalar theories defined by:

$$2\omega(\phi) + 3 = \frac{a}{(1 - G\phi)}$$

(26)

with $a \neq 0$, is rather interesting since $\eta$ and $\eta'$ are then separately expressed in terms of the crucial parameter $\gamma$:

$$\eta_a = (1 - \gamma^2) \frac{(1 + a)}{2a}$$

$$\eta'_a = 2(1 - \gamma^2) \frac{(1 - a)}{2a}.$$  

(27)

For $\gamma \neq \pm 1$, the Yang-Mills conditions (3) imposed on the curvature tensor are not fulfilled and the value of the parameter $a$ characterizes any violation of the SEP (see Fig.1).

The case $a = -1$ corresponds to the well-known “constant-$G$” theory of Barker [22]. Consequently, this theory violates the SEP since $\eta = 0$ but $\eta' \neq 0$. Notice that the same holds true for the complementary case $a = +1$ (i.e., $\eta' = 0$ but $\eta \neq 0$).

For $a = +3$, we recover the “zero-$R$” theory defined in Eq.(23), with $\eta + \eta' = 0$ in the weak field approximation. We have seen from Eq.(25) that this covariant theory provides a smooth interpolation between the pure tensor and the pure scalar theories respecting the SEP (see Fig.1). A minimal violation of the SEP is thus expected here.
Figure 1: Tensor-scalar theories versus the Strong Equivalence Principle. Only the Einstein (E) and Nordström-Einstein-Fokker (NEF) theories comply with the gauge conditions (14). The dashed vertical line corresponds to the Brans-Dicke (BD) models while the dashed curves represent the Barker theory ($a = -1$) and its complementary ($a = +1$) defined by Eq.(26). Arrows indicate possible attractors.

In fact, an attractor mechanism toward $G\phi = 1$ for $a \gg 1$ in Eq.(26) has already been discussed in the Friedmann-Lemaître-Robertson-Walker cosmology (i.e. with a time-dependent evolution for the scalar auxiliary field) \cite{23}. This mechanism is most easily described in the so-called Einstein frame where the scalar and not the ordinary particle follows the geodesic determined by the metric. The corresponding action

$$S_E = -\left(\frac{1}{2\kappa}\right) \int d^4x \sqrt{-g} \left\{ R - 2g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right\} + S_{\text{Matter}} \left\{ A^2(\phi) g_{\mu\nu}, \psi \right\}$$

is obtained through a suitable conformal transformation on the metric originally defined in the Jordan frame (see Eq.16):

$$g_{\mu\nu} \rightarrow A^2(\sigma) g_{\mu\nu}$$

with $G\phi = A^{-2}(\sigma)$ and $2\omega(\phi) + 3 = (A'/A)^{-2}$. In the Einstein frame (28), the $\sigma$ field’s cosmological evolution is just analogous \cite{24} to the damped motion of a particle in the potential $\ln A(\sigma)$.

For the sub-class of tensor-scalar theories defined by Eq.(26), we obtain the corresponding
conformal factor
\[ A(\sigma) = \cosh \left( \frac{\sigma}{\sqrt{a}} \right), \tag{30} \]
such that general relativity \((\sigma = 0)\) is the only point of equilibrium. If \(a < 0\), a singular relaxation toward \(A = 0\) is generic. Yet, for \(a > 0\), \(A(\sigma) \geq 1\) and only a smooth relaxation toward \(A = 1\), i.e. general relativity, is possible. The “constant-\(G\)” theory \((a = -1)\) exhibits indeed a singular attractor toward pure scalar gravity \((A \to 0)\). On the other, the “zero-\(R\)” theory \((a = +3)\) with positive \(\omega(\phi)\) turns out to contain a natural attractor toward general relativity \((A \to 1)\). Consequently, cosmological dynamics may provide a way to disentangle the two theories respecting the SEP in the static approximation (see arrows in Fig.1). In particular, a tensor-scalar theory which obeys the scalar equation \(R = 0\) appears to be the ideal prototype to study a minimal violation of the SEP at the present time. If a small violation of the SEP was observed, such an attractor mechanism would obviously be welcome.

**Conclusion**

The weak equivalence principle (WEP) settles the kinematics of gravity (space-time tells mass how to move), but not its non-linear dynamics (how mass tells space-time to curve). So it is quite remarkable that a covariant formulation of the strong equivalence principle (SEP) applied to generalized Brans-Dicke gravity models singles out one tensor theory \((R_{\sigma\nu} = 0)\) and one scalar theory \((W^{\sigma\lambda\mu\nu} = 0, R = 0)\). They may be considered as the analogs of the non-abelian QCD and abelian QED gauge theories, respectively: gluons carry colours \((D_\mu F^{\mu\nu} = 0)\) but photons do not carry electric charge \((\partial_\mu F^{\mu\nu} = 0)\). From a theoretical point of view, the abelian alternative to general relativity should not come as a surprise since the WEP alone already implies that all non-gravitational fields couple in the same universal way to gravity. The strong version of the equivalence principle simply extends this universality of coupling to the gravitational field itself. The survival of a scalar theory with respect to the SEP becomes then rather obvious once we realize that the photon (like any massless particle) never couples to a conformally flat metric field. This last feature precisely rules out such a metric theory at the phenomenological level.

If the SEP proves to be fundamental, then Eq.(6) itself should also arise from a variational principle, as it is the case for a pure Yang-Mills theory where the action functional is an integral over the square of the curvature, \(F^{\mu\nu} F_{\mu\nu}\). Here, no attempt has been made to obtain the full set of field equations of the gravitational theory from a variational principle. We simply notice that if one considers, again by analogy, the conformally invariant Weyl theory :

\[ S_{\text{Weyl}} = \xi \int d^4x \sqrt{-g} \{ W^{\sigma\lambda\mu\nu} W_{\sigma\lambda\mu\nu} \}, \tag{31} \]

the resulting fourth-order differential equations for the metric \([25]\):

\[ 2D_\nu D_\sigma W^{\sigma\lambda\mu\nu} + W^{\sigma\lambda\mu\nu} R_{\sigma\nu} = 0 \tag{32} \]
contain non-trivial solutions [26] but reduce to the same alternative as for the tensor-scalar models once the constraints (14) are imposed by hand. Thus, the SEP might be a useful guiding gauge principle to single out a higher-order action functional for gravitational interactions. But any analogy with Yang-Mills theories has to break down at some point since the affine connection is itself constructed from the first derivatives of the metric tensor, while the gauge fields are not expressed in terms of more fundamental fields.

On the other hand, if the SEP turns out to be only approximate, the covariant condition (6) suggests a simple and natural way to implement a minimal violation of it within a sub-class of tensor-scalar theories characterized by an attractor mechanism toward general relativity.

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References


