Neutrino Parameter Space for a Vanishing $ee$ Element in the Neutrino Mass Matrix

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Abstract

The consequences of a texture zero at the $ee$ entry of neutrino mass matrix in the flavor basis, which also implies a vanishing effective Majorana mass for neutrinoless double beta decay, have been studied for Majorana neutrinos. The neutrino parameter space under this condition has been constrained in the light of all available neutrino data including the CHOOZ bound on $s_{13}^2$.

One of the fundamental goals of neutrino physics is to determine the neutrino mass matrix given by

$$ (m_\nu)_{\alpha \beta} = (U m_\nu^{\text{diag}} U^T)_{\alpha \beta}; \quad \alpha, \beta = e, \mu, \tau $$

for Majorana neutrinos where $m_\nu^{\text{diag}} = \{m_1, m_2, m_3\}$. Since $m_\nu$ is a symmetric matrix, it is specified by nine physical parameters viz. the three neutrino masses $(m_1, m_2, m_3)$, three mixing angles $(\theta_{12}, \theta_{23}, \theta_{13})$, the Dirac-type CP-violating phase $(\delta)$, and two Majorana-type CP-violating phases $(\alpha, \beta)$. Out of these nine parameters, only the first six have been constrained to a reasonable degree of precision but the Dirac-type CP-violating phase and the two Majorana-type CP-violating phases remain unconstrained at present. While the Dirac-type CP-violating phase is expected to be constrained from the study of CP-violation in long baseline neutrino oscillations, the two Majorana-type CP-violating phases can, hopefully, be constrained from neutrinoless double $\beta$ decay ($0\nu\beta\beta$).

The rate for $0\nu\beta\beta$ decay is proportional to the effective mass defined as

$$ |m_{ee}| = \left| \sum_i m_i U_{ei} \right|^2 $$

which, in fact, is the magnitude of the first element of the neutrino mass matrix in the charged lepton flavor basis. Thus, the $0\nu\beta\beta$ decay provides us an unique opportunity

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to probe directly one of the elements of the neutrino mass matrix. The non-observation of $0\nu\beta\beta$ decay constrains the effective mass $|m_{ee}|$ to be close to zero. The possibility $m_{ee} = 0$ has been examined in the literature [2, 3, 4, 5, 6, 7] earlier under certain special conditions. However, a general study of the neutrino parameter space for $m_{ee} = 0$ with the full interplay of the Majorana phases is still lacking.

In the present work, we examine the consequences of the vanishing effective Majorana mass for the neutrino parameter space. A study of this condition has important implications for the texture specific phenomenological neutrino mass matrices. In particular, the phenomenological conclusions obtained from the condition of vanishing effective Majorana mass will be valid for all texture zero schemes in the charged lepton basis in which the first element is zero.

The neutrino mixing matrix i.e. the PMNS matrix $U$ can be parameterized in terms of three mixing angles ($\theta_{12}, \theta_{23}, \theta_{13}$), the Dirac CP-violating phase ($\delta$), and the two Majorana CP-violating phases ($\alpha, \beta$) in the following manner [8]:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i(\beta+\delta)} \end{pmatrix}$$

(3)

where $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$. This parameterization has the advantage that the Dirac CP-violating phase $\delta$ is formally absent in the expression of the effective Majorana mass $m_{ee}$ probed in $0\nu\beta\beta$ decays. The neutrino mass matrix is a complex symmetric matrix described by nine parameters as discussed above. For the above parameterisation, the effective Majorana mass is given by

$$m_{ee} = c_{13}c_{12}m_1 + c_{13}s_{12}m_2e^{2i\alpha} + s_{13}m_3e^{2i\beta}. \quad (4)$$

The current experimental bound on the effective Majorana mass is [9, 10]

$$|m_{ee}| \leq 0.35\,\text{eV}, \quad (5)$$

where the uncertainty in the calculation of the nuclear matrix element of $0\nu\beta\beta$ is contained in the factor $\zeta = O(1)$. The element $m_{ee}$ of neutrino mass matrix depends on seven (out of a total of nine) parameters viz. $m_1, m_2, m_3, \theta_{12}, \theta_{13}, \alpha$ and $\beta$. The mixing angle $\theta_{12}$ is known from the solar and KamLAND neutrino data. The masses $m_2$ and $m_3$ can be calculated from the two mass-squared differences $\Delta m^2_{12}$ and $\Delta m^2_{23}$ which has been measured experimentally in the solar and atmospheric neutrino experiments. For normal hierarchy (NH), $m_1$ is the lightest neutrino mass and the masses $m_2$ and $m_3$ can be expressed in terms of $m_1$ in the following way

$$m_2 = \sqrt{m_1^2 + \Delta m^2_{12}}, \quad (6)$$

and

$$m_3 = \sqrt{m_2^2 + \Delta m^2_{23}}. \quad (7)$$

For inverted hierarchy (IH), $m_3$ is the lightest neutrino mass and the other two masses are given by

$$m_1 = \sqrt{m_3^2 - \Delta m^2_{23}}, \quad (8)$$
and
\[ m_2 = \sqrt{m_1^2 + \Delta m_{12}^2}. \] (9)

Thus, two neutrino masses are known from the solar and atmospheric mass-squared differences viz. \( \Delta m_{12}^2 \) and \( \Delta m_{23}^2 \) in terms of the lightest neutrino mass \( m_1 \) for NH and \( m_3 \) for IH.

The best fit, 1 and 3 sigma values of the oscillation parameters are [11]:

\[
\begin{align*}
\Delta m_{12}^2 &= 7.9^{+0.3,1.0}_{-0.3,0.8} \times 10^{-5} eV^2, \\
s_{12}^2 &= 0.31^{+0.02}_{-0.03,0.07}, \\
\Delta m_{23}^2 &= \pm 2.2^{+0.37,1.1}_{-0.27,0.8} \times 10^{-3} eV^2, \\
s_{23}^2 &= 0.50^{+0.06,0.18}_{-0.05,0.16}, \\
s_{13}^2 &< 0.012(0.046).
\end{align*}
\]

The best-fit value of \( s_{13}^2 \) is zero. The positive and negative signs of \( \Delta m^2 \) correspond to the normal and inverted hierarchy, respectively.

Thus, the element \( m_{ee} \) is now a function of four unknown parameters viz. the mixing angle \( \theta_{13} \) and the two Majorana CP-violating phases \( (\alpha, \beta) \) and the lightest neutrino mass \( m_1 \) for NH and \( m_3 \) for IH. The study of the element \( m_{ee} \) is, therefore, important in order to decode information about these unknown parameters.

For \( s_{13}^2 = 0 \), the effective Majorana mass \( m_{ee} \) is given by
\[ m_{ee} = c_{12}^2 m_1 + s_{12}^2 m_2 e^{2i\alpha} \] (11)

which vanishes if \( \alpha = \left( n + \frac{1}{2} \right) \pi \) (12) and
\[ m_1 / m_2 = s_{12}^2 / c_{12}^2. \] (13)

Substituting the value of \( m_2 \) from Eq. (6) or Eq. (9) in Eq. (12), we obtain
\[ m_1 = s_{12}^2 \sqrt{\frac{\Delta m_{12}^2}{\cos 2\theta_{12}}} \] (14)

both for normal and inverted hierarchies. Thus, there exists a point on the \( (\alpha, m_1) \) plane at which \( m_{ee} \) vanishes for \( s_{13}^2 = 0 \) for both the hierarchies which is a consequence of the fact that the effective Majorana mass \( m_{ee} \) is independent of \( m_3 \) and, hence, independent of the hierarchy for \( s_{13}^2 = 0 \). This case has been examined earlier [2].

Now we examine the consequences of a vanishing effective Majorana mass for non-zero \( s_{13}^2 \). The element \( m_{ee} \) depends on the four parameters viz. the lightest neutrino mass \( m_1 \) for NH and \( m_3 \) for IH), the mixing angle \( \theta_{13} \) and the two Majorana CP-violating phases \( \alpha \) and \( \beta \). Now, we demand that \( m_{ee} = 0 \) which requires that both the real and imaginary parts of \( m_{ee} \) vanish which yields two conditions on the neutrino parameter space viz.
\[ Re(m_{ee}) = c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 m_2 \cos 2\alpha + s_{13}^2 m_3 \cos 2\beta = 0 \] (15)
and

\[ Im(m_{ee}) = c_{13}^2 s_{12}^2 m_2 \sin 2\alpha + s_{13}^2 m_3 \sin 2\beta = 0. \]  (16)

so that we can express any two variables in terms of the remaining two variables. In fact, the condition \( m_{ee} = 0 \) is more restrictive than the actual measurement of this element from 0νββ decay which yields only one constraint on the above four-dimensional neutrino parameter space. In the following, we express \( \beta \) and \( s_{13} \) in terms of \( m_1 \) and \( \alpha \) to examine the neutrino parameter space allowed by the condition \( m_{ee} = 0 \) on the \((\alpha, m_1)\) plane. We have, already, obtained a point on the \((\alpha, m_1)\) plane corresponding to \( s_{213}^2 = 0 \) and arbitrary values of \( \beta \) [cf. Eqs. (12,14)]. Next, we examine the whole region for \( m_{ee} = 0 \) about this point on the \((\alpha, m_1)\) plane for different values of \( \beta \) and for \( s_{13}^2 \) within the CHOOZ bound.

From Eqs. (15) and (16), we obtain

\[ s_{13}^2 = \frac{m_1 c_{12}^2 + m_2 s_{12}^2 \cos 2\alpha}{m_1 c_{12}^2 + m_2 s_{12}^2 \cos 2\alpha - m_3 \cos 2\beta} \]  (17)

and

\[ s_{13}^2 = \frac{m_2 s_{12}^2 \sin 2\alpha}{m_2 s_{12}^2 \sin 2\alpha - m_3 \sin 2\beta} \]  (18)

respectively.

From Eqs. (17) and (18), one can obtain the mass ratios

\[ \frac{m_1}{m_2} = \frac{s_{13}^2 \sin 2(\alpha - \beta)}{c_{12}^2 \sin 2\beta} \]  (19)

and

\[ \frac{m_2}{m_3} = -\frac{s_{13}^2 \sin 2\beta}{c_{13}^2 s_{12}^2 \sin 2\alpha}. \]  (20)

For \( m_{ee} = 0 \) to hold, Eqs. (17) and (18) must hold simultaneously which implies

\[ \sin 2\beta = \pm \frac{s_{12}^2 m_2}{M} \sin 2\alpha \]  (21)

where

\[ M = \sqrt{m_1^2 c_{12}^4 + m_2^2 s_{12}^4 + 2m_1 m_2 s_{12}^4 c_{12}^2 \cos 2\alpha}. \]  (22)

and \( s_{13} \) is given by

\[ s_{13}^2 = \frac{M}{M \mp m_3}. \]  (23)

where \( \pm \) signs in front of \( m_3 \) correspond to the two signs of \( \sin 2\beta \) in Eq. (21), respectively. The physical requirement \( s_{13}^2 < 1 \) implies that \( m_{ee} = 0 \) if

\[ \sin 2\beta = -\frac{s_{12}^2 m_2}{M} \sin 2\alpha, \]  (24)

and

\[ s_{13}^2 = \frac{M}{M + m_3}. \]  (25)
Figure 1: The contours of constant \( \sin 2\beta \) on \((\alpha, m_1)\) plane as demanded by \( m_{ee} = 0 \). The central vertical line is for \( \sin 2\beta = 0 \). Immediately right (left) to it is the line \( \sin 2\beta = \frac{1}{2}(\frac{-1}{2}) \) followed by the line for \( \sin 2\beta = \sqrt{3}(\frac{-\sqrt{3}}{2}) \).

It is interesting to note that \( \beta \) becomes indeterminate when \( \frac{m_1}{m_2} = \frac{s_{12}^2}{c_{12}^2} \) and \( \alpha = (n + \frac{1}{2})\pi \) for \( s_{13}^2 = 0 \) since \( M = 0 \) at this point. This is consistent with our earlier result that \( m_{ee} \) can vanish for arbitrary values of \( \beta \) when \( s_{13}^2 = 0 \).

Substituting the value of \( \beta \) from Eq. (24) in Eq. (19), we obtain

\[
m_3 = \frac{c_{13}^2 M}{s_{13}^2}
\]  

(26)

Note that R.H.S. of Eq. (26) becomes indeterminate at the point \( \frac{m_1}{m_2} = \frac{s_{12}^2}{c_{12}^2} \) for \( s_{13}^2 = 0 \) since \( M = 0 \) at this point which is consistent with our earlier remark that \( m_{ee} \) can vanish for \( s_{13}^2 = 0 \) irrespective of the values of \( m_3 \). One can solve Eq. (26) for \( \alpha \) to obtain

\[
\cos 2\alpha = \frac{m_3^2 s_{13}^4 - c_{13}^4 (m_1^2 c_{12}^4 + m_2^2 s_{12}^4)}{2m_1 m_2 s_{12}^2 c_{12}^2 s_{13}^4}
\]

(27)

which corresponds to

\[
\cos 2\beta = -\frac{m_3^2 s_{13}^4 + c_{13}^4 (m_1^2 s_{12}^4 - m_2^2 c_{12}^4)}{2m_1 m_2 s_{12}^2 s_{13}^2 c_{13}^2 c_{12}^4}.
\]

(28)

Eqs. (24) and (25) can be used to examine the allowed \((\alpha, m_1)\) parameter space for \( m_{ee} = 0 \). Note that \( \sin 2\alpha \) and \( \sin 2\beta \) should have opposite signs. A natural choice is to take \( \alpha \) centered around \( 90^\circ \) (corresponding to \( n = 0 \) in Eq. (12)) and \( \beta \) centered around \( 0^\circ \). With this
particular choice, \( \beta \) should lie in the fourth quadrant (i.e. \( \beta < 0^\circ \)), if \( \alpha \) lies in the first quadrant (i.e. \( \alpha < 90^\circ \)) and \( \beta \) should lie in the first quadrant (i.e. \( \beta > 0^\circ \)), if \( \alpha \) lies in the second quadrant (i.e. \( \alpha > 90^\circ \)). In other words, \( \alpha \) should be ahead of \( \beta \) by one quadrant. This is apparent in Fig. 1 where we have plotted the contours of constant \( \beta \) on the \((\alpha, m_1)\) plane. The central vertical line corresponds to \( \alpha = 90^\circ \) and \( \beta = 0^\circ \). The left half of the plot corresponds to \( \alpha < 90^\circ \) and \( \beta < 0^\circ \) whereas the right half of the plot corresponds to \( \alpha > 90^\circ \) and \( \beta > 0^\circ \). All the curves corresponding to different values of \( \beta \) intersect at a certain point on the central vertical line. This point has been obtained earlier while discussing the \( s_{13}^2 = 0 \) case. At this point, \( m_{ee} = 0 \) irrespective of the value of \( \beta \). For \( \alpha = 90^\circ \), Eq. (25) gives

\[
s_{13}^2 = \frac{m_2 s_{12}^2 - m_1 c_{12}^2}{m_2 s_{12}^2 - m_1 c_{12}^2 + m_3}.
\]

For \( m_1 = s_{12}^2 \sqrt{\frac{\Delta m_{12}^2}{\cos 2\theta_{12}}} \) [cf. Eq. (14)], \( s_{13}^2 \) becomes zero for which arbitrary values of \( \beta \) are allowed. This is the point through which all the curves corresponding to different values of \( \beta \) pass in Fig. 1. Different points on the \((\alpha, m_1)\) plane correspond to different values of \( s_{13}^2 \). However, the CHOOZ bound allows only a limited region on the \((\alpha, m_1)\) plane, so we shall constrain the \((\alpha, m_1)\) parameter space in the light of the CHOOZ bound. However, the Majorana CP-violating phase \( \beta \) can not be constrained. Since, \( M \) is bounded from above \((\alpha = 0^\circ)\) and from below \((\alpha = 90^\circ)\) (cf. Eq. (22)), \( s_{13}^2 \) is, also, bounded from above and below, i.e.

\[
\frac{m_2 s_{12}^2 - m_1 c_{12}^2}{m_2 s_{12}^2 - m_1 c_{12}^2 + m_3} < s_{13}^2 < \frac{m_1 c_{12}^2 + m_2 s_{12}^2}{m_1 c_{12}^2 + m_2 s_{12}^2 + m_3}.
\]

Thus, the region allowed by \( m_{ee} = 0 \) on the \((m_1, s_{13}^2)\) plane will be bounded by the two limiting values of \( s_{13}^2 \) corresponding to \( \alpha = 0^\circ \) and \( 90^\circ \). As \( m_1 \to 0 \), \( s_{13}^2 \) becomes independent of \( \alpha \) and is given by

\[
s_{13}^2 = \frac{\sqrt{\Delta m_{12}^2 s_{12}^2}}{\sqrt{\Delta m_{12}^2 s_{12}^2} + \sqrt{\Delta m_{23}^2}}.
\]

This situation has been depicted in Fig. 2 where the upper and lower bounds on \( s_{13}^2 \) [cf. Eq. (30)] have been plotted as functions of \( m_1 \). The upper \((\alpha = 0^\circ)\) and lower \((\alpha = 90^\circ)\) curves come closer as \( m_1 \to 0 \) and, eventually, merge for [cf. Eq. (31)]

\[
s_{13}^2 = 0.055^{+0.010, 0.033}_{-0.009, 0.024}.
\]

The central value of \( s_{13}^2 \) for \( m_1 = 0 \) given above is above the CHOOZ bound but is consistent with the CHOOZ bound at about 2.3\( \sigma \). Therefore, a vanishing \( m_{ee} \) element is disallowed at 2.3\( \sigma \) C.L. for \( m_1 = 0 \) contrary to the results reported in Ref. [3] where the special case \( m_1 = m_{ee} = 0 \) has also been examined and it has been found to be consistent with the neutrino oscillation data. However, our results are consistent with the recent results reported by Chauhan et al. [4] who conclude that the CHOOZ bound is violated when \( m_1 = m_{ee} = 0 \) in vanishing determinant neutrino mass matrix scenarios where \( m_1 = 0 \) is a natural choice. As \( m_1 \) increases, the lower curve (corresponding to \( \alpha = 90^\circ \)) dips to its minimum value \( s_{13}^2 = 0 \) for \( m_1 = s_{12}^2 \sqrt{\frac{\Delta m_{12}^2}{\cos 2\theta_{12}}} = 0.045eV \) [cf. Eq. (14)]. At this point, \( m_{ee} = 0 \) irrespective
Figure 2: The \((s_{13}^2, m_1)\) parameter space allowed by \(m_{ee} = 0\) for \(0^o < \alpha < 90^o\). The upper solid line corresponds to \(\alpha = 0^o\) and the lower solid line corresponds to \(\alpha = 90^o\). We have also shown the \(1\sigma\) and \(3\sigma\) CHOOZ bounds as dotted lines.

of the value of \(\beta\). With further increases in \(m_1\), \(s_{13}^2\) increases and, eventually, becomes larger than the CHOOZ bound. Thus, there exists a range of \(m_1\) centered around 0.0045 eV and a corresponding range for \(\alpha\) around 90°, for which the condition \(m_{ee} = 0\) and the CHOOZ bound hold simultaneously. Consequently, the condition \(m_{ee} = 0\) combined with the CHOOZ bound yields both an upper and a lower bound on \(m_1\). With further increase in \(m_1\), we approach the quasi-degenerate (QD) region for normal neutrino mass ordering. For a certain value of \(m_1\) \((\simeq 10^{-2}\text{eV}\) for the central values of neutrino oscillation parameters given in Eq. (10)), the value of \(s_{13}^2\) becomes larger than the CHOOZ bound. Thus, a vanishing \(m_{ee}\) element is not allowed in QD hierarchy with normal ordering of neutrino masses and, therefore, a texture zero at the \(ee\) entry in the neutrino mass matrix in the flavor basis cannot accommodate a QD mass spectrum and small \(\theta_{13}\) simultaneously. When \(m_1\) becomes very large as compared to \(\sqrt{\Delta m_{23}^2}\), Eq. (31) becomes

\[
\frac{s_{12}^2 - c_{12}^2}{s_{12}^2 - c_{12}^2 + 1} < s_{13}^2 < \frac{1}{2}.
\]

(33)

The upper \((\alpha = 0^o)\) and lower \((\alpha = 90^o)\) curves in Fig. 2 approach the upper and lower bounds on \(s_{13}^2\) given above asymptotically with \(m_1\).

In Fig. 3, the contours for \(1\sigma, 2\sigma\) and \(3\sigma\) values of \(s_{13}^2\) allowed by the CHOOZ bound have been plotted on the \((\alpha, m_1)\) plane. The allowed range of \(m_1\) in Fig. 3 is essentially the same as in Fig. 2. The values of \(\alpha\) which give \(m_{ee} = 0\) are centered around \(\alpha = 90^o\) and the
Figure 3: The parameter space on \((\alpha - m_1)\) plane constrained by \(|m_{ee}| = 0\) and the CHOOZ bound. The regions in the increasing order of lighting correspond to 1\(\sigma\), 2\(\sigma\) and 3\(\sigma\) bounds on \(s^{2}_{13}\). The \((\alpha, m_1)\) parameter space becomes smaller with the decrease in the upper bound on \(s^{2}_{13}\). At two standard deviations, the allowed range of \(\alpha\) is \(75^\circ - 105^\circ\) approximately and the best fit point on the \((\alpha, m_1)\) plane is approximately \((90^\circ, 0.0045eV)\). The exact value of \(m_1\) for which \(m_{ee}\) vanishes is given by Eq. (14) and this happens for \(s_{13} = 0\) and \(\alpha = 90^\circ\).

While plotting Fig. 2 and Fig. 3, we used the central values of the neutrino oscillation parameters. Now, we incorporate the experimental errors in these parameters in the numerical analysis to fix the allowed ranges of \(m_1\) and \(\alpha\). The results of this analysis have been summarized in Table 1. The allowed ranges for \(m_1\) and \(\alpha\) shown in Fig. 3, however, come from the 1\(\sigma\), 2\(\sigma\) and 3\(\sigma\) ranges for \(s^{2}_{13}\) and central values for the other oscillation parameters and, therefore, are more restrictive as compared to the ranges given in Table 1.

Thus, the effective Majorana mass can be zero in normal hierarchy if \(m_1\) and \(\alpha\) lie within

<table>
<thead>
<tr>
<th>C.L.</th>
<th>(m_1)</th>
<th>(\alpha)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1(\sigma)</td>
<td>0.0029 - 0.0063eV</td>
<td>83.9(^{\circ}) - 96.1(^{\circ})</td>
</tr>
<tr>
<td>2(\sigma)</td>
<td>0.0011 - 0.0093eV</td>
<td>71.2(^{\circ}) - 108.8(^{\circ})</td>
</tr>
<tr>
<td>3(\sigma)</td>
<td>0 - 0.0129eV</td>
<td>33.7(^{\circ}) - 146.3(^{\circ})</td>
</tr>
</tbody>
</table>

Table 1: The allowed values of \(m_1\) and \(\alpha\) for \(m_{ee} = 0\) from the neutrino oscillation data at various confidence levels.
Figure 4: The parameter space allowed on \((s_{13} - m_1)\) plane by \(|m_{ee}| = 0\) for \(0^\circ < \alpha < 90^\circ\) for the inverted hierarchy. The upper solid line corresponds to \(\alpha = 0^\circ\) and the lower solid line corresponds to \(\alpha = 90^\circ\). We have also shown the CHOOZ 1\(\sigma\) and 3\(\sigma\) bounds as dotted lines.

the ranges shown in Table 1. The inter-relationship between the hierarchy of the neutrino mass spectrum and the resulting texture structure of the neutrino mass matrix is apparent from this analysis. The mass matrices with a texture zero at the \(ee\) entry naturally lead to a mass spectrum with normal hierarchy and small mixing angle \(\theta_{13}\).

Finally, we examine the case of inverted hierarchy where the lightest neutrino mass is \(m_3\). It is clear from Eq. (25) that \(s_{13}^2 = 1\) for \(m_3 = 0\). As \(m_3\) increases, \(s_{13}^2\) decreases and attains its minimum value asymptotically. This feature is apparent from Fig. 4 where \(s_{13}\) has been plotted as a function of \(m_3\) for inverted hierarchy. For \(m_3\) very large, we obtain

\[
\frac{s_{12}^2 - c_{12}^2}{s_{12}^2 - c_{12}^2 + 1} < s_{13}^2 < \frac{1}{2}
\]  

from Eq. (31). The lower and upper bounds on \(s_{13}^2\) given above correspond to the asymptotic values of the upper \((\alpha = 0^\circ)\) and lower \((\alpha = 90^\circ)\) curves in Fig. 4 respectively. It is easily seen that the CHOOZ bound is much below the values of \(s_{13}^2\) required for vanishing \(m_{ee}\) in inverted hierarchy as well as in quasi-degenerate hierarchy with the inverted ordering of neutrino masses. Therefore, neutrino mass matrices with a texture zero at the \(ee\) entry cannot yield a mass spectrum with inverted hierarchy and small \(\theta_{13}\) simultaneously. This happens because the first mass eigenvalue will be smaller than the second and third eigenvalues if the \(ee\) element of the mass matrix is zero unless a very large 1-3 rotation is effected to make it larger than the third mass eigenvalue.
In conclusion, the consequences of a vanishing effective Majorana mass have been examined in detail. It has been concluded that the effective Majorana mass can be zero only for normal hierarchy for certain ranges of values of $m_1$ and $\alpha$. The Majorana phase $\beta$ is left unconstrained but it should be one quadrant behind $\alpha$. It is found that effective Majorana mass is not allowed to vanish at 2.3 $\sigma$ C.L. when $m_1 = 0$ in normal hierarchy. Mass matrices with a texture zero at the $ee$ entry naturally lead to a normal ordered neutrino mass spectrum and small $\theta_{13}$. However, a neutrino mass spectrum with inverted or even quasi-degenerate hierarchies is not allowed by the condition of a vanishing effective Majorana mass in the light of the current neutrino data.

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