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APPROACH TO ASYMPTOTIA FROM s-CHANNEL DYNAMICS

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ABSTRACT

It is suggested that two asymptotic regimes exist in critical point theories of the Pomeron. The single fireball regime of some absorptive models is joined to the strong coupling solution of Reggeon field theory by renormalization group equations.

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Recent results in Reggeon field theory ¹⁾⁻³⁾ and absorptive models ⁴⁾⁻⁶⁾ have suggested that "critical point" theories for the Pomeron are possibly consistent with both t- and s-channel unitarity.

The two above-mentioned approaches differ, however, in various points. First, the s-channel approach deals with different classes of production events [i.e., the fireball expansion ⁷⁾ for the imaginary part of the amplitude or cut Pomeron $\mathbb{P}_C$ ⁶⁾] and includes absorptive corrections due to the rescattering Pomeron $\mathbb{P}_R$ ⁶⁾. Therefore, several kinds of Pomeron interaction vertices, related to different processes, contribute to $\mathbb{P}_C$. For instance, if only elastic rescattering events are considered one has three ⁶⁾: g_A , g_D and g_R , related to $\mathbb{P}_C \mathbb{P}_C \mathbb{P}_R$, $\mathbb{P}_C \mathbb{P}_R \mathbb{P}_R$ and $\mathbb{P}_R \mathbb{P}_R \mathbb{P}_R$ vertices (Fig. 1).

In the t-channel approach ¹⁾⁻³⁾, on the other hand, one has equations for the amplitude (uncut Pomeron $\mathbb{P}$). All classes of events contribute therefore on equal footing to the determination of the critical indices η , ν and γ ¹⁾, which parametrize the departure of the amplitude from pure Regge pole behaviour. If a field theory of the Pomeron is valid, there is only one coupling g related to the $\mathbb{P} \mathbb{P} \mathbb{P}$ vertex.

As a consequence, the single fireball absorptive model ⁶⁾ (corresponding to $g_D = g_R = 0$) has critical indices ($\eta_C = \nu_C - 1 = \gamma_C = \epsilon/4$) different from the ones of RFT ^{2),3)} [$\eta = 2(\nu - 1) = 1/4(\gamma) = \epsilon/12$] to leading order in the ϵ expansion around four transverse dimensions ($\epsilon = 4 - d$).

In this note we want to show that the absorptive model results may hold in a subasymptotic regime, where perturbation theory in g_D is still valid, while for rapidities much larger than the average diffractive gap Δ perturbation theory in g_D breaks down and the RFT results may emerge. Our method is based on writing renormalization group equations for $\mathbb{P}_C$ and $\mathbb{P}_R$ propagators and g_A , g_D , g_R vertices ^{*)}, without requiring them to be a priori related (or equal) as they would be if coming from discontinuities of a single Feynman diagram.

The physical motivation for $g_A \neq g_D$ is based on two remarks. a) The coupling constant g_A occurring in absorptive corrections is to be considered as soon as the Pomeron is built up because it comes from final-state interactions. Its measurement could be performed at intermediate accelerator energies, on the basis of precise calculations for the related correlation effects. b) The coupling constant g_D has on the other hand been measured by diffractive production of large masses. These diffractive events become

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observable at high energies when rapidity gaps Δ (due to Pomeron exchange) are probable. In a different language, the value of $\ln s$ which sets the scale for the renormalization point $\omega_0 = J_0 - 1$ at which the physical couplings are measured, is to be taken larger for g_D than for g_A . At a given energy, renormalization effects are therefore likely to be more important for g_A than for g_D . Since we have some evidence for a perturbative theory of the Pomeron in g_D at ISR energies, we analyse the possibility that g_A is much larger than g_D .

If g_R is uncorrelated to g_A and g_D , renormalization effects will induce a difference between $\mathbb{P}_C$ and $\mathbb{P}_R$ in an intermediate energy range. The unitarity equation requires, however, that $\mathbb{P}_C \simeq \text{Im } \mathbb{P}_R$ asymptotically. Two different attitudes are discussed in the following. 1) $\mathbb{P}_C \simeq \text{Im } \mathbb{P}_R$ at infinite energy only: this possibility is allowed because $\mathbb{P}_R$ and $\mathbb{P}_C$ are evaluated in our equations at different energy values and corresponds to constructing $\mathbb{P}_C$ out of $\mathbb{P}_R$ in a sort of iterative fashion ⁸⁾. 2) $\mathbb{P}_C = \text{Im } \mathbb{P}_R$ identically. In this case there cannot be an independent renormalization scheme for $\mathbb{P}_R$: in other words g_R is related to g_A and g_D .

1) $\mathbb{P}_C \simeq \text{Im } \mathbb{P}_R$ asymptotically. To be specific, we assume for the various vertices the renormalization scheme illustrated in Fig. 2. In principle, the counting of the various vertex corrections can be determined either by specifying the rescattering interaction in the initial ⁹⁾ or final state ^{4), 10)}, or by giving a discontinuity rule for Feynman diagrams ¹¹⁾. Neither approach has been developed so as to reach a clear-cut conclusion ^{12), 13)}. We shall therefore use the absorptive recipe 1:2 for the ratio of diffractive versus absorptive discontinuities, and shall comment on the alternative of Ref. 11) later on.

Note first that the equations for g_R are decoupled from the others and the same as for RFT. By introducing the effective charges $[\alpha'_C(\alpha'_R)]$ is the $\mathbb{P}_C(\mathbb{P}_R)$ slope]

$$u_A = g_A \omega_0^{-\varepsilon/4} \left(\frac{2}{\alpha'_R + \alpha'_C} \right)^{d/4}, \quad u_{D,R} = g_{D,R} \omega_0^{-\varepsilon/4} \left(\frac{1}{\alpha'_R} \right)^{d/4}, \quad (1)$$

and the ratio $\rho = \alpha'_R / \alpha'_C$ of the slopes, we get -- to first order in $\varepsilon = 4-d$ -- the $\mathbb{P}_C$ wave function and slope renormalization constants

$$Z_C = 1 + \frac{1}{2} a (2 u_A^2 - u_D^2) + \dots, \quad a = 1/3 u^{*2},$$

$$\frac{\alpha'_C}{\alpha'_{0C}} = R_C = 1 + \frac{1}{2} a \left[\frac{2 u_A^2}{1+\rho} - \left(1 - \frac{\rho}{2}\right) u_D^2 \right] + \dots, \quad (2)$$

and the vertex renormalization constants

$$Z_A^{-1} = 1 - a \left[u_A^2 + u_A u_R \frac{1+3\rho}{1+\rho} - u_D^2 \frac{u_R}{u_A} \frac{2\rho}{1+\rho} \right] + \dots, \quad (3)$$

$$Z_D^{-1} = 1 - 2a u_A u_R + \dots$$

The renormalization group equation for the Π_C propagator is therefore

$$D_C(\omega, k^2, \alpha_c^1, g_A, g_D, g_R, \rho, \omega_0) = \omega^{-1} e^{\int_0^{-\ln \omega} d \ln \omega' \eta_c(\omega')} d\left(\bar{u}_i(\omega), \bar{\rho}(\omega), \frac{k^2}{\omega \gamma_c(\omega)}\right) \quad (4)$$

where from Eqs. (2) and (3)

$$\eta_c = - \frac{\partial \ln Z_c}{\partial \ln \omega} = \frac{\varepsilon}{4} a \left(2\bar{u}_A^2 - \bar{u}_D^2 \right)$$

$$\gamma_c^{-1} = - \frac{\partial \ln R_c}{\partial \ln \omega} = \frac{\varepsilon}{4} a \left[\frac{2u_A^2}{1+\rho} - \left(1 - \frac{\rho}{2}\right) u_D^2 \right] \quad (5)$$

and the equations of motion for the various effective charges are

$$\frac{d \bar{u}_i}{d \ln \omega} = \beta_i(\bar{u}_i, \bar{\rho}) \quad , \quad \frac{d \bar{\rho}}{d \ln \omega} = \beta_\rho(\bar{u}_i, \bar{\rho}) \quad ,$$

$$\beta_\rho = - \frac{\varepsilon}{4} a \rho \left[\frac{1}{2} u_R^2 + \left(1 - \frac{1}{2}\rho\right) u_D^2 - \frac{2}{1+\rho} u_A^2 \right] = - \frac{\varepsilon}{4} a \rho X_\rho \quad ,$$

$$\beta_A = - \frac{\varepsilon}{4} a u_A \left\{ 1 - 2a \left[u_A u_R \frac{1+3\rho}{1+\rho} + u_D^2 \left(\frac{1}{2} - \frac{u_R}{u_A} \frac{2\rho}{1+\rho} \right) - \frac{X_\rho}{2(1+\rho)} \right] \right\} \quad , \quad (6)$$

$$\beta_D = - \frac{\varepsilon}{4} a u_D \left[1 - 2a \left(2u_A u_R - \frac{1}{4} u_R^2 - \frac{1}{2} u_A^2 + \frac{1}{4} u_D^2 \right) \right] \quad ,$$

$$\beta_R = - \frac{\varepsilon}{4} a u_R \left(1 - 3a u_R^2 \right) \quad .$$

The physical problem is now to investigate how -- starting from pure pole behaviour for $\mathbb{P}_R^{(0)}$ and $\mathbb{P}_C^{(0)}$ ($\rho^{(0)} = 1$) -- the renormalization group equations build up the various critical exponents as $\omega \rightarrow 0$, i.e., as $\ln s$ increases. Owing to the special role of g_D mentioned before, we shall study what happens when g_D is switched off and on.

If $g_D = 0$ exactly, β_D plays no role in Eqs. (5) and (6). It is then easy to see that there is a stable fixed point at

$$\rho = 0, \quad u_A = u_A^0 = x_A u^*, \quad x_A = \sqrt{2} - \frac{1}{2}, \quad (7)$$

from which we obtain the critical indices

$$\gamma_c - 1 = \eta_c = 2 x_A^2 \eta, \quad \gamma_c = \frac{7}{2} \eta \quad \left(\eta = \frac{\epsilon}{12} \right). \quad (8)$$

This is essentially the single fireball solution of Ref. 6) ^{*)}. Note that in this case $\mathbb{P}_C$ shrinks more than $\mathbb{P}_R$, corresponding to the fact that absorptive corrections alone tend to steepen the small t distribution and its shrinkage. As shown in Ref. 6) this solution has exactly logarithmic multiplicity and integrable correlations.

If, however, g_D is switched on, it is easily realized that $u_D = 0$, $\rho = 0$, $u_A = u_A^0$ is no longer a stable fixed point of the coupled problem because $\partial \beta_D / \partial u_D < 0$ at that point. Therefore u_D tends to increase away from $u_D = 0$ as $\ln \omega$ decreases ^{**)} .

Let us now note that Eqs. (5) and (6) admit a fixed point at

$$\mathcal{U} \equiv (u_D, u_A, u_R, \rho) = (u^*, u^*, u^*, 1) \equiv \mathcal{U}_0$$

for which $\mathbb{P}_C = \mathbb{P}_R$, corresponding to the critical indices of RFT. However, this is not a priori stable, because the matrix of the derivatives

*) In Ref. 6) we limit ourselves to two-body rescattering, so that $u_R = 0$ and $\eta_C^0 = \nu_C^0 - 1 = \gamma_C^0 = \epsilon/4$. The inclusion of many-body rescattering considered here changes the critical indices, but not the multiplicity distributions.

**) In the iterative approach of Ref. 6) the exchanged Pomeron is identified with the single fireball solution just obtained. In such a case the probability distribution for large gaps is damped enough for perturbation theory to be applicable, and diffractive events give only an intercept renormalization. In the present approach, on the other hand, the exchanged Pomeron is related to $\mathbb{P}_R$, and according to the renormalization scheme of Fig. 2, diffractive events contribute eventually to the critical indices

of the β 's is not positive definite; for instance, $\partial^2 \rho / \partial \rho^2|_{u_j=u^*, \rho=1} = 0$. Therefore an investigation of the surfaces of zero of β_1 is needed, in order to see whether the RFT fixed point is really reached starting from $\rho^{(0)} = 1$, g_D small.

The intersections of the surfaces $\beta_\rho = 0$ and $\beta_D = 0$ with that of $\beta_A = 0$ are drawn in Fig. 3, by assuming for simplicity that u_R is already at its fixed point u^* . Since $\beta_A = 0$ turns out to be close to the plane $u_A = u^*$, we project the curves in the (u_D, ρ) plane. It appears that apart from U_0 and the single fireball solution $U_A = (0, x_A u^*, u^*, 0)$ there are (in this model) other fixed points at $U_1 = (x_1 u^*, x_A u^*, u^*, 0)$ and at $U_2 = (x_2 u^*, x_A' u^*, u^*, \rho_2)$, where $x_A = \sqrt{2 - \frac{1}{2}} \simeq 0.91$, $x_1^2 = 2x_A^2 - 8x_A + 7 \simeq 1.35$, and $x_2 \simeq 1.2$, $x_A' \simeq 0.9$, $\rho_2 \simeq 0.1$. From the direction of the derivative vector we see that U_1 is repulsive, but U_2 can give rise to spirals, and therefore compete with the U_0 attraction.

We have not determined explicitly the region of initial points U which end up in U_0 . Two possible paths are, however, shown which start at $\rho^{(0)} = 1$, and u_D small. We can see that U (which always has u_A close to u^*) is first attracted towards $\rho = 0$, and then pushed upwards. The slope ratio ρ can start increasing towards $\rho = 1$ after crossing $\beta_\rho = 0$. Path (a) is bound to occur if U is too close to $\beta_A = 0$ because the region between β_ρ and $\beta_A = 0$ with $\rho_2 < \rho < 1$ cannot be crossed upwards on this surface. If, however, U is still some distance away from $\beta_A = 0$, path (b) can occur in a three-dimensional picture.

The path $U(w)$, $(w \rightarrow 0)$ just discussed represents, in our opinion, an appealing interpolation between the single fireball regime of Ref. 6) and the ultra-asymptotic regime of RFT. Substituting $u_D^2(w)$ and $\rho(w)$ into Eq. (5) for η_c and v_c (with u_A close to $\beta_A = 0$ and therefore to u^*) we get the following qualitative features:

- a) there is a rapidity region $1 \lesssim \ln s \lesssim \Delta$ (or a related neighbourhood of $J = 1$) for which the fireball expansion, as a perturbative theory of the Pomeron, makes sense. In this region absorptive corrections are possibly important and the correct single fireball description (if $g_A^2 \Delta / \alpha'$ is big enough) is the one given in Ref. 6), with all implications on the total cross-section ($\eta_c \simeq 2\eta_c$) and on multiplicity distributions (multicomponent picture);
- b) in the rapidity region $\ln s \gg \Delta$ ($w \ll 1/\Delta$) renormalization effects to the fireball expansion are overwhelming and there is no simple picture of the Pomeron in the s-channel. At infinite energy, the completely self-consistent Pomeron of the t-channel picture is obtained.

2) $\mathbb{P}_C = \text{Im } \mathbb{P}_R$ identically. The independent renormalization scheme for $\mathbb{P}_R$ is in this case replaced by a "bootstrap" relation between the propagators of the two Pomerons ($D_C = D_R$). In this case $\mathbb{P}_R$ is treated as an external field for which an effective coupling $u_R(\omega)$ is defined by the relation between the two propagators ^{*}). As an example suggested by Fig. 2a, we consider to first order

$$u_R^2(\omega) = 2u_A^2(\omega) - u_D^2(\omega) \quad (9)$$

This implies that $v_C(\omega) - 1 = \frac{1}{2} \eta_C(\omega)$ in Eq. (5) and that only two independent equations (for u_D and u_A) exists, in which

$$\begin{aligned} \beta_A &= -\frac{\epsilon}{4} u_A \left[1 - 2a \left(2u_A u_R + u_D^2 \left(\frac{1}{2} - \frac{u_R}{u_A} \right) \right) \right], \\ \beta_D &= -\frac{\epsilon}{4} u_D \left[1 - 2a \left(2u_A u_R - \frac{1}{2} u_R^2 \right) \right], \\ u_R^2 &= 2u_A^2 - u_D^2. \end{aligned} \quad (10)$$

The lines of zero of β_A , β_D in the $(u_A^2, u_D^2) \equiv U$ plane are given in Fig. 4. There are two relevant fixed points $U_0 = (u^{*2}, u^{*2})$ and $U_A = [(3/4\sqrt{2})u^{*2}, 0]$ corresponding to RFT, and single fireball, respectively. As before U_A can be reached only if $g_D = 0$, while for g_D small, all paths approach U_A first and then reach U_0 . Two regimes in ω (or $\ln s$) can still be envisaged in this case: the single fireball regime with $U \simeq U_A$, $\eta_C = (3/2\sqrt{2})\eta$ ($\eta = \epsilon/12$) and logarithmic multiplicities, and the RFT regime with $\eta_C = \eta$.

The conclusions reached in both approaches 1) or 2) rely on the following assumptions. i) $g_A^2 \Delta/\alpha' \geq 1$, $g_D^2 \Delta/\alpha' \ll 1$. This means that at ISR energies, we take seriously critical point effects only for absorptive corrections. ii) The renormalization scheme for $\mathbb{P}_C$ is the one given in Fig. 2a,c. Note firstly that the existence of the RFT fixed point U_0 is built in, in these equations. Secondly, it is conceivable that by changing the counting, or including other effects like inelastic rescattering, both the position of U_A and the attraction region of U_0 can change. This can be used in order to select either the initial point, or the model itself. The choice of the model is particularly important also for multiplicity distributions because, e.g., in our examples the multiplicity tends

^{*}) We do not require of course the $\mathbb{P}_C$ and $\mathbb{P}_R$ vertices to be identically equal.

to increase less than logarithmically ($\langle n \rangle \sim \ln s^{1-\eta}$)¹⁴⁾ at U_0 . If, on the other hand, we had assumed a renormalization scheme¹³⁾ consistent with Ref. 11, the opposite situation would hold^{2),14)}.

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FIGURE CAPTIONS

Fig. 1 : Production amplitudes for

- a) the bare-cut Pomeron $\mathbb{P}_C$ and bare couplings
- b) g_A
- c) g_D
- d) g_R .

Fig. 2 : Second-order corrections to the

- a) $\mathbb{P}_C$ propagator
- b) $\mathbb{P}_R$
- c) g_A
- d) g_D
- e) g_R vertices.

Fig. 3 : Possible paths of (u_D, ρ) for two small values of $u_D^{(0)}$. The $\beta_\rho = 0$ lines are dashed and the $\beta_D = 0$ lines are dotted. The arrows denote the flow direction in different regions.

Fig. 4 : The lines of zero of β_A (dashed) and β_D (dotted) in the u_A, u_D plane for the absorptive "bootstrap" model. The full lines denote possible paths for small g_D .

FIG.1

FIG.2

FIG.3

FIG.4