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CONSTITUENT QUARK ANALYSIS OF HADRONIC TRANSITIONS *)

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ABSTRACT

Local operators may be expanded in terms of non-local constituent quarks. If, as we argue, this expansion is rapidly convergent, a useful phenomenological scheme results. We apply this scheme to study the pionic and electromagnetic transitions of hadrons.

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In theories such as the Nambu Jona-Lasinio model ¹⁾ or the σ model ²⁾, massless fermion fields can spontaneously acquire a non-zero effective mass m . The transition from massless to massive fields is described in such models by some analogue of the Bogoliubov transformation ^{3),4)}. This transformation is, in general, non-local with a spatial dispersion of order $a = 1/m$. In this paper we exploit this idea to expand various local operators in terms of non-local constituent quark fields. Essential to this approach is the idea that a is a fundamental parameter which uniquely sets the scale of hadronic physics. This idea is of course suggested by many features of the hadron spectrum ⁵⁾.

In the limit of very strong couplings, one expects the spontaneously generated quark mass to become extremely large. We identify the limit of infinite m with the world of exact $SU(6) \times SU(6) \times O(3)$, where particles are grouped in mass degenerate $SU(6) \times O(3)$ ⁶⁾ multiplets and are coupled to currents with exact $SU(6)_W$ symmetry ⁷⁾. For the realistic case of finite m we will assume that hadrons are still grouped in $SU(6) \times O(3)$ multiplets. These multiplets will not, however, be mass degenerate, and their couplings to currents will deviate from exact $SU(6)_W$ symmetry.

To describe these deviations we can introduce constituent quark fields ⁸⁾ which transform irreducibly under the $SU(6)$ classification group, and expand all current operators in terms of these fields. Since the transformation ⁹⁾ from the canonical fields to these constituent fields is non-local ¹⁰⁾, this expansion will involve the fundamental length $a = 1/m$. Thus a local operator A , of naive dimension μ , will be represented by the expansion

$$A = B_{\mu} + \frac{1}{m} B_{\mu+1} + \frac{1}{m^2} B_{\mu+2} + \dots \quad (1)$$

Here B_{μ} represents an operator of naive dimension μ , which carries the same momentum and internal quantum numbers as A and which is constructed from the constituent quark fields and their derivatives ¹¹⁾.

In hadronic matrix elements of B_{μ} , derivatives of the quark fields measure the momenta of constituent quarks within the hadron. These momenta are typically of order R^{-1} , where R describes the dimensions of the hadronic state in question. Thus the expansion (1) involves powers of the ratio of the quark size a to the hadronic size R ¹²⁾. It is quite plausible that $a/R < 1$ ¹³⁾ and hence that the series (1) is convergent. If convergence is fairly rapid, then we may truncate the series and develop a useful approximation scheme for the matrix elements of any local observable A .

In this paper we will discuss the $1/m$ expansion of several well-measured "good" operators. Each operator will be expanded, as in Eq. (1), in terms of the independent components $q(x)$ of the constituent quark fields on the null plane $x^+ = 0$

1. AXIAL CHARGE

For the axial current density $A_1^+(x)$, Eq. (1) has the explicit form

$$\begin{aligned} A_1^+(x) = & q^+(x) \frac{\lambda_i}{2} \sigma_3 q(x) + \frac{ia_1}{m} q^+(x) \frac{\lambda_i}{2} (\vec{\gamma} \times \vec{\partial})_3 q(x) \\ & + \frac{a_2}{m^2} q^+(x) \frac{\lambda_i}{2} \sigma_3 \partial_\perp^2 q(x) \\ & + \frac{ia_3}{m^2} q^+(x) \frac{\lambda_i}{2} q(x) q^+(x) \sigma_3 \frac{1}{\partial_-} q(x) + \dots \end{aligned} \quad (2)$$

The terms of Eq. (2) are listed for convenient reference in Table 1. Note that all terms, but a_3 , belong to 35-dimensional representations of the classification group $SU(6)$ (constituents). In particular, the first term (integrated over $d^2x_\perp dx^-$) is just a generator of this group. Under the collinear $SU(3) \times SU(3)$ subgroup it belongs to a representation $(\underline{1}, \underline{8})_0 - (\underline{8}, \underline{1})_0$, as does the term a_2 . The term a_1 by contrast belongs to a representation $(\underline{3}, \underline{\bar{3}})_1 + (\underline{\bar{3}}, \underline{3})_{-1}$. The four-quark term a_3 has an exotic $SU(6)$ structure involving the representations $\underline{35} \oplus \underline{189} \oplus \underline{280} \oplus \underline{405}$. Its $SU(3) \times SU(3)$ structure is also rather complex.

The transverse derivatives in a_1 and a_2 will introduce factors of $1/R$ in matrix elements of these terms. Their contributions are thus of order a/R and $(a/R)^2$, respectively. Longitudinal derivatives, on the other hand, are intrinsically of order m . From the term a_3 we can thus deduce a general rule: additional $q\bar{q}$ pairs are suppressed by factors of order $(a/R)^3$.

To fully characterize the $SU(6) \times O(3)$ structure of the operators of Eq. (2), we must discuss their angular momentum [or $O(3)$] content. This requires the explicit construction of the "bad" $O(3)$ generators L_1 and L_2 . For the present purpose it is sufficient to assume that $\vec{L}$ is described as in the Pauli-Lubanski vector and hence in the rest frame of a given state it has the simple form $\vec{L} = \sum_i \vec{x}_i \times \vec{p}_i$ ¹⁴⁾. This implies that $\vec{x}_\perp$ and $\vec{p}_\perp$ are the transverse components of vectors. Thus the term a_1 can only induce transitions with $\Delta L = 1$, while a_2 carries $L = 0$ and $L = 2$. In the lowest order the term a_3 carries $L = 0$. To higher orders,

however, the factor $1/\partial_-$ will introduce both odd and even values of the angular momentum.

It is apparent from Table 1 that leading terms in the $1/m$ expansion have an algebraic structure $(8,1)_0 - (1,8)_0 \oplus (3,\bar{3})_1 + (\bar{3},3)_{-1}$. This structure ¹⁵⁾ is precisely that deduced by Melosh ¹⁶⁾ in the context of the free quark model and applied successfully (via the PCAC hypothesis) to the study ¹⁷⁾ of the pionic decays of hadron resonances. Thus to leading orders in (a/R) the present approach is equivalent to the Melosh scheme. Higher-order terms in a/R provide corrections to the Melosh form, but -- as we will show -- these corrections are only a 10% effect.

More important phenomenologically is the fact that terms with different algebraic structure enter the $1/m$ expansion of A^+ at different orders. Thus for any specific matrix element we can always determine which type of term is dominant ^{*)}. A simple rule of this sort is the fact that -- with the neglect of exotic terms -- even L transitions have the algebraic structure $(8,1)_0 - (1,8)_0$ while odd L transitions have the structure $(3,\bar{3})_1 + (\bar{3},3)_{-1}$. Such a rule has been suggested by several authors ¹⁷⁾ on phenomenological grounds. Table 1 illustrates this rule for $L=0,1$ and 2 transitions.

In what follows we will outline the successful application of the even/odd rule and point out further ways in which our results supplement those obtained in the Melosh approach. Consider first the matrix elements of Eq. (2) between states of the (56 $L=0$) multiplet. This is an even L transition ($\Delta L=0$), so the term a_1 cannot contribute. Thus Eq. (2) implies ¹⁶⁾ that

$$g_A = \langle p | A_+^+(0) | n \rangle = \frac{5}{3} \left(1 - \mathcal{O}\left(\frac{a}{R}\right)^2 \right) \quad (3)$$

which indicates that a/R is of order $1/2$ ¹³⁾. Hence the $(a/R)^3$ contribution from the exotic term a_3 is indeed only a 10% correction. Similarly, for the transition $N^*(1470) \rightarrow N\pi$, we expect

$$g^* = \langle N^{*+}(1470) | A_+^+(0) | n \rangle \simeq \frac{5}{3} \left(\frac{a}{R} \right)^2 \simeq 0.4 \quad (4)$$

*) Apart from a/R factors, the reduced matrix elements of the various algebraic terms are assumed to be of the same order of magnitude.

since a_2 is the first term in Eq. (2) which can contribute to this transition. [The leading term -- being a generator of SU(6) -- does not connect distinct multiplets.] Experimentally $|g^*| \simeq 0.4$ to 0.5.

For the transitions $(\underline{35} \text{ } L=1) \rightarrow (\underline{35} \text{ } L=0) + \pi$ and $(\underline{70} \text{ } L=1) \rightarrow (\underline{56} \text{ } L=0) + \pi$ the dominant contribution is of order a/R and is provided by the $(\underline{3}, \underline{\bar{3}})_1 + (\underline{\bar{3}}, \underline{3})_{-1}$ term a_1 . Corrections to this algebraic structure will come from the exotic term a_3 which contains an $L=1$ piece of order $(a/R)^4$. These contributions thus provide corrections of 10% or so to the dominant $(\underline{3}, \underline{\bar{3}})_1 + (\underline{\bar{3}}, \underline{3})_{-1}$ term. Meson decay analyses and phase-shift analyses for $\pi N \rightarrow \pi N$ and $\pi N \rightarrow \pi \Delta$ scattering support this general picture ¹⁷⁾.

For the decay $(\underline{56} \text{ } L=2) \rightarrow (\underline{56} \text{ } L=0) + \pi$ a discussion paralleling that just given shows that the $(\underline{8}, \underline{1})_0 - (\underline{1}, \underline{8})_0$ term a_2 should be dominant (with exotic correction terms of order 10%). Here also, phenomenological analyses ¹⁷⁾ show the data to be consistent with this prediction.

2. DIPOLE OPERATOR

The magnetic dipole operator

$$D^+(x^+) = \int d^2x_\perp dx^- (x_1 + i x_2) V_{EM}^+(x) \quad (5)$$

has a $1/m$ expansion involving terms of the type listed in Table 2. The labels A, B, C, D and E refer to the algebraic structure of each term. Thus the terms A-D transform as 35-dimensional representations of SU(6) and have -- under the collinear SU(3) $\times$ SU(3) group -- the respective transformation properties $(\underline{8}, \underline{1})_0 + (\underline{1}, \underline{8})_0$, $(\underline{3}, \underline{\bar{3}})_1$, $(\underline{8}, \underline{1})_0 - (\underline{1}, \underline{8})_0$ and $(\underline{\bar{3}}, \underline{3})_{-1}$. These terms are precisely those allowed in the Melosh analysis ¹⁶⁾ of D^+ . Table 2 also indicates the angular momenta carried by the various terms. We find once again a rule which specifies for any transition the algebraic structure of the dominant terms: even L transitions are dominated by $(\underline{3}, \underline{\bar{3}})_1$ and $(\underline{\bar{3}}, \underline{3})_{-1}$ terms; odd L transitions, by $(\underline{1}, \underline{8})_0$ and $(\underline{8}, \underline{1})_0$ terms. It is clear from Table 2 that there are corrections to this rule even at order a/R , i.e., at the 50% level. In addition, there are corrections from exotic terms, but these terms are as usual only of order 10%.

For matrix elements of Eq. (5) between states of $(\underline{56} \text{ } L=0)$ multiplets, the terms A, C and D cannot contribute. The term B should thus dominate these transitions with correction terms of only the exotic variety. Thus, to order 10% we reproduce predictions of the Melosh approach to these processes. For transitions from $(\underline{56} \text{ } L=0)$ to $(\underline{70} \text{ } L=1)$, D cannot contribute. The even/odd rule predicts that A and C should be dominant,

with B smaller by a factor of 2 or so. Phenomenological analyses ¹⁸⁾ again confirm this general picture. Exotic corrections to these results will introduce into the picture violations of the Moorhouse selection rule for the $(8, S=\frac{3}{2})$ states of the 70 . Present data certainly allow for the 10% violation we expect to be present.

Photoproduction of the $(56, L=2)$ provides an interesting test of our model. The even/odd rule suggests for this process the dominance of B and D , with A and C contributions of roughly half the size of the dominant terms. Previous analyses ^{18), 19)} of this process have tended to neglect the D term entirely. We anticipate that with the improved photoproduction data now available, this situation will no longer be tenable.

3. CHARGE RADII

The second moment of the vector current can be analyzed just as the axial charge or the dipole operator. To the leading order we recover the predictions of Melosh ¹⁶⁾, e.g., $\langle r_n^2 \rangle = 0$. Corrections to this result arise from exotic terms in the expansion and are expected to be of order $(a/R)^3$, or 10%. To this figure we may compare the experimental value of $\langle r_n^2 \rangle / \langle r_p^2 \rangle$ variously estimated ²⁰⁾ to be in the range -0.14 to -0.20.

4. DEEP INELASTIC SCATTERING

In this process one probes the light-cone structure of the commutator of two local currents ²¹⁾. Given a form for the operator product expansion of this commutator, we can apply the $1/m$ expansion to each of the local operators entering the operator product expansion. Assuming the spin-two term in the expansion to have the simple structure suggested by the free-quark model, one obtains the predictions

$$\int d\xi F_{2n}(\xi) / \int d\xi F_{2p}(\xi) = \frac{2}{3} \left[1 + \mathcal{O}\left(\frac{a}{R}\right)^3 \right] \quad (6)$$

and

$$\sigma_{\bar{\nu}} / \sigma_{\nu} = \frac{1}{3} \left[1 + \mathcal{O}\left(\frac{a}{R}\right)^3 \right] \quad (7)$$

for electron and (charged current) neutrino scattering. The respective experimental values of these ratios are 0.76 ± 0.05 ²²⁾ and 0.36 ²³⁾.

Extension of this approach to the higher spin components of the operator product expansion is difficult for several reasons. First of all, it is less likely for these operators that the free-quark model is a reliable guide. Secondly, even if the naive scaling of this model were exact, it is unlikely that convergence of the $1/m$ expansion would be uniform in ξ .

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Table 1

1/m expansion of the axial current

Coefficient	Term	$SU(3) \times SU(3)$	Order	L
1	$q^\dagger \frac{\lambda_1}{2} \sigma_3 q$	$(\underline{8}, \underline{1})_0 - (\underline{1}, \underline{8})_0$	1	0
a_1	$\frac{1}{m} q^\dagger \frac{\lambda_1}{2} (\vec{\gamma} \times i\vec{\partial})_3 q$	$(\underline{3}, \underline{\bar{3}})_1 + (\underline{\bar{3}}, \underline{3})_{-1}$	a/R	1
a_2	$\frac{1}{m^2} q^\dagger \frac{\lambda_1}{2} \sigma_3 \partial_\perp^2 q$	$(\underline{8}, \underline{1})_0 - (\underline{1}, \underline{8})_0$	$(a/R)^2$	0, 2
a_3	$\frac{1}{m^3} q^\dagger \frac{\lambda_1}{2} q q^\dagger \frac{\sigma_3}{i\partial_-} q$	complex	$(a/R)^3$	0
			$(a/R)^4$	1

Table 2

1/m expansion of the dipole operator D^+

Type	Term	$SU(3) \times SU(3)$	Order	L
A	$q^\dagger (x_1 + ix_2) \frac{\lambda_{em}}{2} q$	$(\underline{8}, \underline{1})_0 + (\underline{1}, \underline{8})_0$	1	1
	$\frac{1}{m^2} q^\dagger (x_1 + ix_2) \partial_\perp^2 \frac{\lambda_{em}}{2} q$		$(a/R)^2$	1, 2, 3
B	$\frac{1}{m} q^\dagger (\gamma_1 + i\gamma_2) \frac{\lambda_{em}}{2} q$	$(\underline{3}, \underline{\bar{3}})_1$	a/R	0
	$\frac{1}{m} q^\dagger (\gamma_1 + i\gamma_2)(x_1 + ix_2)(i\partial_1 + \partial_2) \frac{\lambda_{em}}{2} q$		a/R	0, 1, 2
C	$q^\dagger \sigma_3(x_1 + ix_2) \frac{\lambda_{em}}{2} q$	$(\underline{8}, \underline{1})_0 - (\underline{1}, \underline{8})_0$	1	1
	$\frac{1}{m^2} q^\dagger \sigma_3(x_1 + ix_2) \partial_\perp^2 \frac{\lambda_{em}}{2} q$		$(a/R)^2$	1, 2, 3
D	$\frac{1}{m} q^\dagger (\gamma_1 - i\gamma_2)(x_1 + ix_2)(i\partial_1 - \partial_2) \frac{\lambda_{em}}{2} q$	$(\underline{\bar{3}}, \underline{3})_{-1}$	a/R	2
E	$\frac{1}{m^2} q^\dagger \frac{1}{i\partial_-} q q^\dagger (x_1 + ix_2) \frac{\lambda_{em}}{2} q$	$(\underline{8}, \underline{1})_0 + (\underline{1}, \underline{8})_0$	$(a/R)^3$	1
			$(a/R)^4$	2

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