CM-P00060606

Ref.TH.1958-CERN

Archives

AN OPEN-SHELL FINITE FERMI SYSTEMS APPROACH TO
THE STRUCTURE OF $A = 12$ NUCLEI *)

John D. Immele

Department of Physics and Astronomy
 University of Maryland, College Park
 and Lawrence Livermore Laboratory +)
 University of California, Livermore

and

Nimai C. Mukhopadhyay x)
 CERN -- Geneva

A B S T R A C T

An extension of Migdal's theory of finite Fermi systems is specialized to an open-shell calculation of the $T = 1$ states in ^{12}C . The over-all fit of the low energy excitation spectrum is found to be satisfactory. The rates of the weak transitions

$$^{12}_{\text{B}}_{\text{gs}} \xrightarrow{\beta^-} ^{12}_{\text{C}}_{\text{gs}}$$

are accurately predicted, yielding a considerable improvement over the closed-shell RPA calculation. Overestimation of the isovector magnetic dipole sum rule strength is observed and discussed.

 *) Work supported in part by the Center for Theoretical Physics, University of Maryland and the US Atomic Energy Commission (JDI), and by the CERN Theoretical Study Division (NCM).

+) Present address.

x) Address from February 1975 : Theory Group, SIN, Villigen, Switzerland.

This letter discusses the results of an *open-shell* calculation for the $A = 12$ nuclei, using an extension [1] of Migdal's theory of the finite Fermi systems (FFS) [2]. Conventional Tamm-Dancoff (TDA) and random-phase approximation (RPA)^{†1} calculations deal with particle-hole (ph) excitations of a nucleus with a closed-shell ground-state (gs). Recently, however, Rowe and his collaborators [3] have used an equation of motion approach to extend the RPA to open-shell nuclei attaining considerable success. Our objective here is to evaluate the generalization [1] to the open-shell systems of Migdal's Green function (GF) approach with particular emphasis on the $T = 1$ positive parity states. This work also provides a crucial test of the simple "universal" interaction of Migdal in the *previously untested* open-shell domain.

The $A = 12$ nuclear system is eminently suited for this test for several reasons: (1) The properties of the low-lying states of both parities are experimentally well studied, and reasonably well understood on the basis of the conventional many-particle shell-models [4,5]; (2) the $T = 1$, $J^\pi = 1^+$ level at 15.11 MeV in ^{12}C , and its analogs (^{12}B and ^{12}N gs's), provide a very sensitive test of the configuration mixing [6]; the TDA fails to account for the observed weak and electromagnetic transition moments, studied by such processes as nuclear β -decay, muon and radiative pion capture and electron scattering [7]; (3) the Kurath energy-weighted magnetic dipole sum rule [8] provides an accurate test for the properties of the excited states with $J^\pi T = 1^+1$.

The ^{12}C gs is well known to be deformed [4]. This means that there is a substantial admixture of 2 particle-2 hole (2p-2h) and 4p-4h states into the usual 0p-0h state. This implies the existence of poles of the one-nucleon GF at several energies, both in the $A + 1$ and in the $A - 1$ nuclear systems, A here being 12. These energies correspond to physical states and the residues of the poles indicate spectroscopic factors. These quantities here are taken from the Cohen-Kurath calculation [4]^{†2}.

In the conventional FFS approach, the one-nucleon GF enters explicitly into the theory of ph excitations. Until recently, only one pole has been assumed for the GF, with the resultant matrix equations identical to those of the RPA [9]. However, the more general approach [1], to be followed here, includes the effect of the single-particle strength distributed among many states, both in the $A + 1$ and in the $A - 1$ nuclei. The details of this approach are given elsewhere [1].

^{†1} Hereafter, unless qualified otherwise, we shall mean by RPA the "closed-shell" RPA only.

^{†2} These could be also extracted directly from one-nucleon transfer reaction data. For facility of comparison with the shell-model calculation, we use the shell-model results to idealize the experiment.

The quasi-particle interaction F , defined below in Eq. (1), is a key element of the Migdal approach; it includes renormalizations due to neglected small residues in the GF, as well as irreducible np-nh contributions to the ph interaction and possibly the influence of the configuration space truncation. The transition operators, in the Migdal approach, are renormalized by introducing suitably defined quasi-particle 'charges' [2].

For the $T = 1$ ph interaction, we take the density-dependent "universal" interaction of Migdal [2]:

$$F = \frac{d\epsilon_F}{dn} \delta(\vec{\pi}_1 - \vec{\pi}_2) \vec{\tau}_1 \cdot \vec{\tau}_2 [f' + g' \vec{\sigma}_1 \cdot \vec{\sigma}_2], \quad (1)$$

the density-dependence entering through the f' term:

$$f' = f'_{ex} + (f'_{in} - f'_{ex}) \frac{\rho(\vec{r})}{\rho(0)}; \quad (2)$$

we take $d\epsilon_F/dn = 385 \text{ MeV fm}^3$ (n being the *nuclear matter* density) $f'_{ex} = 0$, $f'_{in} = 0.67$, $g' = 0.5$. Our values of g' and f'_{ex} are taken from Migdal [2]. The values of f'_{ex} and f'_{in} roughly agree with the nuclear symmetry considerations [2], and with the values employed in a calculation of the quadrupole moments [10]. The single-particle wave functions and the density $\rho(r)$ are taken from a Woods-Saxon calculation.

We now focus on selected items from our calculation to demonstrate how the open-shell nature of $^{12}\text{C}_{gs}$ and the consequent appearance of several poles in the single-nucleon GF's affect the usual ph excited states. We shall discuss the quality of the low-energy level spectrum, the natures of the $T = 1$, 1^- states, and those of the $T = 1$, 1^+ states. In the last item, we also examine the test of Kurath's isovector M1 sum rule.

(1) The quality of the calculated spectrum: This is indicated in Table 1 for low-lying ph states. For comparison, we have included results of experimental excitation energies wherever available [11] and the results of several theoretical calculations [5,12]. Notice the over-all good quality of the calculated spectrum, with the special success of the open-shell extended basis. The present calculation, in particular, reproduces the well known 15.11 MeV level.

(2) The nature of the $T = 1$, 1^- states: The $T = 1$, 1^- states have been previously studied by Rowe and Wong [12], and by Skupsky [5]. The present calculation reproduces the essential features of these other open-shell approaches. The main

point is that, relative to the RPA, the E1 strength is fragmented towards higher energies. This reflects primarily the inclusion of configurations like $d_{3/2}^{-1} p_{1/2}^{-1}$, excluded in the RPA by the closed-shell assumption that the $p_{1/2}$ orbit is "empty".

(3) The nature of the $T = 1, 1^+$ states: The lowest $1^+, T = 1$ state at 15.11 MeV in ^{12}C and its analogues in ^{12}B and ^{12}N have been extensively studied by electromagnetic and weak interactions [6,7]. These reactions provide sensitive tests for the configuration mixing in the nuclear wave functions. It is well known (see, for example, Refs. [6,7]) that the TDA fails to reproduce the experimentally observed rates of reactions $^{12}\text{B}_{\text{gs}} \xrightarrow{\beta^-} ^{12}\text{C}_{\text{gs}}$ by a factor of about five.

In Table 2, we display the matrix elements of the tensor operators relevant to the allowed muon capture reaction $^{12}\text{C}_{\text{gs}}(\mu^-, \nu_\mu) ^{12}\text{B}_{\text{gs}}$ [6], as obtained in the TDA, Cohen-Kurath model [4] and in the present FFS approach with two different values of g' , $g' = 0.5$ being favoured by Migdal's β -decay fits [2,9]. The $g' = 0.5$ Migdal estimate of the Gamow-Teller matrix element is seen to be remarkably close to the shell-model value. The calculated $^{12}\text{B}_{\text{gs}} \xrightarrow{\beta^-} ^{12}\text{C}_{\text{gs}}$ ft value and the rate of the inverse muon capture reaction are 1.26×10^4 sec and 6.3×10^3 sec $^{-1}$, compared with the experimental values of $(1.1789 \pm 0.0051) \times 10^4$ sec [14] and $(6.75 \pm \pm 0.3) \times 10^3$ sec $^{-1}$ [15] respectively. In these calculations, we have assumed the axial vector form factor at zero momentum transfer to be 1.25 [16], and, for the muon capture calculation, we have taken all other weak hadron form factors at their "canonical" values [6,17]. For the latter calculation, oscillator radial wave functions are used with the oscillator parameter equal to 1.64 fm [6]. The striking success of the Migdal calculation here is primarily due to the trimming of the dominant p amplitude of the $p_{1/2}^{-1} p_{3/2}^{-1}$ configuration from ~ 1 , in the RPA, to 0.59, in the present approach.

(4) Distribution of the isovector M1 strength: As a further test of the present calculation, we examine the distribution of the isovector M1 strength in the $T = 1, 1^+$ states. The present calculation substantially overestimates the M1 strength in the higher excited $T = 1, 1^+$ states. Thus, taking the Kurath sum-rule [8]

$$\sum_S |\langle S | \sum_{i=1}^A \vec{\sigma}_i \cdot \vec{\tau}_i^z | 0 \rangle|^2 (E_S - E_0) = \frac{1}{2} \langle 0 | [\sum_{i=1}^A \vec{\sigma}_i \cdot \vec{\tau}_i^z, [H, \sum_{i=1}^A \vec{\sigma}_i \cdot \vec{\tau}_i^z]] | 0 \rangle, \quad (3)$$

with H being the nuclear Hamiltonian, our calculation yields the left-hand side that exceeds the Cohen-Kurath result [18] by a factor of five. In the latter, almost all of the isovector M1 strength is concentrated in the lowest $T = 1, 1^+$

^{†3} We have used here the quasi-particle charge $e(\vec{\sigma})$ equal to *unity*, rather than Migdal's prescribed value, 0.90 [2,9]. The latter yields values of β -decay and muon capture rates *lower* than the experimental values in the present case. We may note in passing that shell-model calculations around mass ~ 16 indicate quasi-particle charge $e(\vec{\sigma})$ to be ~ 0.85 , due to the truncation of the Hilbert space [13].

state. Our calculation thus fails to describe the excited states $|s\rangle$ in Eq. (3), yielding thereby a total allowed muon capture strength inconsistent with the result of a careful decomposition of the observed total muon capture rate [17,19]

In understanding the origin of this difficulty, we consider the graphs of Fig. 1, all of which can be important in the calculation of (ph)-states in open-shell nuclei. Figure 1a is automatically included in the present work by using one-nucleon GF's, whose poles and residues reflect the open-shell nature of the $^{12}\text{C}_{\text{gs}}$. Since these have been taken directly from the Cohen-Kurath shell model [4], Fig. 1a is clearly not responsible for the above discrepancy. Figure 1b, the usual "backwards" RPA graph is also included in the present approach. However, in open-shell nuclei [specially in the p and (sd) shells], many (3p-3h) configurations implicit in Fig. 1b violate the Pauli exclusion principle, and the exchange graph, in Fig. 1c, emerges as essential. In principle, this class of graph gives rise to an *energy-dependent* contribution to the effective (ph) interaction. However, such an energy dependence is not within the spirit of simplicity of the Migdal approach; indeed, the present normalization of the (ph) amplitudes {Eq. (66) of Ref. [1]} requires an energy-independent interaction. We believe that the overestimation of the energy-weighted sum in Eq. (3) is a manifestation of this failure to include the effects of Fig. 1c, particularly for the normalization of the (ph) amplitudes.

It is also worth noting that Rowe [2] has shown that any level of approximation in the equations of motion formalism obeys the energy-weighted sum rule. However, the proof of this point includes the assumption of quasi-boson commutation relations [20] -- which is equivalent to the neglect of Fig. 1c.

In conclusion, the open-shell extension of the Migdal approach seems to give a *reasonable description* of the low-energy spectrum of the $A = 12$ system, including the weak and electromagnetic transition moments connecting $^{12}\text{C}_{\text{gs}}$, $^{12}\text{B}_{\text{gs}}$ and its analogue states. An inherent difficulty of the present approach is its *failure* to satisfy the energy-weighted isovector M1 sum rule. This implies, in the context of muon capture problem, an overestimation of the allowed muon capture strength. This is interesting in view of the claims of the Russian group [21] that the Migdal approach systematically reproduces the experimental *total* capture rate throughout the periodic table.

One of us (NCM) has the pleasure of thanking his several colleagues at CERN, particularly J. Bernabeu, J. Blomqvist, R. Eramzhyan and T.E.O. Ericson, for many useful discussions and their critical readings of the manuscript. He is grateful to the CERN Theoretical Studies Division for its gracious hospitality.

REFERENCES

- [1] J.D. Immele and G.L. Struble, Phys. Rev. C9, 460 (1974). Section II-C of this reference details the formulation for the excited states of even-A nuclei.
- [2] See, for example,
A.B. Migdal, Nuclear Theory: The quasi-particle method (W.A. Benjamin, New York, 1968);
G.E. Brown, Rev. Mod. Phys. 43, 1 (1971).
- [3] S.S.M. Wong, D.J. Rowe and J.C. Parikh, Phys. Letters 48 B, 403 (1974), and references therein;
O. Nalcioğlu, D.J. Rowe and C. Ngo-Trong, Nuclear Phys. A218, 495 (1974).
- [4] S. Cohen and D. Kurath, Nuclear Phys. 73, 1 (1965); *ibid* A101, 1 (1967).
- [5] S. Skupsky, Nuclear Phys. A178 289 (1971).
- [6] N.C. Mukhopadhyay and M.H. Macfarlane, Phys. Rev. Letters 27, 1823 (1971); *ibid* 28, 1545E (1972).
- [7] N.C. Mukhopadhyay, Phys. Rev. C7, 1720 (1973).
- [8] D. Kurath, Phys. Rev. 130, 1525 (1963).
- [9] See, for example, M. Rho, Phys. Rev. 161, 955 (1967).
- [10] P. Ring, R. Bauer and J. Speth, Nuclear Phys. A206, 97 (1973).
- [11] F. Ajzenberg-Selove and T. Lauritsen, Nuclear Phys. A114, 1 (1968).
- [12] V. Gillet and N. Vinh-Mau, Nuclear Phys. 54, 321 (1964);
D.J. Rowe and S.S. M. Wong, *ibid.* A153, 561 (1970).
- [13] See D.H. Wilkinson, Nuclear Phys. A225, 365 (1974).
- [14] R.E. McDonald, J.A. Becker, R.A. Chalmers and D.H. Wilkinson, Phys. Rev. C10, 333 (1974).
- [15] R.T. Maier, R.M. Edelstein and R.T. Siegel, Phys. Rev. 133, B663 (1964);
Yu.G. Budyashov et al., Soviet Phys. JETP 31, 651 (1970).
- [16] Particle Data Group, Phys. Letters 50 B, 1 (1974).
- [17] N.C. Mukhopadhyay, Nuclear muon capture, to be published in Phys. Reports.
- [18] N.C. Mukhopadhyay, Phys. Letters 45 B, 309 (1973);
N.C. Mukhopadhyay and F. Cannata, *ibid.* 51 B, 225 (1974).
- [19] J. Bernabeu and F. Cannata, Nuclear Phys. A215, 424 (1973).
- [20] D.J. Rowe, Rev. Mod. Phys. 40, 153 (1968).
- [21] See, for example, G.G. Bunatyan, Soviet J. Nuclear Phys. 3, 613 (1966).

Table 1

Calculated energies (*in MeV*) of low-lying $T = 1$ levels in ^{12}C . RPA and OS-p represent RPA and open-shell calculation, *in the lp-orbit basis*, using the Migdal interaction. OS-ex denotes an open-shell Migdal calculation in an *extended basis* of 1s, lp (2s-1d) and (2p-1f) shell orbits. Other columns of the table represent the following: E, experimental results [11], GV, R and S, calculated values of Gillet and Vinh-Mau [12], Rowe and Wong [12] and Skupsky [5] respectively.

$J^\pi T$	E	RPA	OS-p	OS-ex	GV	R	S
0^+1	-	-	15.92	15.90	-	-	-
1^+1	15.11	17.61	15.10	15.05	16.6	-	-
2^+1	16.11	17.75	15.36	15.22	16.3	-	-
0^-1	-	-	-	19.07	24.8	17.51	19.9
1^-1	17.23	-	-	17.66	17.7	17.49	18.6
2^-1	16.58	-	-	17.77	18.2	17.18	17.9
3^-1	20.64	-	-	17.99	18.4	19.57	-

Table 2

Matrix elements (defined in Mukhopadhyay [6]) $[101]$, $[121]$, $[111p]$ and $[011p]_{A,B}$, relevant for the reaction $^{12}\text{C}_{gs}(\mu^-, \nu_\mu)^{12}\text{B}_{gs}$, calculated in the Tamm-Dancoff approximation (TDA), Cohen-Kurath shell-model (CKM), and the Migdal model with the lp-orbit basis using (A) $g' = 0.725$ and (B) $g' = 0.5$. Notice the agreement between the CKM and the Migdal case B for the Gamow-Teller matrix element $[101]$, which overwhelmingly dominates the muon capture reaction.

	$[101]$	$[121]$	$[111p]$	$[011p]_A$	$[011p]_B$
TDA	-0.18379	0.064974	-0.037513	-0.091889	0
CKM	-0.07694	0.03303	2.1546×10^{-4}	-0.06537	0.02415
A	-0.06316	0.022622	-0.034887	-0.024502	0.013085
B	-0.07722	0.030826	-0.03168	-0.049697	0.009428

Figure caption

Fig. 1 : Graph (a) is included in the present calculation via the residues and poles of the one-nucleon Green's function. Graph (b) is the usual backward going RPA graph, also included in our formalism. Graph (c), obtained by exchanging lines in (b), is not included, as our interaction between (ph) pairs is independent of energy.

(a)

(b)

(c)

Fig. 1