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SELF-CONSISTENT ρ -P' TRAJECTORY FROM THE PLANAR
DUAL BOOTSTRAP *)

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A B S T R A C T

An approximate form of a recently proposed planar bootstrap equation leads to a non-linear integral equation for the ρ -P' exchange degenerate trajectory and its coupling. Both turn out in good agreement with experimental values and with previous theoretical estimates.

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Some interest has been recently focused on the possibility of computing strong interaction parameters on the basis of planar duality and a particular unitarization scheme ¹⁾⁻²⁾. In the approach of Ref. 1) one divides the full unitarization program into two parts : the first is the planar bootstrap of ordinary Regge poles, the second an expansion in the (small) parameter $1/N^2$, which adds perturbatively a bare Pomeron and its interaction with the Reggeon and with itself.

Some qualitative and semi-quantitative results of the planar bootstrap have recently been obtained ³⁾, whereas the Pomeron problem, already studied at lowest order sometime ago ⁴⁾, is presently under further investigation. The recent work of Chan et al. ²⁾ is more phenomenological in spirit, but still it is based on the same general physical ideas. In that direction too preliminary results are very encouraging ⁵⁾.

In this investigation, by going a little further in the direction of Ref. 3), we are able to obtain, under some reasonable assumption, a non-linear integral equation for the exchange degenerate ρ -P' trajectory $\alpha(t)$. In the region $-0.7 \lesssim \alpha't \lesssim 0$, we show that a linear trajectory of the type $\alpha(t) = 0.6 + \alpha't$, when used in the equation, reproduces itself back within less than 0.05. We also obtain a very reasonable value for the coupling constant. Both results fully confirm the general arguments of Ref. 3).

The equation we employ is a non-forward discontinuity equation discussed in Ref. 3) [Eq. (4), to be precise], which, after trivial simplifications, reads :

$$\frac{1}{\Gamma(1+\alpha(t))} = \frac{\alpha' N}{16\pi^2} \sum_{R_1, R_2} \int d^2 K_\perp \frac{G_{R_1 R_2}^2(t, t_1, t_2) \Gamma(-\alpha_{R_1}(t_1)) \Gamma(-\alpha_{R_2}(t_2)) \cos \pi(\alpha_{R_1} - \alpha_{R_2})}{\Gamma^2(2+\alpha(t) - \alpha_{R_1}(t_1) - \alpha_{R_2}(t_2)) (1+\alpha(t) - \alpha_{R_1}(t_1) - \alpha_{R_2}(t_2))^2} \quad (1)$$

We recall that here $\alpha(t)$ is the leading (exchange degenerate) Regge trajectory (ρ -P') and that $G_{R_1 R_2}(t, t_1, t_2) \cdot \Gamma^{-1}[2+\alpha(t) - \alpha_{R_1}(t_1) - \alpha_{R_2}(t_2)]$ is the triple Regge coupling of such trajectory to two arbitrary Reggeons R_1 and R_2 . Finally $d^2 K_\perp = \int dt_1 dt_2 [-\lambda(t, t_1, t_2)]^{\frac{1}{2}} \delta(-\lambda)$ as usual. Equation (1) can be derived quite rigorously within the planar dual bootstrap. However, we shall now make two simplifying assumptions. The first one, which we hope to be valid in some region of t around $t \lesssim 0$, is to keep only the leading

trajectory contribution to the sum over R_1, R_2 . The second assumption is to replace the triple Regge coupling G by its expression in terms of $\alpha(t_1)$, $\alpha(t_2)$, $\alpha(t)$ obtained in the Born approximation. Since the Born approximation gives a non-trivial vertex satisfying all analyticity and crossing requirements ⁶⁾ and exhibits the nonsense zeros necessary in a Regge theory with no fixed poles (we are planar !), we think that G could be at most multiplied by a smooth function of the t_i , which would not affect our results.

Finally we avoid the problem of the tachyon in the ordinary dual model with $\alpha(0) > 0$, by considering the Neveu-Schwarz model which has no tachyon on the ρ -P' trajectory. In conclusion, in Eq. (1), we eliminate the $\sum_{R_1 R_2}$, we set

$$G(t, t_1, t_2) = g (1 + \alpha(t) - \alpha(t_1) - \alpha(t_2)) \quad (2)$$

where g is the coupling to the external particles, and we also make the replacements

$$\begin{aligned} \Gamma^{-1}(1 + \alpha(t_1)) &\longrightarrow \Gamma^{-1}(\alpha(t_1)) \\ \Gamma(-\alpha(t_{1,2})) &\longrightarrow \Gamma(1 - \alpha(t_{1,2})) \end{aligned} \quad (3)$$

We finally obtain the final form of our equation :

$$\begin{aligned} \Gamma^{-1}(\alpha(t)) &= \\ &= \frac{g^2 \alpha' N}{16 \pi^2} \int \frac{dt_1 dt_2 \Theta(-\lambda)}{[-\lambda(t, t_1, t_2)]^{1/2}} \frac{\Gamma(1 - \alpha(t_1)) \Gamma(1 - \alpha(t_2))}{\Gamma^2(1 + \alpha(t) - \alpha(t_1) - \alpha(t_2))} \cos \pi(\alpha(t_1) - \alpha(t_2)). \end{aligned} \quad (4)$$

We have thus obtained a non-linear constraint involving only $\alpha(t)$ and a parameter $\tilde{g}^2 = g^2 N / 16 \pi^2$ which is presumably determined as in an eigenvalue problem. In spite of its simple form, Eq. (4) is terribly non-linear and we know of no general method to solve it. We shall be guided by physics in looking for a solution of an approximately linear form, and we start by making the following observation : the l.h.s. of Eq. (4) has zeros at nonsense value, in particular at $t = t_0$ with $\alpha(t_0) = 0$. On the other hand

the r.h.s. is positive definite except for the "signature factor"
 $\xi = \cos \pi [\alpha(t_1) - \alpha(t_2)] \approx \cos \pi \alpha'(t_1 - t_2)$. In order to have a zero we need ξ to change sign at least once in the important t_1, t_2 region which, because of an exponential cut-off in $t_1 + t_2$, is $|\alpha'(t_1 + t_2)| \lesssim 5$. Due to the integration region in $|t_1 - t_2|$, we see that this cannot happen unless $\alpha't_0 \gtrsim 0.5$, hence we should get $\alpha(0) \approx 0.5$.

Our numerical calculation proceeds as follows : we start with an input trajectory $\alpha_0(t) = \alpha(0) + \alpha't$ inserted in the r.h.s. of Eq. (4). The result of the integration is then represented as $\text{const} \times \Gamma^{-1}[\alpha_1(t)]$ and we demand $\alpha_1(t)$ to be as equal as possible to $\alpha_0(t)$ up to a best choice of the const. We find indeed that the r.h.s. has a zero at $\alpha't = -0.6 \pm 0.03$ for any input $0 < \alpha(0) < 1$. Thus, if we want a fairly linear $\alpha_1(t) \approx \alpha_0(t)$, we need $\alpha_1(0) \approx \alpha(0) \approx 0.6$. This, of course, fixes the value of the constant (or of $\tilde{g}^2$). If we demand $\alpha_1(t)$ and $\alpha_0(t)$ to agree at $t = 0$ and at $\alpha(t) = 0$ we find the solution plotted in the Figure. If we demand instead a minimal deviation of $\alpha_1(t)$ from $\alpha_0(t)$ we get a slightly higher intercept [$\alpha(0) \approx 0.66$, dotted line in the Figure] and the deviation is always less than 0.05 in $-0.7 \leq \alpha't \leq 0$. In any case we find the consistency for $|\alpha't| \leq 0.7$ quite remarkable, given the approximations. More than one trajectory is presumably needed beyond 0.7. A more precise iterative solution of Eq. (4) is being worked out, but we believe it cannot change the above conclusions significantly.

The other interesting prediction concerns the value of $\tilde{g}^2$. The numerical calculation gives

$$\tilde{g}^2 \equiv \frac{g^2 N}{16\pi^2} \approx .17 \pm .03 \quad (5)$$

This gives $g^2 N / 16\pi \approx 0.5$ in agreement with the estimate of Ref. 3) and with experiments. We also expect from a simple Chew-Pignotti-type argument to have $1 - \alpha(0) \approx g^2 N / 16\pi$ and such relation is indeed satisfied by our values.

In conclusion we have found that, provided the Reggeon intercept is shifted from the bare value one to about 0.6 and the coupling is of order one, the dual Born term is amazingly consistent with our planar unitarity constraints. This is certainly an encouraging result for our ambitious program.

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FIGURE CAPTION

Comparison between input and output trajectory from Eq. (4). $\alpha_0(t)$ is the input linear trajectory $(0.6+\alpha't)$; the two output $\alpha_1(t)$ correspond to slightly different values of g , the solid line giving exact agreement at $t = 0$, the dotted line minimizing $|\alpha_1(t)-\alpha_0(t)|$ in the interval $|\alpha't| < 0.7$.