Baryogenesis in the MSSM, nMSSM and NMSSM

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We compare electroweak baryogenesis in the MSSM, nMSSM and NMSSM. We comment on the different sources of CP violation, the phase transition and constraints from EDM measurements.

1. Introduction to Electroweak Baryogenesis

A viable baryogenesis mechanism aims to explain the observed asymmetry in the baryon density, \( \eta = \frac{n_B - n_{\bar{B}}}{n_B + n_{\bar{B}}} \approx 8.7(3) \times 10^{-11} \), and the celebrated Sakharov conditions state the necessary ingredients for baryogenesis: (i) C and CP violation, (ii) non-equilibrium, (iii) B number violation.

B number violation is present in the hot Universe due to sphaleron processes while C is violated in the electroweak sector of the Standard Model (SM). The two important aspects of electroweak baryogenesis (EWBG) are transport and CP violation. EWBG requires a strong first-order electroweak phase transition to drive the plasma out of equilibrium. The CP violation is induced by the moving phase boundary. Hence it is important to derive transport equations that contain CP-violating quantum effects in a genuine manner.

Compared to other baryogenesis mechanisms, EWBG has the attractive property that the relevant energy scale will be accessible by the next generation of collider experiments.

2. Transport equations derived from the Kadanoff-Baym equations

The Kadanoff-Baym equations represent the statistical analog to the Schwinger-Dyson equations and are of the following form:

\[
(k + \frac{i}{2} \partial \theta - P_L m e^{-\frac{i}{2} \vec{\theta} \cdot \partial k} - P_R m^\dagger e^{-\frac{i}{2} \vec{\theta} \cdot \partial k}) g^<(k, X) = \text{collision terms},
\]  

(1)

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and all quantities are functions of the center of mass coordinate \( X_\mu \) and the momentum \( k_\mu \). In the approximation of planar wall profiles, the spin is conserved what can be used to partially decouple the equations.

In the Kadanoff-Baym equations, the exponential derivative operator is usually expanded which corresponds to a semi-classical expansion in \( \hbar \), the so-called gradient expansion. In the following we will keep the first two orders in \( \hbar \).

The simplest example of CP violation in transport equations is given by the one-flavour case with a \( z \)-dependent complex phase in the mass term [1], \( m(z) = |m(z)| \times e^{i\theta(z)} \). In this case the constraint equation (real part of the K.-B. equation) leads to the following Ansatz (the subscript \( s \) is the spin quantum number)

\[
g^\L_s = 2\pi f_s \delta(k_0 - \omega_s), \quad \omega_0 = \sqrt{k_z^2 + k_\parallel^2 + |m|^2}, \quad \omega_s = \omega_0 - s \frac{|m|^2\theta'}{2\omega_0 \sqrt{\omega_0^2 - k_\parallel^2}},
\]

where \( f_s \) denotes the distribution function and \( \omega_s \) the energy of the particle in the presence of the wall. The kinetic equation (imaginary part of the K.-B. equation) is of the Vlasov type

\[
\frac{k_z}{\omega_s} \partial_z f_s + F_s \partial_k z f_s = \text{collision terms}, \quad F_s = -\frac{|m|^2}{2\omega_s} + s \frac{(|m|^2\theta')'}{2\omega_0 \sqrt{\omega_0^2 - k_\parallel^2}}.
\]

Note that the second part of the force \( F_s \) violates CP and hence sources EWBG.

The multi-flavour case can be treated in the linear response approximation, where the Green function is split according to \( g^\L = g^{\text{eq.}} + \delta g \) and leads to a kinetic equation of the form [2] (without using the in general nonalgebraic constraint Eqs. that reproduce the dispersion relations in lowest order)

\[
k_z \partial_z \delta g + i \frac{\hbar}{2} [m^2, \delta g] + k_0 \Gamma \delta g = S(g^{\text{eq.}}).
\]

The third term is a damping term, taking into account the collision terms. Note that the second term will lead to an oscillation of the off-diagonal densities in the mass eigenbasis, similar to neutrino oscillations. The right-hand side of this equation contains contributions of higher order in the gradient expansion that give rise to CP violation and EWBG.

3. EWBG in the MSSM

In the MSSM the dominant contribution to baryogenesis comes from the charginos (Higgsino - Wino - mixing) with the mass matrix

\[
\begin{pmatrix}
\psi_R^+ \\
\psi_L \\
\end{pmatrix} = \begin{pmatrix}
\tilde{W}_R^L \\
\tilde{h}_{1,R} \\
\end{pmatrix}, \quad m(z) = \begin{pmatrix}
M_2 \\
H_1^*(z) \\
\frac{g}{\mu_c} H_2^*(z) \\
\end{pmatrix},
\]

where the SUSY breaking parameters \( M_2 \) and \( \mu_c \) contain complex phases.

The CP-violating sources to first order in gradients only contribute to the off-diagonal terms in flavour space and hence are suppressed by the oscillation effect. They read [3]

\[
S^\mu_\mu = 2g^2 T_- \Im(M_2 \mu_c) (|M_2|^2 - |\mu_c|^2) \partial_\mu (H_1 H_2) \eta_0^3,
S^\mu_\mu = 2g^2 T_- \Im(M_2 \mu_c) (H_1^* - H_2^*) \partial_\mu (H_1 H_2) \eta_0^3,
S^\mu_\mu = -2g^2 T_- \Im(M_2 \mu_c) (H_2 \partial_\mu H_1 - H_1 \partial_\mu H_2) (\eta_0^3 + 4\eta_2^3).
\]
Figure 1. The left plot shows the produced BAU in the MSSM for $M_2 = 200$ GeV. In the right plot, the black area denotes the region in the $(\mu_c, M_2)$ plane, where EWBG is viable.

In addition, there is a CP-violating source of second order in the gradient expansion that contributes to the diagonal elements in flavour space and corresponds to the semi-classical force that appears in the one flavour case in Eq. (3)

$$S^d_0 = 2 v_w g^2 T^{-4}_c \Im(M_2 \mu_c) (H_2 \partial^2 \overline{z} H_1 + H_1 \partial^2 \overline{z} H_2) \zeta^3_{(0)}.$$  \hspace{1cm} (6)

The functions $\eta$ and $\zeta$ denote certain momentum integrals of the equilibrium distribution functions. Fig. 1 shows the produced baryon asymmetry for a maximal CP-violating phase in the chargino mass matrix using the system of diffusion equations suggested in Ref. [5]. In the right plot, the black area denotes the region of the parameter space where EWBG is viable. We conclude that EWBG in the MSSM is only possible if: (i) The charginos are nearly mass degenerate such that mixing effects are not suppressed. (ii) The CP phases in the chargino sector are $O(1)$.

4. EWBG in singlet extensions of the MSSM

The general NMSSM of Ref. [7] consists of the MSSM extended by a gauge singlet and the superpotential

$$W_{NMSSM} = \lambda S H_1 H_2 + \frac{k}{3} S^3 + \mu H_1 H_2 + r S + W_{MSSM}.$$ \hspace{1cm} (7)

Due to the explicit $\mu$ term and the singlet self-couplings, this model provides a rich Higgs phenomenology; however, additional assumptions have to be made to prevent higher-dimensional operators from destabilizing the hierarchy. It does not suffer from a domain wall problem since there are no discrete symmetries.

In the nMSSM, a $Z_5$ or $Z_7$ symmetry is imposed to solve the domain wall problem without destabilizing the electroweak hierarchy. The $\mu$ term is forbidden and only induced after electroweak symmetry breaking. Thus the $\mu$ problem is solved. The discrete symmetries also eliminate the singlet self coupling. A rather large value of lambda is needed in the nMSSM to fulfill current mass bounds on the Higgsinos and charginos, which might lead to a Landau pole below the GUT scale.
4.1. Electroweak phase transition

In contrast to the MSSM, no light stop is needed in the NMSSM or nMSSM, since the additional singlet terms in the Higgs potential strengthen the phase transition \[7\]. In the nMSSM case these terms read:

\[
L = L_{MSSM} + m_s^2 |S|^2 + \lambda^2 |S|^2 (H_1^\dagger H_1 + H_2^\dagger H_2) + t_s (S + h.c.) + (a_\lambda S H_1 \cdot H_2 + h.c.).
\]

(8)

In a simplified scheme without CP violation, a first-order phase transition due to tree-level dynamics occurs if \[8\]

\[
m_s^2 < \frac{1}{\lambda} \left| \frac{\lambda^2 t_s}{m_s} - m_s \tilde{a} \right|, \quad \tilde{a} = a_\lambda \sin 2\beta, \quad \tilde{\lambda}^2 = \frac{\lambda^2}{4} \sin^2 2\beta + \frac{\tilde{g}^2}{8} \cos^2 2\beta.
\]

(9)

Fig. 2 displays Eq. (9) for random nMSSM models with a strong PT and shows that this criterion is also decisive if CP violation and the one-loop effective potential are taken into account.

4.2. EDM constraints and baryon asymmetry

Since the trilinear term in the superpotential contributes to the Higgs mass, \(\tan(\beta)\) is generically of \(O(1)\); Hence two-loop contributions from the charginos to the electron EDM are naturally small. The one-loop contributions to the electron EDM can, as in the MSSM, be reduced by increasing the sfermion masses.

The effective \(\mu\) parameter is dynamical in the nMSSM and NMSSM, and hence its complex phase can change during the phase transition. This leads to new CP-violating sources in the chargino sector that are of second order in the gradient expansion and do not rely on mixing. Thus, these contributions are not suppressed by the flavour oscillations and mass degenerate charginos are not required for viable EWBG.

Additionally, the bubble wall tends to be thinner than in the MSSM and hence it is rather generic to generate the observed baryon asymmetry \[9\]. Fig. 3 shows the binned BAU for a random set of nMSSM models with a strong first order PT.
Another interesting feature of the NMSSM is transitional CP violation. If universality is violated in the singlet sector of the NMSSM, the phase transition can connect a high temperature phase with broken CP and a low temperature phase of conserved CP \cite{7}. Such CP violation cannot be detected by zero temperature experiments! The singlet self coupling is important to stabilize the singlet in this scenario.

5. Conclusions

Singlet extensions of the MSSM provide a framework in which electroweak baryogenesis seems to be possible without fine tuning.

REFERENCES