Time Evolution of Jets and Perturbative Color Neutralization

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In-medium production of leading hadrons in hard reactions, carrying the main fraction of the jet momentum, involves two stages: (i) the parton originated from the hard process propagates through the medium radiating gluons due to the initial hard collision, as well as to multiple interactions in the medium; (ii) perturbative color neutralization, e.g. picking up an anti-colored parton produced perturbatively, followed by evolution and attenuation of the (pre)hadron in the medium. The color neutralization (or production) length for leading hadrons is controlled by coherence, energy conservation and Sudakov suppression. The $p_T$-broadening is a sensitive and model independent probe for the production length. The color neutralization time is expected to shrink with rising hard scale. In particular, we found a very fast energy dissipation by a highly virtual parton: half of the jet energy is radiated during the first Fermi. Energy conservation makes the production of leading hadrons at longer times difficult.

1. Introduction

Modification of jets produced in a medium is currently a hot topic. Data for heavy ion collisions at RHIC \cite{1,2} show that hadrons produced with high $p_T$ in central gold-gold collisions at $\sqrt{s} = 200$ GeV are five times suppressed compared to pp collisions. This observation is an evidence for a dense medium created in heavy ion collisions. However, to be confident of the medium properties resulting from the data analysis one has to employ a reliable theoretical description of jet development in the medium. Unfortunately, this is not the case so far. The current theoretical models relating the observed jet quenching to medium-induced energy loss are based on unjustified ad hoc assumptions: (i) the high-$p_T$ parton initiating the jet always keeps hadronizing and radiating gluons outwards the medium; (ii) the effect of the medium can be reproduced by a simple shift in the argument of the fragmentation function. In this paper both assumptions are challenged.

It is clear that no reliable information can be extracted from data, if the analysis is based on a theoretical model which has never been tested. This hardly can be done in heavy ion collisions in view of too many uncertainties. The density and space-time development of the medium are unknown and model dependent, and the jet energy is also unknown.
A process which looks similar, but which has much less uncertainties, is hadron production in DIS on nuclei, as is illustrated in Fig. 1.

![Figure 1](image1.png)

**Figure 1.** *Left:* high $p_T$ hadron production in a heavy ion collision. The parton propagates through the created medium which modifies its hadronization. *Right:* leading hadron production in DIS on a nucleus. The nuclear density and the kinematics of the reaction are under control.

![Figure 2](image2.png)

**Figure 2.** Two-step process of leading hadron production. On the production length $l_p$ the quark is hadronizing experiencing multiple interactions broadening its transverse momentum and inducing an extra energy loss. Eventually the quark color is neutralized by picking up an antiquark. The produced color dipole (pre-hadron) is attenuating in the medium and developing the hadron wave function over the formation path length $l_f$.

In this case the density and geometry of the medium, the kinematics are known. Moreover, the parton virtuality $Q^2$ and its energy $\nu$ are not correlated allowing diverse tests.

2. Jet quenching in DIS

In this case the fraction $z_h = E_h/E_q$ of the jet energy taken by the hadron is measured, and we are interested in leading hadron production $z_h \gtrsim 0.5$. The space-time development of the hadronization process which ends up with production of the leading hadron should have a two-step structure as is illustrated in Fig. 2 [6].

Since the produced pre-hadron strongly (exponentially) attenuates in the nuclear medium, the position of the color neutralization point is crucial for the resulting nuclear suppression. In the energy loss scenario the color neutralization is assumed to happen always outside the nucleus, $l_p > R_A$, so that the pre-hadron has no chance to interact.

However, if a large fraction, $z_h \rightarrow 1$, of the initial quark energy is taken by the produced hadron, the process of color neutralization cannot last long because of energy conservation. Indeed, while the quark is propagating and radiating gluons either in vacuum, or in a medium, it keeps losing energy and once the quark energy comes below than $z_h E_q$, there is no chance any more to produce a hadron with fractional energy $z_h$. Thus, energy conservation imposes a restriction on the color neutralization time [3],

$$l_p \leq \frac{E_q}{\langle dE/dz \rangle} (1 - z_h) ,$$

(1)
which must vanish at $z_h \to 1$.

All the consideration hereafter is held in the rest frame of the target. We use here the mean rate of energy loss per unit of length, $z$, although its time dependence and fluctuations may be important and should be taken into account [4]. In the simplest case of the string model the rate of energy loss is just the string tension, $-dE/dz \approx 1 GeV/fm$ [3]. In a hard reaction the rate can be much higher, since a highly virtual parton radiates gluons intensively. Integrating over the radiation spectrum one arrives at a time independent rate [5] which rises quadratically with the hard scale [6,4],

$$-\frac{dE}{dz} \approx \frac{2\alpha_s}{3\pi}Q^2. \quad (2)$$

Actually, at large $z_h$ one should also impose energy conservation restrictions on these calculations, as well as introduce a Sudakov factor which emerges due to this restriction. As a result, Eq. (2) holds only over the distance $\Delta z < z_1 = 2(1 - z_h)E_q/Q^2$ after the DIS, and then slows down [6,4].

Thus, the color neutralization time is controlled by the jet energy, virtuality and fractional energy of the detected hadron as,

$$\langle l_p \rangle \propto \frac{E_q}{Q^2}(1 - z_h). \quad (3)$$

3. Perturbative color neutralization

It is widely believed that hadronization and color neutralization are soft processes with a long duration time controlled by the soft scale. This is correct, as far as it concerns the formation of the final hadron wave function,

$$l_f \sim \frac{E_h}{\Lambda_{QCD}^2}. \quad (4)$$

This seems to be in contradiction with the above estimates for color neutralization time limited by energy conservation, but this is just the consequence caused by mixing up two different time scale. The former, the production time or length, $l_p$, is the distance covered by the hadronizing parton until its color is neutralized by another (anti)parton. The colorless system produced is not a hadron yet, since it has no wave function. For instance, it might be a $\bar{q}q$ dipole with a definite separation, which even has no definite mass and can be projected to wave functions of different hadrons. It still needs a rather long time called formation time, Eq. (4), to form the hadron.

The early color neutralization suggested by Eq. (3) is a perturbative process. It has been also known in QED, for instance, in the production of antihydrogen found at CERN, which was predicted in [7] as a perturbative neutralization of the antiproton charge, as is illustrated in Fig. 3. Color neutralization in QCD looks alike [8,6,4] and leads to the creation of a pre-hadron, as is shown in Fig. 3.

Early perturbative color neutralization is an essential ingredient of the phenomenon known as color transparency. Production of neutral pions in $\pi^-A$ collisions at 40 GeV was measured in the limit $z_h \to 1$ in [9]. A strong signal of color transparency was detected in agreement with prediction [10]. This is a direct proof of the perturbative nature of the produced pre-hadron.
4. Mapping the color neutralization length with nuclear broadening

A quark originated from DIS on a free proton may have a transverse momentum relative the initial photon direction. This happens due to the intrinsic motion of quarks in the proton. In the case of a nuclear environment the quark propagating through the medium increases its transverse momentum due to multiple interactions. This leads in $p_T$ broadening,

$$\Delta p_T^2 = \langle p_T^2 \rangle_A - \langle p_T^2 \rangle_p .$$  

Notice that an inclusive hadron with transverse momentum $k_T$ is experimentally observed, rather than the quark. If the hadron carries fraction $z_h$ of the quark longitudinal momentum, they are related as $p_T = z_h k_T$, when the transverse momentum of the hadron relative to the quark direction is neglected. This relation, called sometimes seagull effect, is well confirmed by data. Thus, the quark $p_T$ broadening is related to the observed hadronic broadening as,

$$\Delta p_T^2 = \frac{1}{z_h^2} \Delta k_T^2 .$$  

This relation does not need the above assumption that the hadron and quark fly in the same direction. Indeed, the mean transverse momentum squared between the quark and hadron cancels in (5) if the jet shape is not disturbed by the medium. The jet shape might be affected by the medium, but this is a small, second order correction.

From the relation Eq. (6) we conclude that binning of data in $z_h$ is very important, while data averaged over $z_h$ is difficult to compare with theory.

After the quark color is neutralized and a colorless pre-hadron is created, we should forbid further inelastic (color-exchange) interactions. One may think that if the pre-hadron experienced an inelastic collision, the detected hadron could be recreated again. However, its momentum would be substantially reduced, i.e. it would feed down the small $z_h$ part of the spectrum. Direct calculations confirm that the contribution of such a two-fold interaction is indeed vanishingly small at large $z_h$.

Elastic interactions of the pre-hadron are possible, but hardly happen. Indeed, even for pion the mean free path for elastic collisions is longer than 20 fm, and it is $\langle r_\pi^2 \rangle / r^4$ times longer for a pre-hadron of transverse size $r$.

We conclude that the main source of transverse momentum broadening is the multiple interaction of the original quark during the hadronization stage along the pathlength $l_p$. 

Figure 3. Perturbative charge neutralization and production of an antihydrogen in QED [7] (left figure) versus perturbative color neutralization and production of a (pre)hadron in QCD [8,6,4] (right figure).
Time Evolution of Jets and Perturbative Color Neutralization

(see Fig. 2. Since broadening is proportional to the pathlength, it is probably the most direct way to measure $l_p$.

Thus, we can check our ideas regarding energy, scale and $z_h$ dependence of $l_p$ using broadening, which according to Eq. (3) is expected to display the following features:

- $\Delta p_T^2$ must rise proportionally to energy $\nu$, unless $l_p$ exceeds the nuclear size, then this dependence will saturate and level off;
- $\Delta p_T^2$ is expected to fall steadily down towards $z_h = 1$, where it should be zero;
- $\Delta p_T^2$ should decrease with rising scale $Q^2$. This is important to test in DIS, since jets produced at mid rapidities in hadronic (and heavy ion) collisions have maximal virtuality relative to their energy, therefore $l_p$ should be especially short [4].

Theoretical tools for calculation of broadening are currently well developed and have a high predictive power [11–13]. The light-cone dipole approach was able explain in a parameter-free way data on Cronin effect and correctly predict this effect for RHIC [14]. In this approach broadening on a pathlength $L$ is given by a simple relation,

$$\Delta \langle p_T^2(L) \rangle = 2C(s) \int_0^L dz \rho_A(z),$$

(7)

where $\rho_A(z)$ is the nuclear density as function of longitudinal coordinate $z$ (and implicitly of impact parameter). The coefficient $C$ reads [12,13],

$$C(s) = \left. \frac{d\sigma_{\bar{q}q}(r_T, s)}{dr_T^2} \right|_{r_T=0}. \tag{8}$$

The cross section $\sigma_{\bar{q}q}(r_T, s)$ for a $\bar{q}q$ dipole with separation $r_T$ interacting with a proton, introduced in [15], cannot be reliably calculated but it has to be fitted to data. There are currently several popular parametrization fitted to DIS data for $F_2^p(x, Q^2)$ and photoabsorption data.

Notice that broadening in Drell-Yan reaction, calculated in [13] and relying on Eq. (7), overestimates data from the E772 experiment [16] by a factor of two. However, the recent revision of the data analysis [17], including the brand-new data from the E866 experiment concluded that the theoretical prediction was correct.

Thus, the measurement of broadening in semi-inclusive hadron production in DIS on nuclei should provide clear and direct information on the production length $l_p$. This would be a much better source of information than nuclear attenuation measured so far, where different effects may be easily mixed up.

The preliminary data from the CLAS spectrometer at JLab [18] depicted in Fig. 4 indeed demonstrate a steep energy dependence of broadening in a few GeV region. This confirms the rise of $l_p$ with energy in accordance with Eq. (3). It also shows that in this range of energy and $z_h$ the color is neutralized mainly inside the nucleus, i.e. $l_p < R_A$, otherwise $\Delta p_T^2$ would be energy independent.

Notice that maximal broadening for a quark is about the same as was found from Drell-Yan data [17] at much higher energy of 800 GeV. This is a surprise, since one should expect factor $C(s)$ to rise with energy.
With this data for broadening one can try to extract $l_p$ from data in a least model-dependent way, using relation,

$$\Delta p_T^2 = \frac{2C z_h^2}{A} \int d^2b \int_{-\infty}^{\infty} dz \rho_A(b, z) \int_{z}^{z+l_p} dz' \rho_A(b, z')$$

Comparing this calculation with data one can determine $l_p$ corresponding to each data point for $\Delta p_T^2$. Simultaneously, minimizing the difference between values of $l_p$ on different nuclei one can also determine factors $C$, which is not reliably known at such low energies. We found $C = 4.48 \pm 0.13$, which is in a good accord with extrapolation from high energies (see details in [19]). The results of the analysis [19] employing the data for all three nuclei are shown in Fig. 5. This results of model-independent extraction of $l_p$ are to be confronted with contemporary models for hadronization, those which are based on perturbative QCD [6,4], or those which employ the ideas of the string model [3,20].

It worth reminding that this is the color neutralization length at $z_h = 0.5 - 0.6$, and one expects a contraction of $l_p$ with rising $z_h$. Unfortunately no data testing this expectation has been released so far.

Figure 4. Preliminary data from the CLAS EG2 experiment at Jlab [18] for broadening (in GeV$^2$) of $\pi^+$ produced with $z_h = 0.5 - 0.6$ in DIS on nuclei (Pb, Fe, C from top to bottom) at $Q^2 = 1 - 2$ GeV$^2$ as function of photon energy $\nu$.

Figure 5. Results of model-independent extraction of the production length $l_p$ using Eq. 9 from data depicted in Fig. 4.
5. Long production length: upper bound for nuclear modifications

Let us make the production length (3) much longer than the nuclear size, for instance by increasing the energy. Thus, gluon radiation continues through the nucleus and outside. The rate of induced energy loss linearly rises with the pathlength up to the medium surface as is shown in Fig. 6. What happens afterwards? According to the Landau-Pomeranchuk principle, radiation at longer times does not resolve the structure of the interaction at the initial state. What is important is the accumulated kick, and it does not matter whether it was a single or multiple kicks. Therefore, the vacuum energy loss is continuing with a constant rate increased due to final state interactions.

Lacking a good knowledge of the hadronization dynamics, one can impose an upper bound for the medium-induced suppression. This bound can be calculated precisely with no ad hoc procedures.

Let us increase the amount of induced energy loss assuming that its rate does not rise up to the maximal value near the medium surface, but starts with this maximal rate from the very beginning, as is illustrated in Fig. 7. Since the induced energy loss is increased, the resulting suppression of leading hadrons can only be enhanced.

We arrive at a constant rate of energy loss corresponding to hadronization in vacuum, but with increased scale $Q^2 \Rightarrow Q^2 + \Delta p_T^2$. The scale dependence of the fragmentation function can be calculated perturbatively via of DGLAP equations

$$\bar{D}_i^h(z_h, Q^2) = D_i^h(z_h, Q^2) + \frac{\Delta p_T^2}{Q^2} \sum_j \int_{z_h}^1 \frac{dx}{x} P_{ji}[x, \alpha_s(Q^2)] D_j^h(z_h/x, Q^2).$$

(10)

The medium induces a harder scale which makes the energy loss more intensive. The difference is the induced energy loss which is $\propto \Delta p_T^2$ and present implicitly in the DGLAP.
6. Time evolution of a high-$p_T$ jet

How much energy is radiated over path length $L$?

$$\Delta E(L) = E \int_{\Lambda^2}^{Q^2} \int_0^1 dx \frac{dn}{dx dk^2} \Theta(L - l_c),$$

where

$$l_c = \frac{2E x (1 - x)}{k^2}; \quad \frac{dn}{dx dk^2} = \frac{\gamma}{x k^2}; \quad \gamma = \frac{3\alpha_s}{\pi^2}$$

The rate of energy loss is constant for each interval of $k^2$,

$$\frac{dE}{dL dk^2} = \frac{1}{2} \gamma$$

Radiation of gluons with given transverse momentum $k$ is continuing with the constant rate $\gamma/2$ until the maximal length $L_{\text{max}}(k^2) = 2E/k^2$ is reached.

The total energy radiated over this maximal path length is

$$\Delta E_{\text{tot}} = \int_{\Lambda^2}^{Q^2} dk^2 \frac{1}{2} \gamma \frac{2E}{k^2} = \gamma E \ln \frac{Q^2}{\Lambda^2}$$

How long does it take to radiate fraction $\delta$ of the total emitted energy? The answer depends on how large is $\delta$. For $Q = E$

$$L = \frac{4}{E} \ln \frac{E}{\Lambda} \quad \text{if} \quad \delta < 1/\ln \left(\frac{Q^2}{\Lambda^2}\right)$$

$$L = \frac{2}{Ee} \left(\frac{E}{\Lambda}\right)^{2\delta} \quad \text{if} \quad \delta > 1/\ln \left(\frac{Q^2}{\Lambda^2}\right)$$
Thus, more than a half of the total energy is lost within 1 fm! So, the color neutralization, or production length shrinks with the jet energy.

Notice that this result does not contradict the fact that the mean time of radiation of a gluon is long and rises with jet energy,

$$\langle l_c \rangle = \int \frac{Q^2}{\Lambda^2} \int_0^1 dx \, \frac{dn}{dx} \, l_c(x, k^2) = \frac{E}{\Lambda^2} \frac{1}{\ln(Q/\Lambda) \ln(Q\Lambda/4E^2)}$$

(16)

The medium suppression factor $R_{AA}(p_T)$ is a result of the interplay of two phenomena which act in opposite directions: as $l_p$ shrinks with $p_T$, the amount of induced energy loss reduces, and this should lead to a rising $R_{AA}(p_T)$.

However, contraction of the production length makes the path available for absorption of the colorless pre-hadron longer. This leads to a reduction of $R_{AA}(p_T)$. Usually attenuation caused by absorption is quite a strong effect, however one should incorporate it with precaution.

7. Summary

• Production of leading hadrons in hard reactions involves two stages of time development: (i) propagation of a parton though the medium accompanied with vacuum and induced gluon radiation; (ii) perturbative color neutralization followed by evolution and attenuation of the (pre)hadron in the medium.

• Theoretical tools describing both stages are well developed and do not need ad hoc fits to the data to be explained.

• The production length of leading hadrons is controlled by coherence of radiated gluons and energy conservation

• $p_T$-broadening is a sensitive probe for the production length

• Shortness of the production (color neutralization) length is the main source of nuclear suppression of leading hadrons observed in DIS. There is no room for induced energy loss.

• Maximizing the induced energy loss one can reach a calculable upper bound for the modification of the fragmentation function. It shows that the effects of induced energy loss are far too weak to explain the observed nuclear suppression of leading hadrons.

• The time scale of vacuum gluon radiation in high-$p_T$ jets is very short, less than 1 fm. The production time of leading pre-hadrons is even shorter by a factor of $(1 - z_h)$.

Acknowledgments: We are grateful to Hans-Jürgen Pirner for numerous inspiring and informative discussions. This work was supported in part by Fondecyt (Chile) grants, numbers 1030355, 1050519, 1050589, by DFG (Germany) grant PI182/3-1, and by the Slovak Funding Agency, Grant No. 2/4063/24.
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