Jackiw and Rajaraman Respond: Hagen¹ criticizes our analysis² of the chiral Schwinger model and also points out that he had analyzed a related model in 1973.³ While we were unaware of his work when we published our paper, we rectified our omission through an erratum.⁴

As for our analysis, we remain content with it. The spectrum that Hagen and we find is indeed the same. Our new point is that chiral anomalies which threaten the consistency of this model need not be cured "by hand" with a vector-meson bare mass, as in Hagen's work.³ Rather, such a mass can be radiatively induced by virtual fermions. That radiative corrections are perturbatively finite does not alter the fact that they are polynomially ambiguous. This ambiguity, which we parametrize by a , can be exploited to obtain consistency and unitarity.

Hagen's other objection, based on counting the degrees of freedom of the "Maxwell field," is wrong when applied to this chiral model, since gauge invariance is lost. For $a > 1$, although some constraints prevail, they only fix A_0 , leaving A_1 a true dynamical field.⁵

Turning to Das's work,⁶ he too fails to exploit the full regularization freedom permitted by the chiral model. The classical Lagrangean of the model couples the chiral fermion only to the combination $A_- \equiv (A_0 - A_1)/\sqrt{2}$. Das's evaluation demands that the quantum fermionic determinant also depend only on A_- . Such a regularization is certainly permissible. In fact, it has appeared in earlier literature⁷ and was noted by us² below our Eq. (6). This regularization corresponds to $a = 0$, and as was shown by us, leads to a nonunitary theory. However, a host of other regularizations are also possible, yielding arbitrary nonvanishing values of a . (See for instance Leutwyler⁸ for the $a = 1$ case.) With $a > 1$, a consistent unitary theory emerges.

The need for other ($a \neq 0$) regularizations may also

be seen in the context of the normal Schwinger model. There the classical action again separates into its chiral parts, with the left- and right-chirality fermions coupling only to A_+ and A_- , respectively. If the full fermionic determinant were treated as a product of separate chiral parts, each evaluated by Das's prescription, then no $A_+ A_-$ term can arise. The result would not be gauge invariant. The gauge-invariant result⁷ can be achieved, however, by use of an $a = 1$ regularization for each of the chiral parts. In the chiral model, where the demand of gauge invariance cannot be met anyhow, we showed² that the demands of unitarity and consistency *can* be met by the choice $a > 1$.

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