Forte Replies: In his Comment [1], Hagen presents a lowest-order perturbative evaluation of the current-current correlator $\Pi^{\mu\nu}(q)$, the Fourier transform of $\Pi^{\mu\nu}(x, y) = \langle J^\mu(x) J^\nu(y) \rangle$, and shows explicitly that in the absence of a chemical potential $\Pi^{\mu\nu}(q=0) = 0$; he then argues that this result should persist also when the chemical potential $\mu \neq 0$. Because in the weak-field limit, as is well known [2], $\Pi^{\mu\nu}(0) \neq 0$ is a necessary (and sufficient) condition for the Meissner effect, he then concludes that the (nonperturbative) computation presented in Ref. [3] which claims a Meissner effect for QED with chemical potential in $2 + 1$ dimensions must fail.

Taken at face value, Hagen's argument would also imply the vanishing of the axial anomaly in QED in two dimensions, because in two dimensions, using Eq. (5) of Ref. [3], $\langle \partial_i J_i^5 \rangle = \langle \epsilon^{ij} \partial_i J_j \rangle \propto \Pi(q=0)$ for weak fields. However, as Hagen recognizes, his naive argument fails in the presence of infrared divergences, and such is the case in two dimensions. Now, as shown in Ref. [3], in the presence of a chemical potential the determination of the curl of the vacuum-expectation value of the current in $2 + 1$ dimensions reduces to an effectively two-dimensional problem, a similar infrared divergence appears, and again Hagen's naive argument fails.

To see how this works, notice that in the presence of a chemical potential, the free Fermion propagator $S(p)$ has simple poles located at $p_0 = -\mu \pm |\mathbf{p}|$. Thus, if, according to Eq. (3) of Ref. [1], the correlator is perturbatively expressed in terms of a loop integral, $\Pi^{\mu\nu}(q) = L_1^{\mu\nu}(q; \mu) + L_2^{\mu\nu}(q; \mu)$, then the energy integration in

$$L_1^{\mu\nu}(q; \mu) = \int \frac{d^3 p}{(2\pi)^3} \text{tr} \gamma^\mu S(p + q/2) \gamma^\nu S(p - q/2) \quad (1)$$

has singularities along the path of integration at $p^0 = -\mu \pm (q_0 - |\mathbf{q}|)/2 \pm |\mathbf{p}|$, which ought to be treated using an energy-dependent $i\epsilon$ prescription [4]. This is equivalent to Wick rotating to Euclidean space; then, the integrand in Eq. (2) will have simple poles at $p^0 = -i\mu \pm (q_0 - i|\mathbf{q}|)/2 \pm i|\mathbf{p}|$. It is then easy to show, by shifting the integration variable $p^0 \rightarrow p^0 + \mu$ [4,5], that

$$L_1(q; \mu) = L_1(q; \mu = 0) + \int \frac{d^2 \mathbf{p}}{(2\pi)^2 2|\mathbf{p}|} \times \Theta(\mu - |\mathbf{p}|) \text{tr} \not{p} \gamma^\mu S(p + q) \gamma^\nu. \quad (2)$$

However, the other contribution to the correlator:

$$L_2(q; \mu) = \int \frac{d^3 p}{(2\pi)^3} \text{tr} \gamma^\mu S(p) \gamma^\nu S(p) \quad (3)$$

suffers an infrared divergence because the integrand has a *double* pole at $p^0 = -\mu \pm |\mathbf{p}|$. (This pole is absent when $\mu = 0$ because of the measure of integration.) This pole cannot be treated with a standard $i\epsilon$ prescription, so that the analytic continuation to Euclidean space is not uniquely defined, and renders the perturbative result ill defined.

The origin of this divergence can be traced to the perturbative nature of the derivation presented in Ref. 1,

which is based on an expansion of the effective action $\Gamma = \ln \text{Det} [\not{D}(A; \mu)]$ (where $\not{D}(A; \mu)$ is the Dirac operator with chemical potential and background field μ):

$$\Gamma = \ln \text{Det} \not{D}(0; \mu) + \text{Tr} \sum_n \frac{1}{n} [-S(\mu) \not{A}]^n. \quad (4)$$

The expansion is then truncated at second order, and the result is used in the definition $\langle j^\mu \rangle = \delta \Gamma / \delta A^\mu$.

This derivation is known [5] to fail in the presence of chemical potential when the Hamiltonian corresponding to the Dirac operator $\not{D}(A)$ admits bound states, essentially because the bound states, which contribute directly to Γ provided they have energy $\epsilon < \mu$, are lost from the spectrum if the Dirac operator is expanded perturbatively as in Eq. (5). Now it is also known [6] that in $2 + 1$ dimensions bound states of the Hamiltonian are always present due to an index theorem if the magnetic field is nonzero. The inclusion of such states is required in order to achieve an infrared-finite definition of the expectation value of the current.

In conclusion, this shows that Hagen's perturbative expression for $\Pi^{\mu\nu}(q)$ does not lead to a vanishing result when $q \rightarrow 0$ if $\mu \neq 0$, but rather to an ill-defined one. This is due to the fact that a nonperturbative approach to the computation of the current is not "needlessly involved" but actually mandatory.

Finally, it should be noticed that the results of Ref. [3] are established using a manifestly gauge-invariant regularization of gauge-invariant expressions. Indeed, contrary to a surprising statement by Hagen in [1], the spectrum of eigenvalues λ_k of the Dirac operator is manifestly gauge invariant; this is the main motivation behind the use of ζ -function regularization [7]. The extra point-splitting regularization required in the definition of current matrix elements cited by Hagen [8] is only required when axial anomalies of the current are present, as explained in Ref. [8]. This is not the case in odd dimensions, where such anomalies do not exist.

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