like $p^3 + 2\text{Re}\alpha$ (where p is the barycentric momentum in the u channel), b should be essentially constant, and the threshold unitarity relation,

$$\text{Im}\alpha \cong \frac{-p^{3+2\alpha}}{8\pi^2} b \frac{(m+\lambda)}{\lambda^2} \frac{\sqrt{\pi}\Gamma(\alpha+1)}{(2\alpha+1)\Gamma(\alpha+\frac{1}{2})}, \quad (9)$$

should hold. Presumably, the higher corrections fix up the situation at threshold, as in potential theory.

As the photon mass, λ , tends to zero all of our expressions become infrared divergent. This comes as no surprise, of course, since the divergence of the Compton amplitude in fourth order must be present in order to cancel the corresponding soft photon emission in the double Compton cross section. We have not fully analyzed the requisite modifications of the Regge formalism for this limiting situation and hope to return to the question elsewhere.

For finite photon mass, the photon is presumably on a Regge trajectory if the electron is. If we consider the correction to the amplitude for the exchange of a photon between two electrons (order e^2) arising from the exchange of three photons (order e^6), the motion of the photon pole should be apparent. Again infrared divergences may arise as $\lambda \rightarrow 0$.

The S-matrix or dispersion relation approach to relativistic quantum theory employs dispersion and unitarity formulas abstracted from conventional field theory. If it is really true that the Regge pole boundary conditions are also contained in conventional field theory, as the present work suggests, then there is no evidence for any conflict between the two points of view.

One of us (M.G.-M.) would like to acknowledge the great value of conversations with Professor Gribov, Professor Pomeranchuk, and Professor

Feynman.

¹T. Regge, *Nuovo cimento* **14**, 951 (1959); **18**, 947 (1960).

²B. Lee and R. Sawyer (to be published).

³D. Amati, S. Fubini, and A. Stanghellini (to be published).

⁴S. Mandelstam, *Proceedings of the 1962 International Conference on High-Energy Physics at CERN* (to be published).

⁵V. N. Gribov and I. Ya. Pomeranchuk, *Proceedings of the 1962 International Conference on High-Energy Physics at CERN* (to be published).

⁶R. Blankenbecler and M. L. Goldberger, *Phys. Rev.* **126**, 766 (1962).

⁷G. F. Chew and S. C. Frautschi, *Phys. Rev. Letters* **7**, 394 (1961).

⁸G. F. Chew and S. C. Frautschi, *Phys. Rev. Letters* **8**, 41 (1962).

⁹S. C. Frautschi, M. Gell-Mann, and F. Zachariasen, *Phys. Rev.* **126**, 2204 (1962).

¹⁰S. W. MacDowell, *Phys. Rev.* **116**, 774 (1959).

¹¹S. C. Frautschi and J. D. Walecka, *Phys. Rev.* **120**, 1486 (1960).

¹²V. N. Gribov and I. Ya. Pomeranchuk (to be published).

¹³N. Dombey (private communication).

¹⁴Here s_0 is an arbitrary constant with the dimensions of mass squared; in reference 9 it was set equal to $2m_\pi m_N$.

¹⁵Thus for large s and then large u , the correction to the lowest order result goes like $(\ln s)(\ln u)$. In the u channel, we have the motion of a Regge pole with $\Delta\alpha(u) \propto \ln u$ for large u . In the s channel, a subsidiary Regge pole is likewise moving with $\Delta\alpha(s) \propto \ln s$ for large s .

¹⁶V. V. Sudakov, *J. Exptl. Theoret. Phys. (U.S.S.R.)* **30**, 87 (1956) [translation: *Soviet Phys. - JETP* **3**, 65 (1956)].

¹⁷R. P. Feynman (private communication). In the limit $\lambda \rightarrow 0$, this result is implicit in the calculation of radiative corrections to the Compton effect by L. M. Brown and R. P. Feynman, *Phys. Rev.* **85**, 231 (1952).

COMMENTS ON CHEW'S BOOTSTRAP RELATIONSHIP*

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In a recent Letter,¹ Chew has shown the existence of a certain reciprocity between the nucleon and the (3,3) resonance. Essentially, he calculates the reduced width γ_{33} in terms of the "reduced width" of the nucleon, $\gamma_{11} = 3f^2$, and finds²

$$\gamma_{33} \cong A_{33,11}\gamma_{11}/(1 - A_{33,33}) = \frac{1}{2}\gamma_{11}, \quad (1)$$

where $A_{\alpha\beta}$ is the P -wave pion-nucleon crossing matrix. Reciprocally, he may calculate γ_{11} in terms of γ_{33} ; he finds

$$\gamma_{11} \cong A_{11,33}\gamma_{33}/(1 - A_{11,11}) = 2\gamma_{33}. \quad (2)$$

Since $\frac{1}{2} \times 2 = 1$, Eq. (1) and Eq. (2) are consistent with each other (and with experiment), from which

his conclusions follow.

We would like to make the following two comments. The first somewhat broadens the validity of Chew's numerical results. The second points out the existence of an alternative interpretation of them.

(I) One may interpret Eq. (1) as a calculation of γ_{33} in terms of γ_{11} , and Eq. (2) as a calculation of γ_{11} in terms of γ_{33} . This calculation is exact within the approximation of a two-level formula for the scattering amplitude. That is, Chew writes equations for the P -wave amplitudes,

$$f_{\alpha} = \frac{1}{\pi} \int_{(s \text{ channel})} \frac{d\omega'}{\omega' - \omega} \text{Im} f_{\alpha}' + \frac{1}{\pi} \int_{(u \text{ channel})} \frac{d\omega'}{\omega' - \omega} \text{Im} f_{\alpha}',$$

$$= \frac{1}{\pi} \int_{(s \text{ channel})} \frac{d\omega'}{\omega' - \omega} \text{Im} f_{\alpha}' + \frac{1}{\pi} A_{\alpha\beta} \int_{(s \text{ channel})} \frac{d\omega'}{\omega' + \omega} \text{Im} f_{\beta}' \quad (3)$$

(where we have left out terms coming from ρ exchange, since they are not essential to any of the arguments), and attempts to solve them by inserting the $(3, 3)$ and N states in crossed channels and otherwise ignoring all cuts in what remains. We can extend his result to include cuts by making the approximation $\frac{1}{9} \cong 0$ and changing the crossing matrix from

$$A = \frac{1}{9} \begin{pmatrix} -1 & -4 & -4 & 16 \\ -2 & -1 & 8 & 4 \\ -2 & 8 & -1 & 4 \\ 4 & 2 & 2 & 1 \end{pmatrix} \quad (4)$$

to

$$A' = \begin{pmatrix} 0 & 0 & 0 & \alpha \\ -\frac{2}{9} & 0 & 1 & 2\alpha/9 \\ -\frac{2}{9} & 1 & 0 & 2\alpha/9 \\ 1/\alpha & 0 & 0 & 0 \end{pmatrix}, \quad (5)$$

with $\alpha = 2$ for this case. We can then solve exactly for the $(1, 1)$ and $(3, 3)$ scattering amplitudes (in the one-meson approximation, say, or with a given ratio of inelastic to elastic scattering in each partial wave) for the following three cases: (a) nucleon pole inserted explicitly, $(3, 3)$ pole [for the case of a bound $(3, 3)$ state] calculated by summing the power series in γ_{11} ; (b) $(3, 3)$ pole inserted explicitly with its previously calculated energy and width, and nucleon pole calculated by

summing the power series in γ_{33} ; (c) both poles inserted explicitly, i.e., double power series summed.

This system is a simple generalization of one recently considered by the author³ (in which one had $\alpha = 1$). The result found there may be extended to arbitrary α . It is that the scattering is the same in all three cases. In particular, not only are the widths reciprocal, as shown in reference 1 for the slightly different crossing matrix A , but also the position of the nucleon pole will automatically come out right in calculation (b) above, provided the $(3, 3)$ energy and width have been properly inserted from calculation (a). Of course, one needs a cutoff for this calculation.

Thus the validity of Chew's reciprocity is wider than claimed in his letter. On the other hand, it is evidently independent of the energies and coupling constants of the states, and is therefore more a property of the form of the equations than of the particular solution found in nature.

(II) How must this reciprocal property of the equations be interpreted? This question is subtle. The calculation described in I(b) above, but with the original crossing matrix A , corresponds to a Lagrangian theory in which one has an interaction

$$L^I = \bar{\Psi}_{33} \gamma_{33}^{(0)\frac{1}{2}} \Gamma \Psi_N \Phi_{\pi} + \text{H.c.}, \quad (6)$$

where Γ is whatever is needed to balance spin, parity, and isotopic spin. One then calculates π - N scattering, finds a pole in the $(1, 1)$ amplitude, and tries to interpret this pole as due to the appearance of an N - N - π vertex. However, the Lagrangian (6) possesses a symmetry, which one might call $(3, 3)$ parity,⁴ $\Psi_N \rightarrow -\Psi_N$, $\Phi_{\pi} \rightarrow -\Phi_{\pi}$, $\Psi_{33} \rightarrow \Psi_{33}$, which forbids an N - N - π vertex. Therefore, the pole in this model can not be a consequence of an N - N - π vertex, but rather must result from an N - N' - π vertex, where N' is a nucleon-like particle, but with even $(3, 3)$ parity.

If one accepts the Lagrangian interpretation given above, then, since neither the N' nor $(3, 3)$ parity appears to exist, one should seek the desired reciprocity starting from a study of the $(3, 3) + \pi$ system, explicitly inserting a $(3, 3)$ - $(3, 3)$ - π vertex to avoid the appearance of a non-existent quantum number.

Although the Lagrangian model just described leads to the same equations as those of Chew in reference 1, it is in principle a different model, since in the latter one simply searches for con-

sistent solutions of the equations, and therefore does not see a selection rule that has no consequences for the channel under consideration. The difference between the Lagrangian and Chew theories here appears to be almost a matter of words, as indeed it should in the subspace of π - N scattering. In a larger space, including for example N - N scattering, the difference would no longer be a matter of words if it existed at all, since in the N - N channel the presence of the N' would be clear not only in the Lagrangian theory but also in the S matrix. In the absence of a complete solution of the dispersion equations, how-

ever, it would be interesting to have a criterion for how to count bootstrapped particles.

¹G. F. Chew, Phys. Rev. Letters 9, 233 (1962).

²The small term $A_{33,11} = \frac{1}{9}$ is left out of the denominator of Eq. (1) in reference 1, although it should consistently be there.

³F. E. Low, Nuovo cimento 25, 678 (1962).

⁴This quantum number corresponds to $\exp(i\pi S)$, where S is strangeness, in the model of reference 3. In that model the analogues of N' , N , $(3,3)$, and π are, respectively, N , Λ , Ξ , and κ . The degeneracy of N and N' is therefore essentially accidental.

E R R A T A

OPTICAL-MODEL FRINGE ABSORPTION AND THE NEUTRON STRENGTH FUNCTION. P. A. Moldauer [Phys. Rev. Letters 9, 17 (1962)].

On p. 17, second column, lines 7 and 8, the words " $2^{-1/2}$ - F dispersion" should read " $2^{-3/2}$ - F standard deviation". This means that in the usual optical model notation the Gaussian width employed was $b = \frac{1}{2}F$.

ONE-PION-EXCHANGE MODEL AND EVIDENCE THAT THE SPIN OF THE ζ IS EVEN. Allen E. Everett [Phys. Rev. Letters 9, 74 (1962)].

The basic conclusion of the paper was that the ζ is not coupled to two pions by the strong interactions. In addition to states of even spin and parity referred to in the paper, this would also

allow states of odd spin and parity, provided the G -parity is also odd. Thus, 1^{--} is also a possible assignment for the ζ . Under this assignment, G -parity conservation means that the 2π decay of $\zeta^{\pm}$ must be electromagnetic. By arguments similar to those of Reference 7 (where the 1^{--} possibility is also not considered), odd G -parity would mean that the 2π decay of the (so far unobserved) ζ^0 would be forbidden by charge conjugation invariance. The 1^{--} assignment would make a 3π -decay mode of the ζ allowed by strong interactions, though somewhat suppressed by phase space and angular momentum barrier effects, which could possibly account for the peak in the 3π mass spectrum at about 625 MeV observed by Pickup *et al.*¹

¹E. Pickup, D. K. Robinson, and E. O. Salant, Phys. Rev. Letters 8, 329 (1962).