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SOME APPLICATIONS OF THE JOST-LEHMANN-DYSON THEOREM
TO THE STUDY OF THE GLOBAL ANALYTIC STRUCTURE
OF THE N POINT FUNCTION OF QUANTUM FIELD THEORY *)

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A B S T R A C T

The Jost-Lehmann-Dyson parametrization of distributions with support contained in the union of two opposite light cones is reviewed and applied in particular to the derivation of global analyticity properties for the p space analytic functions of quantum field theory.

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INTRODUCTION

Whereas the theory of hyperfunctions and microfunctions ¹⁾ is able to produce local information on the problem posed by the structure of the p space n point function of quantum field theory ²⁾, it is quite obvious that the problem physicists are confronted with is of a global nature.

Most global results that are known at present can be derived from a few ingredients :

- the tube theorem,
- the semi-tube theorem,
- the edge-of-the wedge theorem,
- the vanishing of the first cohomology group of a Stein manifold,

several combinations of which are able to yield partial answers of a global nature. In some cases, more sophisticated applications of the continuity theorem were able to produce non-trivial analytic completions.

Standing aside is a beautiful piece of work due to Jost, Lehmann and Dyson (JLD) ³⁾, which is by itself able to produce many of the results known by the previously mentioned methods and has, besides, a touch of originality and exotism which has made it almost impossible to generalize, excepting an unpublished work by Glaser. This can be taken as a proof that, if there is a general idea behind it, no one has been so far able to grasp it.

We shall furthermore see here some applications ⁴⁾ of JLD's trick which have so far not been recovered by the more conventional methods.

In what follows, we shall first give a summary of JLD's work, then make the bridge between some of its consequences and results which can be obtained by analytic methods, and finally give some applications which, at present, cannot be treated otherwise.

1. - THE JOST-LEHMANN-DYSON (JLD) REPRESENTATION

A) - Let C be a temperate distribution in Minkowski configuration (x) space, with support the union of two closed opposite light cones

$$\bar{V}^{\pm} = \left\{ x \mid x^2 = (x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2 \equiv (x^0)^2 - \vec{x}^2 \geq 0, x^0 \geq 0 \right\}$$

The Fourier transform $\tilde{C}$ of C is the restriction to the hyperplane $\kappa = 0$ of a tempered distribution Γ in five-dimensional momentum Minkowski space (p, κ) , where κ is an additional space-like co-ordinate, even under reflections through the hyperplane $\kappa = 0$, and solution of the wave equation

$$\square_{p, \kappa} \Gamma = 0$$

where

$$\square_{p, \kappa} = \frac{\partial^2}{(\partial p^0)^2} - \frac{\partial^2}{(\partial p^1)^2} - \frac{\partial^2}{(\partial p^2)^2} - \frac{\partial^2}{(\partial p^3)^2} - \frac{\partial^2}{(\partial \kappa)^2}$$

Conversely, every solution Γ of the wave equation that is even in κ has a restriction to the hyperplane $\kappa = 0$ whose Fourier transform has support $\bar{V}^+ \cup \bar{V}^-$. C 's and Γ 's with the stated properties are in a one-to-one correspondence

A*) - This situation can be generalized to the case where $\bar{V}^{\pm}$ is replaced by $\bar{V}_M^{\pm} = \left\{ x \mid x^2 \geq M^2, x^0 \geq 0 \right\}$, the wave equation being replaced by the Klein-Gordon equation

$$KG \Gamma \equiv (\square + M^2) \Gamma = 0.$$

The variable κ can be replaced by a set $\vec{\kappa}$ of space-like variables and the evenness requirement by that of invariance under the orthogonal group in $\vec{\kappa}$ space.

Sketch of proof

(i) From C to Γ

Let (x, κ) be the variables conjugate to (p, κ) . The product $C(x) \cos \kappa \sqrt{x^2}$ exists as a distribution in (x, κ) , even in κ , which is infinitely differentiable in κ , polynomially bounded in κ , and coincides with C for $\kappa = 0$, because $\cos \kappa \sqrt{x^2}$ is polynomially bounded together with all its derivatives with respect to x , on the support of C, and the support of C is regular. Γ is defined as the Fourier transform with respect to x of $C(x) \cos \kappa \sqrt{x^2}$ and obviously is an even solution of the wave equation.

(ii) From Γ to C

Since Γ fulfils the wave equation, its Fourier transform $\tilde{\Gamma}$ has its support within the union of two opposite light cones in (p, κ) space. Hence, from the regularity property of this support, one can decompose $\tilde{\Gamma}$ according to

$$\tilde{\Gamma} = \tilde{\Gamma}^+ - \tilde{\Gamma}^-$$

$\tilde{\Gamma}^\pm$ having its support on the $\begin{cases} \text{positive} \\ \text{negative} \end{cases}$ closed light cone. Γ can thus be written as the difference of boundary values of two polynomially bounded analytic functions holomorphic in tubes with these two self-dual cones as imaginary bases. The restrictions of these analytic functions to the analytic hyperplane $\kappa = 0$ exist, are polynomially bounded and analytic, in the remaining variables, in tubes with light cone basis, hence the support properties of the Fourier transforms of their boundary values.

B) - We now wish to take into account support properties of $\tilde{C}$, in p space, and derive from it support properties of Γ .

B.1. - The double cone property

If $\tilde{C}$ vanishes in a real neighbourhood of a time-like segment :

$$p = p_0 + t n \quad n^2 > 0 \quad -1 \leq t \leq +1$$

Γ vanishes in the double cone subtended by it :

Using the wave equation and the evenness of Γ , one finds that Γ is zero together with all its derivatives where $\tilde{C}$ vanishes. One can then use the decomposition of Γ as a difference of boundary values of functions holomorphic in opposite tubes and the Kolm-Nagel version of the edge-of-the-wedge theorem ¹⁾, together with a completion by disks belonging to hyperbolae going through the end points of the time-like segment and asymptotic to light-like directions as indicated on Fig. 1.

One thus gains points by application of the continuity theorem, up to completely filling in the expected double cone. This is a geometric version of a proof first constructed by Borchers ⁵⁾.

B.2. - Huyghens principle

Knowing some support properties for Γ , one solves the Cauchy problem on a hypersurface best adapted to the support of Γ : let Σ be a hypersurface which is spacelike, so that it is compactly intersected by an arbitrary light cone (one may even allow lightlike pieces provided they are bounded). Let Σ_+, Σ_- be the closed half-spaces defined by Σ . One can decompose Γ according to

$$\Gamma = \Gamma_+ - \Gamma_-$$

$\Gamma_{\pm}$ having support in $\Sigma_{\pm}$. Then

$$\square \Gamma = \square \Gamma_+ - \square \Gamma_- = 0$$

Hence

$$\square \Gamma_+ = \square \Gamma_- = \sigma$$

where σ has its support on Σ . If a different decomposition is chosen, σ changes into $\sigma + \square\tau$, where τ has its support on Σ . We call $\sigma \bmod \square\tau$ the Cauchy data of Γ on Σ . They are distributions with support Σ defined modulo the Dalembertian of distributions with support Σ , and one can check that, locally where Σ is a manifold, this notion coincides with the usual one. If Γ vanishes in some open set Ω , it is clear that one can choose σ to vanish in $\Omega \cap \Sigma$. Let now D_{ret} , D_{adv} be the elementary retarded and advanced solutions of the wave operator. One can solve the inhomogeneous wave equations :

$$\Gamma_{\pm} = \overset{\circ}{\Gamma}_{\pm} + D_{\text{ret/adv}} * \sigma$$

where $\overset{\circ}{\Gamma}_{\pm}$ are solutions of the homogeneous wave equation which, under the geometrical assumptions we made, have supports $\Sigma_{\pm}$, bounded from below or above, hence identically zero. We thus obtain the representation

$$\Gamma = \Gamma_+ - \Gamma_- = D * \sigma$$

where $D = D_{\text{ret}} - D_{\text{adv}}$ vanishes outside the union of two opposite light cones. As expected, Γ only depends on its Cauchy data $\sigma \bmod \square\tau$, since $\square D = 0$. The representation of Γ is able to yield further support properties : Γ vanishes outside the future and past shadows of the support of its Cauchy data (Huyghens principle), an information which has to be combined with the double cone property.

A more refined property ⁶⁾ is the following. D is real analytic except on the skin of the light cone. Hence, if Γ vanishes on an open subset ω of a connected region $\mathcal{R}$ where no characteristic from a point of the support of σ goes, it vanishes identically in $\mathcal{R}$ (cf. Fig. 2).

Whereas the double cone and Huyghens principle are enough to obtain complete information when the input is the vanishing of $\tilde{C}$ between two space-like surfaces (the problem solved by JLD ³⁾), the analyticity principle was applied by Greenberg ⁶⁾ to cases where the region of vanishing of $\tilde{C}$ is non connected.

As is well known, the JLD problem is connected with the following edge-of-the-wedge problem : let us decompose C into

$$C = R - A$$

Support $R = \bar{V}_+$, support $A = \bar{V}_-$.

Then $\tilde{R}, \tilde{A}$ are boundary values of functions analytic in opposite tubes, and the statement

$$\tilde{C} = 0 \quad \text{in } \Omega$$

is converted into

$$\text{b.v. } \tilde{R} = \text{b.v. } \tilde{A} \quad \text{in } \Omega,$$

a statement to be exploited by use of the edge-of-the-wedge theorem and analytic completion. It turns out that all analyticity properties derivable by the JLD procedure could be obtained by analytical geometric methods including those which require the use of the analyticity principle. Some known results can only be obtained by geometrical methods ⁷⁾. Whatever analyticity follows from the JLD theorem can be obtained as follows : D can be decomposed in a non-unique way as the difference of boundary values of two functions analytic in tubes :

$$D = D^{(+)} - D^{(-)}$$

It turns out, however, that these functions have as sole singularities the points belonging to the hyperboloids

$$p^2 - \kappa^2 = \rho, \quad \rho \geq 0, \quad \rho \text{ real.}$$

A careful choice of the decomposition of D allows to express the corresponding analytic functions T^+, T^- as

$$T^\pm = D^{(\pm)} * \sigma$$

which yields the analyticity described in Section 2, after investigation of the support of σ . This way, the JLD procedure solves an analytic problem "with growth", whereas the geometrical methods solve it without growth ⁷⁾.

2. - PROBLEMS OF ANALYTIC COMPLETION ⁷⁾

The analytic result that stems from JLD's analysis can be summarized as follows. Let $\mathcal{C}^\pm$ be the tubes with imaginary basis $V^\pm$, $\mathcal{R}$ a coincidence region, open, connected, bounded by two spacelike surfaces. The holomorphy envelope of the associated edge-of-the-wedge problem ($f^\pm$ holomorphic in $\mathcal{C}^\pm$, $bvf^+ = bvf^-$ in $\mathcal{R}$) is the complement of the set of complex points of real "admissible" hyperboloids $\mathcal{H}_{u,\kappa}$:

$$(p-u)^2 = \kappa^2 \quad u, \kappa \text{ real}, \kappa^2 \geq 0$$

which do not intersect $\mathcal{R}$.

We now look at a few special cases.

A) - $\mathcal{R}$ is unbounded from above (cf. Fig. 3). The only admissible hyperboloids are spacelike planes which do not intersect $\mathcal{R}$. Hence $\mathcal{R}$ has to be convex ("re-entrant nose" phenomenon). The domain of analyticity is made up with :

- the real points of the convex hull of $\mathcal{R}$;
- all complex points of the real straight lines which intersect $\mathcal{R}$ (no such point lies in a real plane not intersecting $\mathcal{R}$.)

B) - The previous result can be applied to the following situation :

$\mathcal{R}$ as above, but $V^\pm$ replaced by arbitrary (convex) cones.

Using two-dimensional hyperplanes and putting together all analyticity points deduced from A), one has the same result as in A). In particular, $V^\pm$ are enlarged to the $\begin{cases} \text{future} \\ \text{past} \end{cases}$ asymptotic cones of $\mathcal{R}$.

C) - Same as B) but $\mathcal{R} = \mathcal{R}^+ \cup \mathcal{R}^-$ (cf. Fig. 4). The union of the domains pertaining to $\mathcal{R}^+, \mathcal{R}^-$ is natural and is the complement of the real and complex points of real hyperplanes intersecting neither $\mathcal{R}^+$ nor $\mathcal{R}^-$ (Hahn-Banach).

Remark

All these domains are of the following type : given an open set Ω in real space, the set of complex points of straight lines intersecting Ω , together with the real points of Ω is natural if Ω is convex. If the real points of Ω are deleted, one obtains a generalization of two opposite tubes.

From this point of view, the re-entrant nose phenomenon is not essentially different from the double cone phenomenon : in two dimensions let $\mathcal{R} = \Gamma_a^+ \cup \Gamma_b^+$ (cf, Fig. 5)

Within the domain pertaining to Γ_a^+ , take the complex points of straight lines intersecting αa : make a real projective transformation which takes $a\alpha$ into the straight line at ∞ and apply the double cone theorem to a half-straight line in Γ_b^+ . One could, of course, also apply the continuity theorem to a family of straight lines parallel to ab , starting high enough.

D) - A more general application ⁷⁾ allows to describe in geometrical terms the holomorphy envelope associated with two arbitrary double cones in two dimensions, combining projective transformations and inversions [see E] which transform finite double cones into infinite double cones. For instance, in the example shown on Fig. 6, the JLD result can be applied to situation II. There, the union of the two holomorphy envelopes is natural. In situation I, the adapted admissible hyperbolae go through the apex of the infinite cone. The solution to this problem was found during an attempt to understand the analytic equivalent to the analyticity principle in the JLD approach ⁷⁾.

E) - $\Gamma^{+,-}$ is a topological product of $V^{+(-)}$'s, $\mathcal{R} = \Gamma_a^+$. The inversion

$$p \rightarrow p' = \{p'_i\} \quad p'_i = b_i - \frac{p_i - b_i}{(p_i - b_i)^2}$$

transforms Γ_a^+ into the double cone of diagonal ab if b is in the past of a . It preserves the tubes of imaginary basis $\Gamma^{+,-}$. It transforms straight lines

$$p_i = u_i + v_i t, \quad u_i, v_i, \text{ real}, \quad t \in \mathbb{C}$$

into curves which were called Q curves. In the JLD case, the Q curves are hyperbolae which allow to describe the domain from inside. The JLD domain consists of all complex points of doubly inadmissible hyperbolae, i.e., real hyperbolae asymptotic to the light cone with an upper sheet and a lower sheet which both intersect $\mathcal{R}$, together with its suitable completions (by double cone and re-entrant nose enlargements).

In the more general case of a double cone, one describes the domain in terms of complex points of doubly inadmissible Q curves. The description of the domain from outside, in more general cases, is due to Epstein and Glaser.

3. - SOME PROBLEMS OF GLOBAL DECOMPOSITIONS

All the problems which are dealt with here are solved with the help of the JLD theorem : they are problems "with bounds" but no solution is so far known without bound. They are all connected with the structure of the p space n point function of quantum field theory ²⁾ summarized in these proceedings.

The starting point is the decomposition of the generalized retarded functions associated with a given channel ("hypercell") in terms of Bros trees ²⁾. The next step is to use the discontinuity formula for neighbouring geometric cells : the notations being the same as in the lecture by Epstein, Glaser and Stora (this volume), the discontinuity formula

$$\Delta_{\mathcal{S}_I} = t_{\mathcal{S}_I^+} - t_{\mathcal{S}_I^-} = (\Omega, [T_{\mathcal{S}_I}(I'), T_{\mathcal{S}_I}(I)] \Omega)$$

contains both x space and p space support information, thus suggesting the use of the JLD theorem. Similar problems were first treated by Streater ⁸⁾ in the context of the structure of Wightman functions. From Bros' tree decomposition, one sees that the difference $t_{\mathcal{S}_I^+} - t_{\mathcal{S}_I^-}$ involves only trees for which one of the defining equations is $S_I > 0$ or $S_I < 0$. Thus, one has the following situation :

$$\tilde{\Delta}_{\mathcal{S}_I} = \text{b.v.}_{p_I=0} \left(\sum_{i \in \mathcal{J}^+} \tilde{f}_i^+ - \sum_{i \in \mathcal{J}^-} \tilde{f}_i^- \right) = 0, \quad p_I^2 < M_I^2$$

where the $f_i^\pm$ are holomorphic in simplicial tubes with imaginary basis contained in $\text{Imp}_I \in V^\pm$ and M_I is a mass associated with the spectrum properties of the theory. Analyticity is furthermore deduced from x space support properties.

Let us choose a set of variables, one of which we denote ξ , $\xi = x_i - x_j$ where $i \in I, j \in I'$. By JLD, Δ_{ξ_I} is the restriction to $\kappa = 0$ of a distribution Γ in all variables together with κ , solution of the Klein-Gordon equation

$$KG \Gamma \equiv \left(\square_{\xi} - \frac{\partial^2}{\partial \kappa^2} + M_I^2 \right) \Gamma = 0$$

Using the double cone principle, we see that at least in the applications given here,

$$\text{supp. } \Gamma = \text{supp. } \Delta_{\xi_I} \times (-\infty < \kappa < +\infty)$$

because all other variables being fixed, the support in ξ is a union of $V^{\pm}$'s with different origins, whose complement contains arbitrary large timelike segments between the upper and lower boundary and all apices of V^+ cones are in the past of all apices of V^- cones. We can therefore split

$$\Gamma = \sum_{i \in \mathcal{I}^+} \Gamma_i^+ - \sum_{i \in \mathcal{I}^-} \Gamma_i^-$$

where

$$\text{supp } \Gamma_i^{\pm} = \text{supp } f_i^{\pm} \times (-\infty < \kappa < +\infty)$$

Hence

$$KG \Gamma = \sum_{i \in \mathcal{I}^+} KG \Gamma_i^+ - \sum_{i \in \mathcal{I}^-} KG \Gamma_i^- = 0$$

There exist therefore distributions $\Delta_{ij} = -\Delta_{ji}$, $i, j \in \mathcal{I}^+ \cup \mathcal{I}^-$ such that

$$\pm KG \Gamma_i^{\pm} = \sum_{j \in \mathcal{I}^+ \cup \mathcal{I}^-} \Delta_{ij}^{\pm\pm}$$

where

$$\text{supp. } \Delta_{ij}^{\pm\pm} = \text{supp. } \Gamma_i^{\pm} \cap \text{supp } \Gamma_j^{\pm}$$

hence

$$\Gamma_i^\pm = \overset{\circ}{\Gamma}_i^\pm + D_{adv}^{ret} * \sum_{j \in J_0^+ J^-} \Delta_{ij}^{\pm\pm}$$

where the convolution exists, has its support contained in the support of $\overset{\circ}{\Gamma}_i^\pm$, hence, also that of $\overset{\circ}{\Gamma}_i^\pm$. This support being bounded from above or from below, $\overset{\circ}{\Gamma}_i^\pm$ vanishes identically. Putting pieces together, we get

$$\Gamma = D * \sum_{i \in J^+, j \in J^-} \Delta_{ij}^{+-}$$

with restriction to $\mathcal{K} = 0$ equal to $\Delta_{\mathcal{I}}$. A suitable decomposition $D = D^{ret} - D^{adv}$ yields a common analytic decomposition in p space, for

$$t_{\mathcal{I}}^\pm = \sum_{i \in J^+, j \in J^-} D^{ret} * \Delta_{ij}^{+-}$$

It is of the form (in p space)

$$\tilde{t}_{\mathcal{I}}^\pm = \sum_{i \in J^+, j \in J^-} \tilde{f}_{ij}$$

where $\tilde{f}_{ij}$ is holomorphic in the tube convex hull of the tubes defining the trees f_i, f_j indented by the cut $p_I^2 = M_I^2 + \rho$, ρ real positive. The ambiguity of this decomposition is of the form

$$\delta \tilde{f}_{ij} = \sum_{k \in J_0^+ J^-} \tilde{f}_{ijk}$$

where $\tilde{f}_{ijk}$ is the common analytic continuation of

$$\widetilde{D^{ret} * \Delta_{ijk}^{+-}}, \quad \widetilde{D^{adv} * \Delta_{ijk}^{+-}},$$

the support of Δ_{ijk} being the intersection of those corresponding to trees f_i, f_j, f_k . Hence $\tilde{f}_{ijk}$ is analytic in the convex hull of the tubes defining f_i, f_j, f_k , indented by the cut $p_I^2 = M_I^2 + \rho$.

We now list a few applications of this decomposition property to low values of n . We shall not write down supports and their duals in detail but rather show the s space corresponding cell. We shall write equations involving cells, each cell symbol representing a function analytic in the corresponding Araki tube - wiggly lines represent cuts. In the following, E.W. means edge-of-the-wedge, JLD means Jost-Lehmann-Dyson.

A) - Three-point function. s space is shown in Fig. 7. Each couple of neighbouring cells gives rise to a holomorphy envelope ⁸⁾ ;

B) - Four-point function. s space is shown in Fig. 8.

1)

2) the quartet ⁸⁾ :

Hence the common decomposition for all four members of the quartet hypercell :

3) putting together both decompositions we get :

hence :

= holomorphy envelope

defining

$$\begin{array}{c} s_1 \\ \diagdown \\ s_4 \quad s_3 \\ \diagup \\ s_2 \end{array} - \begin{array}{c} s_1 \\ \square \\ s_2 \quad s_3 \end{array} = \begin{array}{c} s_1 \\ \diagdown \\ s_4 \quad s_3 \\ \diagup \\ s_2 \end{array}$$

$$\begin{array}{c} s_1 \\ \diagdown \\ s_4 \quad s_3 \\ \diagup \\ s_2 \end{array} - \begin{array}{c} s_1 \\ \square \\ s_2 \quad s_3 \end{array} = \begin{array}{c} s_1 \\ \diagdown \\ s_4 \quad s_3 \\ \diagup \\ s_2 \end{array}$$

we get :

$$\begin{array}{c} s_4 \\ \diagdown \\ s_1 \\ \diagup \\ s_3 \end{array} = \begin{array}{c} s_1 \\ \diagdown \\ s_4 \quad s_3 \\ \diagup \\ s_2 \end{array} = \begin{array}{c} s_1 \\ \diagdown \\ s_4 \quad s_3 \\ \diagup \\ s_2 \end{array} \quad \text{(not explicitly known)}$$

$$\begin{array}{c} s_4 \\ \diagdown \\ s_1 \\ \diagup \\ s_3 \end{array} = \begin{array}{c} s_1 \\ \diagdown \\ s_4 \quad s_3 \\ \diagup \\ s_2 \end{array} = \begin{array}{c} s_1 \\ \diagdown \\ s_4 \quad s_3 \\ \diagup \\ s_2 \end{array} \quad \text{(not explicitly known)}$$

hence the global decomposition

$$\begin{array}{c} s_1 \\ \diagdown \\ s_4 \quad s_3 \\ \diagup \\ s_2 \end{array} + \begin{array}{c} s_1 \\ \diagdown \\ s_4 \quad s_3 \\ \diagup \\ s_2 \end{array}$$

Unfortunately the next step involves the relationship

$$\begin{array}{c} s_1 \\ \diagdown \\ s_4 \quad s_3 \\ \diagup \\ s_2 \end{array} + \begin{array}{c} s_1 \\ \diagdown \\ s_4 \quad s_3 \\ \diagup \\ s_2 \end{array} = \begin{array}{c} s_1 \\ \diagdown \\ s_4 \quad s_3 \\ \diagup \\ s_2 \end{array} + \begin{array}{c} s_1 \\ \diagdown \\ s_4 \quad s_3 \\ \diagup \\ s_2 \end{array}$$

C) - Five-point function. Two types of Bros trees are shown in Fig. 9 : ABCD, BCDE ; there are six of each kind within the hypercell represented by ABCDEF (cf. Fig. 7). Their convex hull ABCDE indented by the cut BCD will be denoted $\overline{ABCDE}$. On our drawing, there is another similar object : $\overline{FABDC}$. Let (EF) denote the holomorphy envelope of these two domains (which is not explicitly known). There are six such domains, each one corresponding to a diagonal of a vertical facet. In terms of these six functions, one has the global decomposition, valid for the analytic continuation of any member of the hypercell :

$$\overline{F} = (AD) + (CF) + (AE) + (BC) + (EF) + (BD)$$

CONCLUSION AND OUTLOOK

The partial answers exhibited here pose the more general following problems :

- For general n , can one reduce the problem which arises from the Steinmann identities within a hypercell to the construction of well-defined holomorphy envelopes ?
- When looking at neighbouring hypercells, one is faced with problems of the following type : let f_i be holomorphic in Ω_i , $i \in I$, let

$$\sum_i f_i = 0 \quad \text{in} \quad \bigcap_{i \in I} \Omega_i$$

what are the implications of such identities ? Do these problems belong to some part of cohomology theory ?

- In other words, can the problem posed by the structure of the n point p space function of quantum field theory be reduced to the construction of well-defined holomorphy envelopes ?

REFERENCES

This list is not exhaustive, but mostly gives a sample of references where more bibliography can be found, besides few originals which are not very well known.

- 1) "Hyperfunctions and Pseudo-Differential Equations", Lecture Notes in Mathematics, Vol. 287, Springer Verlag (1973) ;
"Théorie des Hyperfonctions", by P. Shapira, Lecture Notes in Mathematics, Vol. 126, Springer Verlag (1970) ;
A. Martineau - Lectures at the Meetings of RCP 25, Strasbourg Mathematics Institute, Vol. 3 (1967).
- 2) J. Bros and R. Stora - Lectures at the Meetings of RCP 25, Strasbourg Mathematics Institute, Vol. 3 (1967) ;
H. Epstein - Brandeis Summer Institute (1965) ; Gordon and Breach, New York (1966) ;
H. Epstein, V. Glaser and R. Stora - These Proceedings.
- 3) A.S. Wightman - Lectures at Les Houches Summer School of Theoretical Physics (1960) ; Hermann, Paris (1961) ;
V.S. Vladimirov - "Methods of the Theory of Functions of Several Complex Variables", Cambridge M.I.T. Press (1966).
- 4) R. Stora - unpublished (1964), summarized in Lectures at the Meetings of RCP 25, Strasbourg Mathematics Institute, Vol. 2 (1966).
- 5) H. Borchers - Nuovo Cimento 19, 787 (1961).
- 6) O.W. Greenberg - J.Math.Phys. 3, 859 (1962).
- 7) J. Bros, A. Messiah and R. Stora - J.Math.Phys. 2, 639 (1961) ;
H. Borchers and R. Stora - unpublished (1962).
For another type of solutions with growth, see :
R. Seneor - Commun.Math.Phys. 11, 233 (1968).
- 8) R.F. Streater - Proc.Roy.Soc. A256, 39 (1960).

FIG.1 Analytic proof of the double cone theorem.

FIG. 2

If I has null Cauchy data on R it is real analytic in R .

FIG. 3

FIG. 4

FIG. 5

① is transformed into
 ② by a real projective transformation

FIG.6

FIG. 7

FIG. 8

FIG.9