CM-P00049199

Archives

Ref.TH.1727-CERN

THE NUCLEON AS A BOUND STATE OF THREE QUARKS
AND DEEP INELASTIC PHENOMENA

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ABSTRACT

We present a description of deep inelastic eP and νP scattering. The nucleon is described as a bound state of three quarks in the $p \rightarrow \infty$ frame. The constituent quarks have themselves a structure suggested by the quark gluon parton model. We thus obtain a good fit to existing data. This fit requires that $SU(6)_W \otimes O(3)$ is broken at $p \rightarrow \infty$ by small configuration mixing.

1. INTRODUCTION

The idea that baryons are bound states of three quarks^{1,2)} accounts for a large number of experimental facts. This paper is devoted to investigating the possibility that such a picture holds also in processes which probe deeper into the nucleon structure, such as deep inelastic, electromagnetic, and weak processes. Contact with the parton description³⁾ of deep inelastic processes will be made by assuming that each "constituent" quark is itself a complex object, made out of point-like partons (quarks, antiquarks, and neutral gluons).

The structure functions of nucleons will thus result from two factors, one determined by the wave function of the three constituents as seen in the infinite momentum frame, the other being the structure function of the constituents themselves.

Let us recall²⁾ that in the quark parton model, the structure functions for eP and eN deep inelastic scattering are expressed in terms of six positive functions: $p(x)$, $n(x)$, $\lambda(x)$; $\bar{p}(x)$, $\bar{n}(x)$, $\bar{\lambda}(x)$. These functions give the average number of point-like quarks or antiquarks of each type inside the parton, with a fraction x of proton momentum (in the infinite momentum frame). We thus have

$$\begin{aligned} \frac{1}{x} F_2^{eP}(x) &= 2F_1^{eP}(x) = \frac{4}{9}(p + \bar{p}) + \frac{1}{9}(n + \bar{n} + \lambda + \bar{\lambda}) \\ \frac{1}{x} F_2^{eN}(x) &= 2F_1^{eN}(x) = \frac{4}{9}(n + \bar{n}) + \frac{1}{9}(p + \bar{p} + \lambda + \bar{\lambda}) \end{aligned} \quad (1)$$

A combination of e.m. and neutrino data indicates that about one half of the nucleon momentum has to be associated with neutral objects [conventionally referred to as gluons⁴⁾]. Since we wish to attribute the whole energy and momentum of the nucleon to the three constituents, these cannot be identified with parton quarks.

In our picture the constituent quarks are themselves complex objects, and their weak and electromagnetic structure functions are described by a set of positive functions $\phi_{ab}(x)$ that specify the number of point-like partons of type b , which are present in a constituent of type a with a fraction x of its total momentum. The functions ϕ_{ab} are not all independent; and are variously restricted by isospin and charge conjugation. Later on we will give a simple model for these functions, inspired by the quark-gluon field theory.

The functions $p(x)$, $n(x)$, etc., describing the nucleon structure functions, are expressed in terms of ϕ_{ab} and of the probability distributions $p_0(x)$ and $n_0(x)$ for p and n constituents inside a proton:

$$\begin{aligned}
 p(x) &= \int_x^1 \frac{dz}{z} \left[p_0(z) \Phi_{pp} \left(\frac{x}{z} \right) + n_0(z) \Phi_{np} \left(\frac{x}{z} \right) \right] \\
 n(x) &= \int_x^1 \frac{dz}{z} \left[p_0(z) \Phi_{pn} \left(\frac{x}{z} \right) + n_0(z) \Phi_{nn} \left(\frac{x}{z} \right) \right],
 \end{aligned} \tag{2}$$

etc. $p_0(x)$ and $n_0(x)$ are determined by the three-quark wave function which describes the proton.

In the infinite momentum frame the wave functions of bound states depend on:

- i) transverse momenta of constituents, $\vec{q}_{\perp i}$;
- ii) longitudinal momenta, expressed as fractions x_i of nucleon momentum;
- iii) spin and unitary spin variables.

Complete symmetry of the wave functions under the exchange of this complex of variables for any two quarks will be assumed, this corresponding either to the existence of three identical triplets of quarks, or to quarks obeying a parastatistic of order three⁵⁾.

Our knowledge of baryon and meson spectroscopy indicates that the wave functions should be classified according to representations of $SU(6)_W \otimes O(3)$ ⁶⁾. The $O(3)$ group is associated to the internal orbital angular momentum, and acts on the variables $x_i, \vec{q}_{\perp i}$. A central part of this paper will be devoted to the specification of the way in which the $O(3)$ structure can be superimposed on the $x_i, \vec{q}_{\perp i}$ space. Independently from the detailed solution to this problem, the experimental results on eP and eN deep inelastic scattering already indicate that some breaking of $SU(6)_W \otimes O(3)$ must be present in the form of a configuration mixing, as suggested by earlier investigations⁷⁾. In fact if the nucleon were assigned to a pure $56, \ell = 0$, we would have $p_0(x) = 2n_0(x)$. Equation (2), taking into account isospin and positivity constraints on ϕ_{ab} , would then lead to

$$\frac{3}{2} \geq \frac{F_2^{eN}}{F_2^{eP}} \geq \frac{2}{3}.$$

The lowest bound is already violated by existing data⁸⁾ for values of the scaling variable $x = -q^2/2Mv$ near one. We will adopt a simple mixing of the $56, \ell = 0$ with the states of a $70, \ell = 1$. This is the next simplest possibility for a completely symmetric wave function, and it is also suggested by the accepted assignment of the low-lying baryon resonances.

To summarize this brief survey of our program, the structure functions of the nucleon are determined on the one hand from a knowledge of the three-quark system wave function, which allows the computation of the probability distributions for constituent quarks, $p_0(x)$, $n_0(x)$; on the other hand, by the structure functions of the constituent quarks themselves, described by $\phi_{ab}(x)$. This information is obviously also sufficient for computing structure functions for neutrino or antineutrino³⁾ scattering.

As we will see, this scheme leads to a satisfactory description of the experimental results on deep inelastic phenomena accumulated so far.

Our scheme can be considered as a step in the direction strongly advocated by Gell-Mann⁵⁾. The analysis we will present indicates that the unitary transformation which carries constituent quarks into current quarks (partons) has the character of the relation between bare and dressed particles in field theory. The features which emerge from our discussion of the functions $\phi_{ab}(x)$ seem to indicate that the nature of this relation is much more complicated than that envisaged in the recent proposal by Melosh⁹⁾.

The present approach can also be compared to the work by Kuti and Weisskopf¹⁰⁾, where the proton is assumed to be made out of three "valence" quarks plus a neutral core of $q\bar{q}$ pairs and gluons. In our scheme gluons and $q\bar{q}$ pairs do not form a separate entity, but rather appear as clouds of virtual particles accompanying each constituent.

This picture seems to us theoretically superior; in particular, it leads to a natural understanding of the relations among Regge residues for strong interaction processes, implied by the so-called quark additivity¹¹⁾, or by the algebra of Regge residues¹²⁾. In fact in our scheme the Regge behaviour of nucleon structure functions derives from the Regge behaviour of ϕ_{ab} . Equation (2) then shows that Regge residues of nucleon structure functions are the sum of the corresponding residues for individual constituents, a property which is propagated to strong interaction phenomena by the factorization property of Regge residues.

The plan of this paper is the following.

Section 2 is devoted to discussing the realization of orbital angular momentum in the $p_z \rightarrow \infty$ frame. In Section 3 we discuss the construction of the intrinsic angular momentum operator out of $SU(6)_W \otimes O(3)$ generators, taking into account the possibility of a mixing, at $p_z \rightarrow \infty$, among states of different orbital angular momentum.

Configuration mixing and the explicit form of the wave functions are discussed in Section 4; Section 5 contains the derivation of $p_0(x)$ and $n_0(x)$.

Constituent structure functions ϕ_{ab} are considered in Section 6. Proton and neutron structure functions are then computed (Section 7) and compared to experimental results. Finally, our conclusions and a discussion of possible further developments of the ideas presented here are contained in Section 8.

2. ORBITAL ANGULAR MOMENTUM IN THE $p \rightarrow \infty$ FRAME

The first problem we have to face is that of writing explicit wave functions for the nucleon in the $p_z \rightarrow \infty$ frame, and to classify them according to representations of $SU(6)_W \otimes O(3)$. In doing so, we will have to specify the action of the group generators on the variables which describe the motion of each constituent. This problem is particularly simple in the case of the static $SU(6) \otimes O(3)$, i.e. the symmetry group for the non-relativistic quark model. The solution there is to let the $SU(6)$ generators act on spin and unitary spin variables, while the $O(3)$ generators act as the usual tridimensional rotations on constituent momenta, $\vec{p}_i$. In the $p_z \rightarrow \infty$ frame it is natural, by extension, to keep this separation. We assume that the generators of $SU(6)_W$ act on spin and unitary spin variables of the constituents^{*)}, while the $O(3)$ group acts on the variables describing the space configuration, i.e. longitudinal and transverse momenta. The problem here is how to superimpose on $O(3)$ structure onto a system with an obvious cylindrical symmetry. In fact, while transverse momenta can vary over the full x-y plane, the longitudinal momenta of each quark can only vary between zero and the total momentum of the bound state, p_z .

We expect that, as $p_z \rightarrow \infty$, the wave function reaches a limiting form, where it depends only on transverse momenta, $\vec{q}_{\perp i}$ and on ratios of longitudinal quark momenta, conveniently expressed in terms of the Feynman variables:

$$\begin{aligned} x_i &= \frac{q_{Li}}{p_z} & i = 1, 2, 3 \\ \sum_i x_i &= 1, & x_i \geq 0 \end{aligned} \tag{3}$$

In terms of these nine variables we want to construct three vectors, $\vec{V}_1, \vec{V}_2, \vec{V}_3$, on which $O(3)$ rotations act in the usual way.

Under rotations around the z axis both the $\vec{q}_{\perp i}$ and the transverse components of $\vec{V}_i$ should transform as two-dimensional vectors. It is then natural to assume that:

*) This is in fact a definition of the spin operator.

i) transverse components of $\vec{V}_i$ are linearly related to $\vec{q}_{\perp i}$.

There must be a constraint among the three $\vec{V}_i$, since we have only six independent variables. This constraint has to be invariant under $O(3)$. The simplest possibility, also suggested by (i), is that this is a linear constraint, which can always be cast into the form

$$\text{ii) } \vec{V}_1 + \vec{V}_2 + \vec{V}_3 = 0 .$$

The introduction of three vectors related by Eq. (ii) is a prerequisite for our next condition:

iii) the V_i should have definite symmetry properties under permutations of quarks.

Since the $q_{\perp i}$ can vary over the full x-y plane, the same applies to the transverse components of $\vec{V}_i$ [condition (i)]. The $O(3)$ covariance then implies that:

iv) the allowed range for V_{iz} is the whole real axis.

This condition implies that V_{iz} cannot be simply linear combinations of the x_k [see Eq. (3)]. Finally we require that

v) the $O(3)$ invariant measure $\prod_i d^3 V_i \delta^3(\sum_\ell \vec{V}_\ell)$ coincides up to a constant⁽⁴⁾ with the Lorentz invariant measure, at $p_z \rightarrow \infty$:

$$\prod_k \frac{dx_k}{x_k} d^2 q_{\perp k} \delta^2\left(\sum_\ell \vec{q}_{\perp \ell}\right) \delta(x_1 + x_2 + x_3 - 1)$$

To gain some insight into the implications of these conditions, let us first discuss the simpler case of a quark-antiquark bound state, where we have only six variables related by the conditions

$$\begin{aligned} \vec{q}_{\perp 1} + \vec{q}_{\perp 2} &= 0 \\ x_1 + x_2 &= 1 \quad (x_1, x_2 \geq 0) \end{aligned} \tag{4}$$

We will have two V vectors: $\vec{V}_1$ and $\vec{V}_2 = -\vec{V}_1$. Condition (i) and Eqs. (4) imply that we can put

$$\vec{V}_1 \equiv (\vec{q}_{\perp 1}, f(x_1, |\vec{q}_{\perp 1}|))$$

Condition (v) then implies that

$$f(x_1, |\vec{q}_{\perp 1}|) = m \ln \frac{x_1}{1-x_1} + g(|\vec{q}_{\perp 1}|)$$

Under the exchange $1 \leftrightarrow 2$, $\vec{q}_{\perp 1} \rightarrow \vec{q}_{\perp 2}$, $x_1 \rightarrow (1 - x_1)$. Since transverse components of $\vec{V}_1$ are odd under such an exchange, the same should be true of the longitudinal component, so that $g(|\vec{q}_{\perp 1}|) = 0$. We thus see that in this case conditions (i)-(v) completely determine the $\vec{V}_i$ up to a factor m , which has the dimension of a mass.

In the case of interest, namely that of three-quark bound states, a class of solutions to conditions (i)-(v) is given by the following:

$$\begin{aligned} \vec{V}_1 &= \vec{Q}_1 - \frac{1}{2} (\vec{Q}_2 + \vec{Q}_3) \\ \vec{V}_2 &= \vec{Q}_2 - \frac{1}{2} (\vec{Q}_3 + \vec{Q}_1) \\ \vec{V}_3 &= \vec{Q}_3 - \frac{1}{2} (\vec{Q}_1 + \vec{Q}_2) \end{aligned} \tag{5}$$

where $\vec{Q}_k$ are only functions of the variables of the k th quark, and have the form

$$\vec{Q}_k \equiv [\vec{q}_{\perp k}, m \ln x_k + h(|\vec{q}_{\perp k}|)] \tag{6}$$

m being a parameter with the dimension of a mass. In the Appendix we shall prove that this is, in fact, the most general solution, under the hypothesis that we replace condition (i) by a stronger one, namely:

i') The vectors $\vec{V}_i$ are linear combinations of three vectors $\vec{Q}_k$, each being a function of the variables of the k^{th} quark, of the form

$$\vec{Q}_k \equiv [\vec{q}_{\perp k}, f(x_k, |\vec{q}_{\perp k}|)] .$$

The solution is defined up to an arbitrary function $h(|\vec{q}_{\perp}|)$. (This ambiguity is absent in the case of $q\bar{q}$ bound states, discussed above, where $|\vec{q}_{\perp 1}| = |\vec{q}_{\perp 2}|$.) By a suitable choice of $h(|\vec{q}_{\perp}|)$, the z -component of $\vec{Q}$ can be identified with a multiple of the rapidity. In fact if we choose

$$(\vec{Q})_k = m \ln \frac{2 \times p_k}{\sqrt{m^2 + q_\perp^2}} =$$

$$= \lim_{p_k \rightarrow \infty} \frac{m}{2} \ln \frac{E + q_k}{E - q_k}$$

$(\vec{Q})_z$ is the $p_z \rightarrow \infty$ limit of a quantity that in the non-relativistic limit reduces to q_z . Note that the infinite additive constant, $m \ln p_z$, disappears in building the $\vec{V}_i$'s.

In the construction of the explicit wave functions that we use in the following sections, we have adopted a simpler ansatz, which consists in setting $h(|\vec{q}_\perp|) = 0$, i.e. we use

$$\vec{Q}_k = (\vec{q}_\perp k, m \ln x_k) \quad (7)$$

This choice is only motivated by the simplicity of the computation, but for the particular physical problems we discuss in this note it should not be very different from the rapidity scheme, which will be the object of further study.

To summarize this section, the wave function for a three-quark bound state in the $p_z \rightarrow \infty$ limit, will have the form

$$\Psi_{A_1 A_2 A_3} (\vec{V}_1, \vec{V}_2, \vec{V}_3)$$

Acceptable wave functions will be symmetric under the exchange of any two quarks.

The $SU(6)_W \otimes O(3)$ approximate symmetry is implemented by letting $SU(6)_W$ act on the discrete indices A_1, A_2, A_3 , while $O(3)$ acts on $\vec{V}_1, \vec{V}_2, \vec{V}_3$. The $\vec{V}_i$'s have been constructed in such a way that they have the character of relative coordinates of the quarks, so that $O(3)$ automatically commutes with the total momentum of the bound state.

3. INTRINSIC ANGULAR MOMENTUM AND THE ROLE OF PARITY

We have anticipated in the Introduction that in our scheme it is not possible to obtain a good description of proton and neutron electromagnetic structure functions if nucleons belong to a pure $56, l = 0$, the preferred assignment in the limit of exact symmetry. Some breaking of $SU(6)_W \otimes O(3)$ is needed, in the form of a configuration mixing. It is here that the $p_z \rightarrow \infty$ scheme differs substantially from the non-relativistic quark model. In fact in the latter case mixing can only be allowed among states with the same orbital parity. This forbids, for example, a

mixing between a $56, \ell = 0$ and a $70, \ell = 1$, and the ground state can only mix with higher states. In the $p_z \rightarrow \infty$ frame, physical one-particle states are not eigenstates of parity (which sends $p_z \rightarrow -p_z$) and there is no selection rule on orbital parity, which we define to be the operation P_0 which sends $\vec{V}_1$ into $-\vec{V}_1$. Thus a mixing between $56, \ell = 0$ and $70, \ell = 1$ is allowed, and we expect it to be the most relevant one.

Conservation of parity gives certain restrictions on the possible form of the mixing and on the relation of the intrinsic angular momentum to the quark spin and the orbital angular momentum. The origin of this restriction can be simply illustrated as follows. If the proton is a mixture of a $56, \ell = 0$ and $70, \ell = 1$, which have opposite orbital parity, the longitudinal distribution of partons will be asymmetric in x space. With an appropriate sign of the mixing coefficients, we can arrange a p -quark to carry, on the average, a fraction of proton momentum higher than the mean value $x = 1/3$, while the n -quark will carry a fraction lower than $1/3$. This is in fact a desired effect, needed to explain the ratio F_2^{eN}/F_2^{eP} in the $x \sim 1$ region, which indicates a prevalence of p -type over n -type quarks.

Parity conservation requires then this asymmetry to be the same in the two helicity states of the proton (for higher spin particles, similar asymmetries must be the same for states of opposite helicity). In our language such an asymmetry corresponds to a non-vanishing expectation value for the z -component of the $\vec{V}$ vector of any given quark. We have then to require that:

$$\langle s, \lambda | V_z | s, \lambda \rangle = \langle s, -\lambda | V_z | s, -\lambda \rangle \quad (8)$$

for any physical state at $p_z \rightarrow \infty$, with spin s and helicity λ .

States of opposite helicity are related by the equation

$$e^{-i\pi J_2} |s, \lambda\rangle = (-1)^{s-\lambda} |s, -\lambda\rangle \quad (9)$$

where J_2 is the y -component of the intrinsic angular momentum of the particle. Equations (8) and (9) imply

$$e^{i\pi J_2} V_z e^{-i\pi J_2} = V_z \quad (10)$$

The intrinsic angular momentum $\vec{J}$ must be a combination of the W -spin operator $\vec{\Sigma}$, contained in the $SU(6)_W$ algebra, and of the orbital angular momentum $\vec{L}$, which generates $O(3)$. In particular:

$$J_3 = \Sigma_3 + L_3 \quad (11)$$

The simplest generalization of Eq. (11), $\vec{J} = \vec{\Sigma} + \vec{L}$, is however in contradiction with Eq. (10), since $e^{i\pi(\Sigma_2+L_2)}$ anticommutes with V_z . To obtain a consistent definition of J_2 we can use the orbital parity operator P_0 , which also anticommutes with V_3 , and define

$$e^{i\pi J_2} = P_0 e^{i\pi(\Sigma_2+L_2)} \quad (12)$$

Since both Σ_2 and J_2 have half-integer eigenvalues, this can be realized in two different ways, i.e. by defining^{*)}

$$J_{1,2} = L_{1,2} + P_0 \Sigma'_{1,2} \quad (13)$$

or alternatively

$$J_{1,2} = P_0 (L_{1,2} + \Sigma'_{1,2}) \quad (14)$$

The appearance of the operators $P_0 \Sigma'_{1,2}$ in both definitions gives a generalization of the known connection of the W-spin with quark spin. We have a slight preference for the first definition. In fact, as we have seen in Section 2, the variable $\vec{V}$ can be chosen in such a way (the rapidity scheme) that we can pass smoothly from the $p_z \rightarrow \infty$ to the $p_z = 0$ frame. If we choose the definition given in Eq. (13), the same holds for the orbital part of the angular momentum. In this paper we will adopt Eq. (13), although the results presented would not change if we had taken the other alternative.

4. WAVE FUNCTIONS

As a basis of wave functions we will select the family of symmetric states of three quarks in an harmonic oscillator well. This reproduces the known ordering of states: (56, $\ell = 0$), (70, $\ell = 1$), etc. The correctness of this choice can be checked *a posteriori* by the amount of mixing necessary to describe physical states.

This family includes a ground state 56, $\ell = 0$, followed by 70, $\ell = 1$, the higher states being obtained by the addition of further L and radial excitations. This pattern obviously coincides with the one obtained in the non-relativistic quark model, and is known to be in fair agreement with baryon spectroscopy data²⁾.

Guided by these considerations, we shall describe a three-quarks bound state with a wave function of the form

*) The definition of J_1 follows from that of J_2 and J_3 through O(3) commutation rules.

$$\psi_{ABC}(\vec{V}_1, \vec{V}_2, \vec{V}_3) = \phi_{ABC}(\vec{V}_1, \vec{V}_2, \vec{V}_3) e^{-k \sum_i (\vec{V}_i)^2 / 2} \quad (15)$$

In this formula, A,B,C are $SU(6)_W$ indices, i.e. $A = (a, \alpha)$, $a = 1, 2$; $\alpha = 1, 2, 3$ and ψ is completely symmetric under the exchange of the pairs $(A, \vec{V}_1)$, $(B, \vec{V}_2)$, $(C, \vec{V}_3)$. $\phi_{ABC}(\vec{V}_1, \vec{V}_2, \vec{V}_3)$ is a combination of polynomials in the $\vec{V}_i$'s and $SU(6)_W$ tensors, and determines the $SU(6)_W \otimes O(3)$ properties of ψ . The exponential fall-off of ψ in the constituents transverse momenta $\vec{q}_{\perp i}$ is described by the parameter k . On the other hand, recalling Eq. (6) the dependence on x_i is determined by the independent, adimensional combination:

$$\beta = k m^2$$

As we will see later, since the $(V_i)_z$'s are logarithmic functions of the x_i 's, the exponential fall-off of ψ in $(V_i)_z$ leads essentially to a power behaviour in x -space, for the structure functions, with exponents which are themselves slowly varying (logarithmic) functions of x .

Given the wave function Eq. (15), the probability for finding a constituent of type q (i.e. p , n , or λ) with a fraction x of longitudinal momentum, is given by

$$q_\psi(x) = \int d^3V_1 d^3V_2 d^3V_3 \delta^3(\sum \vec{V}_i) \delta(x - x_1(\vec{V})) \bar{\psi}^{A'BC} q_{A'}^A \psi_{ABC} \quad (16)$$

Here and in the following a summation over repeated indices is understood; $x_1(\vec{V})$ is the value for x_1 obtained by inverting the equations which define the $\vec{V}_i$'s [Eq. (5) together with Eq. (6) or Eq. (7)], with the constraint

$$x_1 + x_2 + x_3 = 1 \quad (17)$$

$q_{A'}^A$ indicates one of the three projectors defined as

$$\begin{aligned} p_{A'}^A &= \delta_{a a'} \delta_{\alpha 1} \delta_{\alpha' 1} \\ n_{A'}^A &= \delta_{a a'} \delta_{\alpha 2} \delta_{\alpha' 2} \\ \lambda_{A'}^A &= \delta_{a a'} \delta_{\alpha 3} \delta_{\alpha' 3} \end{aligned} \quad (18)$$

The wave function ψ will be normalized to the total number of constituents, i.e.

$$\int_0^1 (p_\psi + n_\psi + \lambda_\psi) dx = \int d^3\vec{v}_1 d^3\vec{v}_2 d^3\vec{v}_3 \delta^3(\sum \vec{v}_i) \bar{\psi}^{ABC} \psi_{ABC} = 3. \quad (19)$$

Equations (15) and (16) imply also

$$\int_0^1 (p_\psi + n_\psi + \lambda_\psi) x dx = 1, \quad (20)$$

i.e. the constituents carry all the momentum of the bound state.

The explicit form for unmixed $56, \ell = 0$ and $70, \ell = 1$ wave functions can be obtained from Eq. (15) with

$$\phi_{ABC}^{56,0}(\vec{v}_1, \vec{v}_2, \vec{v}_3) = N_{56} T_{\{ABC\}} \quad (21)$$

$$\phi_{ABC}^{70,1}(\vec{v}_1, \vec{v}_2, \vec{v}_3) = N_{70} (T_{A\{BC\}} \vec{v}_1 + T_{B\{CA\}} \vec{v}_2 + T_{C\{AB\}} \vec{v}_3), \quad (22)$$

where $T_{\{ABC\}}$ is a fully symmetric $SU(6)_W$ tensor, $T_{A\{BC\}}$ is a mixed tensor, symmetric in B and C, and such that

$$T_{A\{BC\}} + T_{B\{CA\}} + T_{C\{AB\}} = 0.$$

The normalization factors N_{56} and N_{70} can be explicitly obtained from Eq. (19), once a normalization is chosen for the $SU(6)_W$ tensors, e.g.

$$\overline{T_{\{ABC\}}} T_{\{ABC\}} = \overline{T^{A\{BC\}}} T_{A\{BC\}} = 1. \quad (23)$$

Within any $SU(6)_W \otimes O(3)$ multiplet, we may define a basis of states with definite $SU(3)$ properties and definite values for L^2 , Σ^2 , L_z , Σ_z , which we will label as follows:

$$|SU(3), \Sigma, \Sigma_z, L, L_z\rangle \quad (24)$$

Out of these states, we then construct the eigenstates of intrinsic angular momentum $\vec{J}$. The definition of $\vec{J}$ given in Eqs. (11) and (13) implies that the usual Clebsch-Gordan coefficients $(JJ_z | \Sigma \Sigma_z, LL_z)$ have to be modified by an extra phase factor, which can be easily computed to be

$$\eta = \begin{cases} 1 & \text{for } P_0 = 1 \\ (-1)^{\Sigma - \Sigma \eta} & \text{for } P_0 = -1 \end{cases}$$

In the complex of the $56, \ell = 0$ and $70, \ell = 1$ states there are three spin $\frac{1}{2}$ octets which, according to this rule, have the explicit expression (for $J_z = +\frac{1}{2}$):

$$\begin{aligned} |A\rangle &= |8, \frac{1}{2}, \frac{1}{2}, 0, 0\rangle_{56} \\ |B\rangle &= -\frac{1}{\sqrt{3}} |8, \frac{1}{2}, \frac{1}{2}, 1, 0\rangle_{70} - \frac{\sqrt{2}}{3} |8, \frac{1}{2}, -\frac{1}{2}, 1, 1\rangle_{70} \\ |C\rangle &= \frac{1}{\sqrt{2}} |8, \frac{3}{2}, \frac{3}{2}, 1, -1\rangle_{70} + \frac{1}{\sqrt{3}} |8, \frac{3}{2}, \frac{1}{2}, 1, 0\rangle_{70} + \frac{1}{\sqrt{6}} |8, \frac{3}{2}, -\frac{1}{2}, 1, 1\rangle_{70} \end{aligned} \quad (25)$$

The physical baryon octet will be a linear superposition of these states.

On the basis of Eqs. (16), (21), (22), and (25) we can explicitly compute the distribution functions $p_0(x)$ and $n_0(x)$, i.e. the average number of p and n constituents inside a proton, with a fraction x of total momentum. In the case of a pure $56, \ell = 0$, the wave function factorizes into a space part and $SU(6)_W$ tensor [see Eqs. (15) and (21)] so that

$$p_0(x) = 2 \eta_0(x) \quad (\text{unmixed } 56, \ell = 0) \quad (26)$$

As mentioned in the Introduction, this leads in our scheme to the bound:

$$\frac{3}{2} \geq \frac{F_2^{eN}}{F_2^{eP}} \geq \frac{2}{3} \quad (\text{unmixed } 56, \ell = 0)$$

which is contradicted by experimental data in the region $x \sim 1$.

To improve the situation, we will assume that the proton is a mixture of states $|A\rangle$ and $|B\rangle$:

$$|P\rangle = a|A\rangle + b|B\rangle, \quad |a|^2 + |b|^2 = 1 \quad (27)$$

We discard the possibility of adding a $|C\rangle$ component, as it can be shown that this would increase the ratio F_2^{eN}/F_2^{eP} near $x = 1$. Only a small admixture of $|C\rangle$ can be allowed, but we neglect it for simplicity.

5. EXPLICIT EVALUATION OF DISTRIBUTION FUNCTIONS FOR CONSTITUENTS

We now proceed to evaluate the distribution functions $p_0(x)$ and $n_0(x)$, on the basis of the equations derived in the previous section.

From Eq. (27) it follows that

$$p_0(x) = |a|^2 p_{0AA} + |b|^2 p_{0BB} + 2 \operatorname{Re}(a^* b) p_{0AB} \quad (28)$$

and a similar formula holds for $n_0(x)$. In the following we shall restrict ourselves to real values of a and b . Putting together Eqs. (15), (16), (21), (22), and (25), we get, by a simple computation,

$$p_{0AA} = 2 n_{0AA} = \frac{6\sqrt{3} k^3}{\pi^3} \int d\Omega \delta(x - x_1(\vec{r})) e^{-k \sum_i (\vec{V}_i)^2}$$

$$p_{0BB} = 2 n_{0BB} = \frac{2\sqrt{3} k^4}{\pi^3} \int d\Omega \delta(x - x_1(\vec{r})) \sum_i |\vec{V}_i|^2 e^{-k \sum_i |\vec{V}_i|^2} \quad (29)$$

$$p_{0AB} = -n_{0AB} = \frac{3\sqrt{3} k^{7/2}}{\sqrt{2} \pi^3} \int d\Omega \delta(x - x_1(\vec{r})) (\vec{V}_1)_z e^{-k \sum_i |\vec{V}_i|^2}$$

where $d\Omega = \prod_i d^3 V_i \delta^3(\sum_i \vec{V}_i)$

For completeness we also record the corresponding expressions for contributions relative to the state $|C\rangle$:

$$\left\{ \begin{array}{l} p_{0CC} \\ n_{0CC} \end{array} \right\} = \frac{3\sqrt{3} k^4}{\pi^3} \int d\Omega \delta(x - x_1(\vec{r})) \left\{ \begin{array}{l} (\vec{V}_2)^2 + (\vec{V}_3)^2 \\ (\vec{V}_1)^2 \end{array} \right\} e^{-k \sum_i (\vec{V}_i)^2} \quad (29')$$

For the particular choice of the $(V_i)_z$ given in Eq. (7), the integrations over transverse momenta can be easily performed. The integrations over the z -components of $\vec{V}_i$'s can be reduced, using the δ -functions, to a single integration, which however cannot be analytically performed. The latter steps require the explicit expression of x_1 in terms of the $\vec{V}$'s, which is:

$$x_1 = [1 + 2 e^\alpha \operatorname{ch} \alpha]^{-1}$$

where

$$m \alpha = (\vec{V}_2 + \vec{V}_3)_{\mu}$$

$$m \gamma = \frac{1}{3} (\vec{V}_2 - \vec{V}_3)_{\mu}$$

We thus arrive at the expressions

$$p_{0AA}(x) = \frac{3\sqrt{3}\beta}{\pi} \frac{1}{x(1-x)} \int_{-\infty}^{+\infty} d\gamma e^{-\beta C(\gamma, x)}$$

$$p_{0BB}(x) = \frac{2}{3} p_{0AA} + \frac{\sqrt{3}\beta^2}{\pi} \frac{1}{x(1-x)} \int_{-\infty}^{+\infty} d\gamma C(\gamma, x) e^{-\beta C(\gamma, x)}$$

$$p_{0AB}(x) = \left(\frac{3}{2}\beta\right)^{3/2} \frac{1}{\pi} \frac{1}{x(1-x)} \int_{-\infty}^{+\infty} d\gamma \ln\left(\frac{2x}{1-x} \cosh \gamma\right) e^{-\beta C(\gamma, x)} \quad (30)$$

$$C(\gamma, x) = \frac{g}{2} \gamma^2 + \frac{3}{2} \left[\ln\left(\frac{2x}{1-x} \cosh \gamma\right) \right]^2.$$

$$\beta = k m^2$$

The integrals appearing in these equations have been computed numerically. An analytic approximation to the functions p_{0AA} , p_{0BB} , p_{0AB} can be obtained by performing the integrations by the method of the steepest descent, i.e. by developing $C(\gamma, x)$ in powers of γ , and retaining only terms up to γ^2 . By this method we find, for example,

$$p_{0AA} \sim \left(\frac{3\beta}{2\pi}\right)^{3/2} \frac{1}{x^2} \left(\frac{1-x}{2x}\right)^{-1 + \frac{3}{2}\beta \ln\left(\frac{2x}{1-x}\right)} \left[1 + \frac{1}{3} \ln\left(\frac{2x}{1-x}\right)\right]^{-1/2} \quad (31)$$

This formula is expected to hold for $x \sim 1$ and not-too-small values of β . In fact we have found it to give a fairly good representation of p_{0AA} down to $x \sim 1/3$, for β in the range of values we will be interested in, i.e. $\beta \sim 0.3$.

Equation (31) shows that the distribution functions, $p_0(x)$ and $n_0(x)$, exhibit an approximate power behaviour in $(1-x)$, the exponent being itself a logarithmic function of x , as we have anticipated in Section 4. The effective exponent, as obtained by taking the logarithmic derivative of p_{0AA} , is

$$\alpha_0(x) = -1 + 3\beta \ln\left(\frac{2x}{1-x}\right). \quad (32)$$

6. POINT-LIKE PARTONS IN A CONSTITUENT QUARK

In this section we discuss the structure of the functions $\phi_{q_1 q_2}(x)$, which express the distribution of type q_2 partons (point-like quarks or antiquarks) of momentum $x p_z$ within a constituent quark of momentum p_z and of type q_1 . These functions are related to the structure functions of constituent quarks. In contrast with the nucleon structure functions, which have a complicated behaviour due to the nucleon composite nature, we expect the structure functions of constituents to have a relatively simple behaviour, to be derived on the basis of general theoretical ideas. In building our ansatz for $\phi_{q_1 q_2}$, we will be guided by

- i) the quark-gluon model, according to which the point-like particles are quarks and neutral $[SU(3) \text{ singlet}]$ mesons;
- ii) Regge behaviour for $x \rightarrow 0$ (corresponding to $\nu \rightarrow \infty$ in deep inelastic scattering off a constituent) and duality ideas.

The structure function of a constituent can be represented as the s-channel absorptive part of a forward current-constituent scattering amplitude (Fig. 1). Scaling contributions to such an absorptive part can be represented as in Fig. 2, and in the gluon model can be separated into three classes.

The first two classes include diagrams where the currents are attached to the fermion line continuously connected to the incoming and outgoing lines (Fig. 2, a and b). In diagrams of the third class, the currents are attached to a fermion loop, connected to the external lines by an arbitrary number of gluon lines (Fig. 2, c). Diagrams of the first type contribute only if the parton q_2 is of the same type as the constituent q_1 . In the second class of diagrams $q_2 = \bar{q}_1$, while diagrams of the third class give contributions that are independent both from q_2 and q_1 .

Both diagrams of type (a) and (b) in Fig. 2 contain a quark-antiquark pair in the t-channel. Thus their Regge behaviour (i.e. their behaviour for $x \rightarrow 0$) is naturally associated to the normal, non-Pomeron trajectories (ρ , A_2 , f_0 , etc.). On the other hand, diagrams of type (b), when cut in the s-channel, contain states which belong to 3, $\bar{6}$, and 15 $SU(3)$ representations, and would lead to s-channel baryonic exotic states when recombined with the other two constituents (which act here as spectator particles, Fig. 3).

Duality requires such contributions to be much smaller than those arising from diagrams (a). The same conclusion can obviously be drawn if one extends to the Regge-quark couplings the requirements of exchange degeneracy of vector and tensor meson trajectories. We will therefore neglect contributions of type (b).

Diagrams of type (c) correspond to a pure SU(3) singlet, C-even exchange in the t-channel, which will be identified with the Pomeron trajectory. In conclusion, writing

$$\phi_{q_1 q_2}^{(a)}(x) = \delta_{q_1 q_2} \phi^{(a)}(x) \quad (33)$$

$$\phi_{q_1 q_2}^{(c)}(x) = \phi^{(c)}(x) \quad (34)$$

we shall assume that, for $x \rightarrow 0$,

$$\phi^{(a)}(x) \underset{x \rightarrow 0}{\sim} B/\sqrt{x} \quad (35)$$

$$\phi^{(c)}(x) \underset{x \rightarrow 0}{\sim} C/x \quad (36)$$

The two contributions (a) and (c) have a simple interpretation in a parton picture of the constituent. At $p_z \rightarrow \infty$, a constituent is supposed to be made up of

- i) a leading parton, of the same type of the constituent itself;
- ii) a "sea" of neutral gluons;
- iii) a "sea" of parton-antiparton pairs.

Contributions (a) and (c) correspond to the leading parton and to the sea pairs distribution, respectively. The dx/x law expressed by Eq. (36) corresponds precisely to the bremsstrahlung-like behaviour suggested by non-covariant, time-independent perturbation theory

A similar behaviour is also expected to hold for the probability distribution of the neutral gluons:

$$\phi_{gg}(x) \underset{x \rightarrow 0}{\sim} \frac{G}{x} \quad (37)$$

The average number of soft sea partons of any kind inside a given constituent can thus be written, on the basis of Eqs. (36) and (37), as

$$dN(x) = A \frac{dx}{x} \quad (x \rightarrow 0) \quad (38)$$

$$A = G + 6C$$

This information can be used to derive the $x \rightarrow 1$ behaviour of the leading parton distribution, $\phi_{q_1 q_2}^{(a)}$. In fact the probability for finding the leading parton with a fraction x of the constituent momentum is equal to the probability that all the other partons carry away exactly a fraction $1 - x$. For $x \rightarrow 1$, the latter partons are pushed into the infra-red region, and the probability we need can be directly computed from Eq. (38), assuming soft partons to be independently emitted. The computation is identical to the computation of infra-red corrections in the Block-Nordsieck approximation¹⁴⁾, and yields

$$\phi^{(a)}(x) \underset{x \rightarrow 1}{\sim} \text{const.} \cdot (1-x)^{A-1} \quad (39)$$

A being the (positive) quantity defined in Eq. (38).

Finally, we note that since the sea is on the average neutral, $\phi^{(a)}$ has to satisfy the normalization condition

$$\int dx \phi^{(a)}(x) = 1 \quad (40)$$

Equation (35), (39), and (40) suggest the simple ansatz:

$$\phi^{(a)}(x) = \frac{\Gamma(A + \frac{1}{2})}{\Gamma(\frac{1}{2}) \Gamma(A)} \frac{(1-x)^{A-1}}{\sqrt{x}} \quad (41)$$

With this ansatz, the average fraction of momentum associated to the leading parton is

$$\int dx x \phi^{(a)}(x) = \frac{1}{2A+1} \quad (42)$$

As for $\phi_{q_1 q_2}^{(c)}$, we have no information concerning its behaviour at $x = 1$, except that it should not dominate over $\phi_{q_1 q_2}^{(a)}$. We will choose the simple form

$$\phi^{(c)}(x) = \frac{c}{x} (1-x)^{D-1} \quad (43)$$

The average fraction of momentum carried by quark-pairs turns out to be

$$6 \int dx x \phi^{(c)}(x) = \frac{6c}{D} \quad (44)$$

We note two inequalities:

$$A \geq 6C$$

$$1 - \frac{6C}{D} - \frac{1}{2A+1} \geq 0$$

which follow respectively from the positivity of G and the fraction of momentum carried by gluons.

7. THE NUCLEON STRUCTURE FUNCTIONS

Given the distribution functions for constituents within a proton [$p_0(x)$, $n_0(x)$] and the distribution functions for point-like quarks within a constituent [$\phi_{q_1 q_2}(x)$], we can now compute the structure functions for protons and neutrons. Nucleon structure functions are simple combinations of the distribution functions for point-like constituents in a proton [Eq. (1)]. On the basis of Section 6, Eq. (2) can be rewritten as

$$p(x) = \int_x^1 \frac{dx}{x} \left\{ p_0(x) \left[\phi^{(a)}\left(\frac{x}{x}\right) + \Phi^{(c)}\left(\frac{x}{x}\right) \right] + n_0(x) \Phi^{(c)}\left(\frac{x}{x}\right) \right\} \quad (45)$$

$$n(x) = \int_x^1 \frac{dx}{x} \left\{ n_0(x) \left[\phi^{(a)}\left(\frac{x}{x}\right) + \Phi^{(c)}\left(\frac{x}{x}\right) \right] + p_0(x) \Phi^{(c)}\left(\frac{x}{x}\right) \right\} \quad (46)$$

$$\bar{p}(x) = \bar{n}(x) = \lambda(x) = \bar{\lambda}(x) = \int_x^1 \frac{dx}{x} \left[p_0(x) + n_0(x) \right] \Phi^{(c)}\left(\frac{x}{x}\right) \quad (47)$$

$\phi^{(a)}$ and $\phi^{(c)}$ have been defined in Eqs. (33) and (34). Comparison with experimental data will be done by using for $p_0(x)$ and $n_0(x)$ the expressions derived in Eqs. (28), (29), and (30), i.e. by using the simplest realization of $O(3)$ algebra obtained in Eq. (7).

We postpone to a future analysis the investigation of alternative schemes, e.g. the rapidity realization of $O(3)$.

Neutrino data indicate that the average momentum carried by $\bar{p}$ and $\bar{n}$ partons is very small⁴⁾:

$$\int dx \ x \left[\bar{p}(x) + \bar{n}(x) \right] = 0.035 \pm 0.015. \quad (48)$$

Accordingly we expect the contribution of pairs to structure functions to be negligible for $x \sim 1$. In this region structure functions must be accounted for by only $p_0(x)$, $n_0(x)$, and $\phi^{(a)}(x)$, which depend upon three free parameters:

- i) b/a and β , which determine the configuration mixing and the shape of wave functions;
- ii) A , which fixes $\phi^{(a)}$.

The same parameters determine the difference $F_2^{eP} - F_2^{eN}$ for all values of x .

Our fitting procedure has been the following. We first determine b/a , β , and A by fitting F_2^{eP} for $x \gtrsim 0.6$ and $F_2^{eP} - F_2^{eN}$ over the full range of x . Having done this, we determine the two additional parameters C and D which appear in the pair contribution $\phi^{(c)}$, by fitting F_2^{eP} over the full range of x .

Comparing Eqs. (44) and (48) we see that

$$\frac{2C}{D} \approx 0.035 ,$$

On the other hand, the average momentum carried by charged partons is known to be approximately 54% ⁴⁾ of the total so that [see Eqs. (42) and (44)]

$$\frac{1}{2A+1} + \frac{6C}{D} \approx 0.54 .$$

From this A can be approximately determined to be:

$$A \approx 0.65 .$$

We have already noticed that p_0 and n_0 have an approximate power behaviour near $x \sim 1$ [see Eqs. (31) and (32)]:

$$p_0, n_0 \sim (1-x)^{\alpha_0}$$

The effect of folding p_0 and n_0 with $\phi^{(a)}$ is that of giving also to $p(x)$ and $n(x)$ an approximate power behaviour, with an effective exponent $\alpha = \alpha_0 + A$. Experimental data ¹⁵⁾ indicate that F_2^{eP} behave approximately as $(1-x)^3$. If we require $\alpha \sim 3$ for $x \sim 0.9$, we get

$$9\beta + A \approx 4$$

i.e.

$$\beta \approx 0.37$$

As for the mixing parameters, we observe that to obtain $F_2^{eN}/F_2^{eP} < 2/3$ in the $x \sim 1$ region, b/a is required to be negative (as can be seen from the equations derived in Section 5).

Finally, the C parameter is related to $F_2^{eP}(0)$ according to

$$F_2^{eP}(0) = 4C \approx 0.20 \quad (\text{see Ref. 15})$$

so that approximate values for C and D are

$$C \approx 0.05$$

$$D \approx 2.8$$

This discussion has been given here to illustrate the physical significance of the parameters, and the criteria used in choosing initial values for the fitting procedure.

Figure 4 shows a comparison of our results for F_2^{eP} with experimental data. The dotted line corresponds to the leading parton contribution [obtained by dropping the $\phi^{(c)}$ term], and the full line represents the complete structure function. In Fig. 5 we report our results for the difference $F_2^{eP} - F_2^{eN}$. The corresponding values for the parameters are:

$$\begin{aligned} A &= 0.80 \\ C &= 0.03 \\ D &= 1.4 \\ \beta &= 0.35 \\ a^2 &= 1 - b^2 = 0.91 \quad (b/a < 0) \end{aligned} \quad (49)$$

The values for A and β which give a good fit to data are very close to their initial values, as given by our previous arguments. On the other hand, to obtain a good fit to F_2^{eP} , we had to lower the parameter C considerably. Its value, as quoted in Eq. (49), now gives

$$F_2^{eP}(x=0) = 4C = 0.12$$

The quality of the fit we obtain for $F_2^{eP} - F_2^{eN}$ is excellent for $x \geq 0.3$, but not so good for smaller values. This discrepancy could indicate the need for a more complex configuration mixing and/or a more complicated behaviour of the structure functions of constituents. For completeness we quote our values for the integrals of F_2^{eP} and F_2^{eN} :

$$\int F_2^{eP} dx = 0.162$$

$$\int F_2^{eN} dx = 0.108$$

In Fig. 6 we have reported the functions $x p(x)$, $x n(x)$, $x \bar{\lambda}(x)$ separately. In various combinations they determine completely the nucleon structure functions in ν and $\bar{\nu}$ deep inelastic scattering³⁾. The available experimental information⁴⁾ on ν and $\bar{\nu}$ processes is, up to now, mainly related to integrated quantities which in the quark-parton model are simple combinations of integrals of the form $\int dx x p(x)$, $\int dx x n(x)$, etc. In our scheme all these quantities can be derived from three basic integrals, which are:

$$\int dx x p(x) = 0.293 \quad (50)$$

$$\int dx x n(x) = 0.133 \quad (51)$$

$$\int dx x \bar{P}(x) = 0.0214 \quad (52)$$

More explicitly, our predictions for the experimentally observed quantities up to now are (for simplicity we set to zero the Cabibbo angle, θ):

$$\frac{1}{2} \int (F_2^{\nu P} + F_2^{\nu N}) dx = 0.469 \quad (53)$$

$$\frac{\sigma(\bar{\nu} P) + \sigma(\bar{\nu} N)}{\sigma(\nu P) + \sigma(\nu N)} = 0.419 \quad (54)$$

For completeness, we finally record our predictions for total $\nu, \bar{\nu}$ cross-sections on nucleons:

$$\begin{pmatrix} \sigma^{\nu P} \\ \sigma^{\bar{\nu} P} \end{pmatrix} = \frac{G^2 M E}{\pi} \int_0^1 dx \begin{pmatrix} n + \frac{1}{3} \bar{P} \\ \frac{1}{3} p + \bar{n} \end{pmatrix} = \frac{G^2 M E}{\pi} \begin{pmatrix} 0.280 \\ 0.238 \end{pmatrix} \quad (55)$$

$$\begin{pmatrix} \sigma^{\nu N} \\ \sigma^{\bar{\nu} N} \end{pmatrix} = \frac{G^2 M E}{\pi} \int dx \, 2x \begin{pmatrix} p + \frac{1}{3} \bar{\pi} \\ \frac{1}{3} n + \bar{p} \end{pmatrix} = \frac{G^2 M E}{\pi} \begin{pmatrix} 0.601 \\ 0.131 \end{pmatrix} \quad (56)$$

In Eqs. (55) and (56) G , M , and E denote the Fermi constant, the nucleon mass, and ν or $\bar{\nu}$ laboratory energy, respectively. Equations (53) and (54) are in very good agreement with the experimental results⁴⁾.

8. CONCLUSIONS AND OUTLOOKS

The results presented in this paper show that a coherent description of deep inelastic phenomena on nucleons can be obtained by assuming that the nucleon is a bound state of three constituent quarks. The resulting formalism is very similar to the impulse approximation used in high-energy nuclear phenomena. The knowledge of the wave functions which we have obtained from deep inelastic processes will allow a better understanding of the static properties of baryons, such as G_A/G_V , the anomalous magnetic moments, etc. We shall discuss this problem in a separate paper, and add here only a few comments.

It is well known that in the limit where constituents are point-like objects, and no configuration mixing is allowed, all anomalous magnetic moments would vanish and $(G_A/G_V)_{\text{nucleon}} = 5/3$. Deviations from this situation arise from two different sources. One is the configuration mixing, as was proposed some years ago⁷⁾. The second source is the non-elementary nature of constituents⁹⁾.

Mixing the $56, \ell = 0$ with $70, \ell = 1$ essentially reduces $(G_A/G_V)_{\text{nucleon}}$ by a factor a^2 [see Eq. (27)]. If this were the only effect we would get

$$(G_A/G_V)_{\text{nucleon}} \approx \frac{5}{3} a^2 \approx 1.52$$

This indicates that a sizeable fraction of axial current renormalization should come from the structure of the constituents. The same conclusion can be reached by considering anomalous magnetic moments. In fact configuration mixing gives contributions to k_p and k_N which have the correct sign and are in the correct ratio ($k_p = -k_N$) but are too small in magnitude by about a factor of three. Moreover, our configuration mixing does not contribute to the N^*-N transition moment.

A process which is usually thought to have a simple description in terms of partons is the production of high mass lepton pairs in nucleon-nucleon collisions. According to Drell and Yan¹⁶⁾ this process is seen as the annihilation into a lepton pair of one parton belonging to the projectile with a corresponding antiparton in the target. Kutli and Weisskopf¹⁰⁾ and Landshoff and Polkinghorne¹⁷⁾ have

computed the cross-section for this process, following this simple description, and have obtained a reasonable agreement with the data at 29.5 GeV. The same computation with our distribution functions $p(x)$, $n(x)$, etc. would lead to a result which is smaller by about a factor of five to ten. The main reason for this reduction is a factor of $\frac{1}{3}$ which follows from the colour quark hypothesis: a quark can only annihilate with an antiquark of the same colour. The further reduction is expected since the number of antiquarks is smaller than in Refs. (10) and (17). It is therefore likely that our fit to data at $E = 29.5$ GeV will not be as good as that of Refs. (10) and (17). We would not consider this as a serious argument against the coloured quarks model, and in particular against our scheme, but only as an indication that strong interaction effects can be particularly important in this process, where at least a violent reshuffling of constituents is to be expected after a q and a $\bar{q}$ have disappeared from the initial projectile and target particles.

APPENDIX

In this appendix we study the problem of finding all possible functions $f(x, q)$ such that, if one defines

$$(V_1)_x = m \left[f(x_1, q_1) - \frac{1}{2} f(x_2, q_2) - \frac{1}{2} f(1-x_1-x_2, q_3) \right] \quad (A.1)$$

$$(V_2)_x = m \left[f(x_2, q_2) - \frac{1}{2} f(x_1, q_1) - \frac{1}{2} f(1-x_1-x_2, q_3) \right],$$

then

$$d(V_1)_x d(V_2)_x = H \frac{dx_1 dx_2}{x_1 x_2 (1-x_1-x_2)}, \quad (A.2)$$

H being an arbitrary constant. To make our formulae simplest, it will be convenient to choose:

$$H = \frac{4}{3} m^2$$

The variables q_i appearing in Eqs. (A.1) are the absolute values of transverse momenta $\vec{q}_{\perp i}$, subjected to the condition

$$\sum \vec{q}_{\perp i} = 0 \quad (A.3)$$

We will prove that all the solutions of this problem are of the form

$$f(x, q) = \ln x + h(q) \quad (A.4)$$

(h being any function of q) provided we restrict ourselves to functions which are analytic in x , for x varying in the physical domain:

$$0 \leq x \leq 1$$

Equation (A.2) can be transformed immediately into

$$\begin{aligned} g(x_1, q_1) g(x_2, q_2) + g(x_2, q_2) g(1-x_1-x_2, q_3) + g(x_1, q_1) g(1-x_1-x_2, q_3) = \\ = \frac{1}{x_1 x_2 (1-x_1-x_2)}, \end{aligned} \quad (A.5)$$

where we have set:

$$g(x, q) = \frac{\partial f(x, q)}{\partial x}$$

In spite of Eq. (A.3), q_1, q_2, q_3 are actually independent variables, only subjected to triangular inequalities. Equation (A.5) then immediately shows that g cannot depend upon q , i.e. that

$$f(x, q) = f(x) + h(q) \quad (\text{A.6})$$

By setting

$$x \frac{df}{dx} = \zeta(x - \frac{1}{3})$$

and by renaming the variables according to

$$x_1 - \frac{1}{3} = x$$

$$x_2 - \frac{1}{3} = y$$

after simple manipulations we get the equation

$$\zeta(x) \zeta(y) \left(\frac{1}{3} - x - y\right) + \zeta(y) \zeta(-x - y) \left(\frac{1}{3} + x\right) + \zeta(-x - y) \zeta(x) \left(\frac{1}{3} + y\right) = 1 \quad (\text{A.7})$$

where the analyticity domain for G should at least include the interval I :

$$I : -\frac{1}{3} < x < \frac{2}{3} \quad (\text{A.8})$$

Equation (A.7) implies $G^2(0) = 1$, and we are free to select the solutions such that

$$\zeta(0) = 1.$$

An obvious solution to Eq. (A.7) is

$$\zeta(x) \equiv 1. \quad (\text{A.9})$$

Equation (A.9) immediately leads to (A.4), and in the following we shall prove that Eq. (A.9) is the only solution to Eq. (A.7), analytic in I.

For our arguments it will be sufficient to replace Eq. (A.7) with two simpler independent equations which can be obtained by setting either $x + Y = 0$, or $X = y$. The two equations read:

$$\frac{1}{3} G(x) G(-x) + G(x) \left(\frac{1}{3} - x\right) + G(-x) \left(\frac{1}{3} + x\right) = 1 \quad (\text{A.10})$$

$$\left[G(x)\right]^2 \left(\frac{1}{3} - 2x\right) + 2 G(-2x) G(x) \left(\frac{1}{3} + x\right) = 1 \quad (\text{A.11})$$

In the following we shall refer to Eqs. (A.10) and (A.11) as the R (for reflection) and B (for bisection) formulae, respectively.

We first observe that

- i) $G(x) > 0$ for x belonging to the interval I' :

$$I' : -\frac{1}{3} < x < \frac{1}{6}$$

In fact from B it follows that if $G(\bar{x}) = 0$ then G has a singularity in $-2\bar{x}$; moreover if $\bar{x} \in I'$, $-2\bar{x} \in I$, where G cannot be singular. A consequence of this is

- ii) if $G(\bar{x}) = 1$ for some $\bar{x} \in I$, $G(-\bar{x}/2) = 1$. In fact B implies that if $G(\bar{x}) = 1$,

$$G\left(-\frac{\bar{x}}{2}\right) = 1 \quad \text{or} \quad \frac{-1}{\bar{x} + 1/3}$$

The second solution is always negative for $\bar{x} \in I$. Since $-(\bar{x}/2) \in I'$, it follows from (i) that only the first solution is acceptable. Thus:

- iii) if $G(\bar{x}) = 1$ for some $\bar{x} \neq 0$, $\bar{x} \in I$, then

$$G(x) \equiv 1 \quad (\text{A.12})$$

In fact the point $\bar{x}$ generates an infinite sequence of points $(-\bar{x}/2, \bar{x}/4, -\bar{x}/8, \dots)$ accumulating around $x = 0$, where G is equal to one. This immediately proves Eq. (A.12). Our proof will be completed by showing that $G(1/3) = 1$.

With this aim in view, let us first let $x \rightarrow -1/3$ in the B-formula. Two cases are possible, namely:

- a) $G(2/3)$ is finite, which implies $G^2(-1/3) = 1$, i.e., by (i), $G(-1/3) = 1$;

b) $G(\frac{2}{3})$ is infinite, which implies $G(-\frac{1}{3}) \neq 1$ (possibly divergent).

First consider (a). Application of the R-formula, for $x = \frac{1}{3}$, shows that

$$G\left(\frac{1}{3}\right) = 1$$

from which Eq. (A.12) follows.

We now show that case (b) is actually inconsistent, thus completing the proof. To this end, we observe that, combining the B- and R-formulae, we can write:

$$G^2(x) \left(\frac{1}{3} - 2x\right) + 6 \frac{1 - G(2x) \left(\frac{1}{3} - 2x\right)}{G(2x) + 1 + 6x} G(x) \left(\frac{1}{3} + x\right) = 1$$

If we now let $x \rightarrow \frac{1}{3}$, (b) implies that

$$G\left(\frac{1}{3}\right) = 1 \text{ or } 3$$

The first alternative leads to $G(x) = 1$, in contrast with (b). Starting from the second solution, $G(\frac{1}{3}) = 3$, we can reach $G(-\frac{1}{6})$ by two different routes. First, by

$$\frac{1}{3} \xrightarrow{B} -\frac{1}{6}$$

we get

$$G\left(-\frac{1}{6}\right) = \frac{-3 + \sqrt{33}}{4}$$

On the other hand, we can follow the route

$$\frac{1}{3} \xrightarrow{R} -\frac{1}{3} \xrightarrow{B} \frac{1}{6} \xrightarrow{R} -\frac{1}{6}$$

which gives

$$G\left(-\frac{1}{6}\right) = \frac{13}{19}$$

in contrast with the previous value.

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Figure captions

- Fig. 1 : General representation of structure functions in the quark-parton model. The vertical lines indicate taking the s-channel absorptive part.
- Fig. 2 : Different types of contributions to structure functions in the gluon-quark model. Inside circles in diagrams (a), (b), and (c), the exchange of arbitrary numbers of gluon lines and/or $q\bar{q}$ loops is understood.
- Fig. 3 : s-channel absorptive part for baryon-current scattering, coming from contributions represented in Fig. 2(b).
- Fig. 4 : Comparison of our fit for F_2^{eP} with experimental results. Experimental data are taken from Ref. (15) with $R = \sigma_L/\sigma_T = 0.18$, selecting data points where $W = \sqrt{s} > 2 \text{ GeV}$, $-q^2 > 1.5 \text{ GeV}^2$. The solid line represents the full structure function. The dashed line corresponds to omitting the $\phi^{(c)}$ ($q\bar{q}$ pairs) contribution.
- Fig. 5 : Comparison of our fit with $F_2^{eP} - F_2^{eN}$. Data are taken from Ref. (8).
- Fig. 6 : Graphs of the functions $x p(x)$, $x n(x)$, $x \bar{p}(x)$.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6