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ANALYTIC PARAMETRIZATION OF
HIGH ENERGY FORWARD SCATTERING AMPLITUDE

II. $K^{\pm}p$ and $\pi^{\pm}p$ scattering

C. Bourrely

Centre de Physique Théorique, CNRS, Marseille

J. Fischer ⁺) and Z. Sekera

CERN - Geneva

A B S T R A C T

The $K^{\pm}p$ and $\pi^{\pm}p$ forward scattering amplitudes are parametrized in terms of functions which are analytic in the cut E plane and which give the asymptotic behaviour of the total cross-sections of the type $c(\log E/E_0)^{\delta}$, E being the laboratory energy. Fits to high-energy data give, in particular, the best value of δ ; around it we find, however, a large interval of positive values of δ which are also admissible on the basis of the χ^2 estimation.

Being equally good from the point of view of the existing experimental data, the different asymptotic growth rates δ do lead to different predictions, e.g., of the real part at high energies. A remarkable common feature is, however, a zero in the real part of the π^+p and π^-p scattering amplitudes, which is predicted by all of our parametrizations to be between 52 and 80 GeV/c of the laboratory momentum.

⁺) Address after September 1, 1973: Institute of Physics,
C.S.A.V., Prague.

1. INTRODUCTION

Evidence for a significant increase of the proton-proton total cross-section obtained recently at the CERN Intersecting Storage Rings ¹⁾⁻³⁾ raises the question whether a similar behaviour is to be expected from the total cross-sections of the other hadron reactions.

Besides the proton-proton total cross-section $\sigma^{pp}(E)$, only $\sigma^{K^+p}(E)$ is experimentally known to be rising at high energies. Nevertheless, $\sigma^{p\bar{p}}(E)$ and σ^{K^-p} are expected to start rising at some higher energies, if validity of the Pomeranchuk theorem is assumed. The $\pi^\pm p$ total cross-sections seem not to show up any high energy growth ⁴⁾, but the data are also compatible with models and parametrizations yielding asymptotically growing $\sigma^{\pi^\pm p}(E)$ [see, for instance, 5), 6)].

We propose to perform a detailed model-independent analysis of the existing forward scattering data on the reactions

$$\begin{aligned} p + p &\rightarrow p + p \\ \bar{p} + p &\rightarrow \bar{p} + p \end{aligned} \quad (1)$$

$$\begin{aligned} K^+ + p &\rightarrow K^+ + p \\ K^- + p &\rightarrow K^- + p \end{aligned} \quad (2)$$

$$\begin{aligned} \pi^+ + p &\rightarrow \pi^+ + p \\ \pi^- + p &\rightarrow \pi^- + p \end{aligned} \quad (3)$$

To this end, we use for $\sigma(E)$ phenomenological parametric expressions which are dominated by $(\log E)^k$ in the asymptotic region and which contain several free parameters. By using optical theorem, crossing symmetry and analyticity, we obtain energy dependence of the imaginary and the real part of the forward scattering amplitude. Experimental data on the total cross-sections and on the real part of the forward amplitude are used to determine the free parameters, among them also γ .

A clear-cut criterion distinguishing between a rising and a constant asymptotic behaviour of the total cross-section was obtained by Khuri and Kinoshita a long time ago ⁷⁾. Starting from very general properties of the forward scattering amplitude, these authors have proved that if the total cross-section diverges the real part must asymptotically be positive. (The statement is not true in the opposite direction, of course.) In other words, if the real part is negative in some energy interval (this is experimentally known to be the case for the π^+p , π^-p and K^+p scattering amplitude), it must have a zero at some higher energy in this case. This general statement is far from being purely academic, at least for pion-proton scattering; indeed, we shall see that the existing experimental data put the zero rather uniquely between 52 and 80 GeV/c both for the π^+p and for the π^-p real part for all y between 0.2 and 2.0, but both of these zeros run towards $E \rightarrow \infty$ for $y \rightarrow 0$. Measurements of the real part in this region can therefore help answering the question whether σ^{π^+p} and σ^{π^-p} are asymptotically rising or not.

2. THE METHOD

The above-mentioned analysis of the reactions (1) was reported in a previous paper ⁸⁾. In the present paper, starting from the same assumptions as those listed in 8), we represent the kaon-proton and the pion-proton scattering amplitude $F(E)$ in two different parametric forms, each of which exhibits all the analytic properties of $F(E)$ in the cut energy plane ^{*)} required by dispersion relations.

Using these parametrizations we perform a set of MINUIT fits to experimental data on $\text{Im } F(E)$ and on $\rho(E) = \text{Re } F(E) / \text{Im } F(E)$ both in the direct and in the crossed channel. In contradistinction to 8), we fit simultaneously all of these four curves, because the number of experimental points in the crossed channel is almost equal to that of the direct channel, unlike to the proton-proton and proton-antiproton scattering.

It is clear that this approach, although based on very general assumptions, is biased by some implicit assumptions connected, for instance, with

^{*)} We denote by p and by $E = \sqrt{p^2 + m_i^2}$ the laboratory momentum and energy of the incident particle respectively, m_i being the rest mass of the incident particle.

the specific choice of the parametrization made. This feature is common to the majority of papers dealing with applications of dispersion theory. It is one of the aims of the present paper to confront predictions of different parametrizations which are equally reasonable and satisfy the same general requirements.

Two different analytic parametrizations of the forward scattering amplitude are considered in more detail:

$$F_{\alpha}(E) = i a_1 p_j \cdot (1 + a_2 L_{\alpha})^{\alpha} - a_4 (m_j - E)^{1/2} + (m_j + E)^{1/2} - \\ - a_5 (E - E_j)^{1/2} + a_6 + a_7 E \quad (4)$$

$$F_{\beta}(E) = i b_1 p_j \cdot (1 + b_2 L_{\beta}^2)^{\beta} - b_5 (E - E_j)^{1/2} + b_6 + b_7 E \quad (5)$$

where

$$L_{\alpha} = \log(1 - i a_3 p_j) \\ L_{\beta} = \log(b_3 - i b_4 p_j) \\ p_j = (m_j^2 - E^2)^{1/2} \quad (6)$$

and subscript j stands for K ($K^{\pm}p$ scattering) or π ($\pi^{\pm}p$ scattering) with

$$E_K = \frac{1}{2m_p} (m_p^2 + m_K^2 - m_{\Lambda}^2 - m_{\pi}^2 - 2 m_{\Lambda} m_{\pi}) \quad (7)$$

$$E_{\pi} = - m_{\pi},$$

m_p , m_K , m_{π} and m_{Λ} being the rest mass of the proton, kaon, pion and the lambda hyperon respectively.

All parameters a_i , b_i , α and β are real and satisfy the following conditions

$$\begin{array}{ll} a_2 > 0 & a_3 > 0 \\ 0 < b_2 < 4/\pi^2 & b_4/b_3 > 0 \end{array} \quad (8)$$

in order to avoid additional singularities on the physical sheet.

The functions $F_\alpha(E)$ and $F_\beta(E)$ are, except for two minor differences, identical to the functions $F_a(E)$ and $F_b(E)$ respectively introduced in Ref. 8). The two differences are:

- i) the pole term has been omitted (due to its negligible rôle at high energies);
- ii) we have added the linear term in E (which was small in the pp scattering amplitude, but is generally non-vanishing).

Further, we have put b_3 equal to a fixed small positive value, $b_3 = 0.001$, because of a very weak dependence of all results on the value of b_3 .

In this way, the four independent measurable quantities (the imaginary and the real part of the forward scattering amplitude on the right- and left-hand physical cut) are expressed in terms of several free parameters. The parametrizations are rather economical; note that $F_\alpha(E)$ and $F_\beta(E)$ allow to express the four curves in terms of eight and seven real parameters respectively.

3. RESULTS

Independent sets of fits were performed to all data above $p_{\min}$, which was put equal to p_σ , $p_{\bar{\sigma}^*}$, p_ρ and $p_{\bar{\rho}}$ for data on $\sigma(E)$, $\bar{\sigma}(E)$, $\rho(E)$ and $\bar{\rho}(E)$ respectively according to the scheme shown in Table I.

*) Quantities without bar pertain to the direct channel, whereas quantities with a bar pertain to the crossed channel. Under the direct and the crossed channel, we understand the (meson⁺)-proton and the (meson⁻)-proton forward scattering respectively.

Results with $p_{\text{e}} = p_{\text{e}} = 10 \text{ GeV/c}$ and with $p_{\text{e}} = p_{\text{e}} = 20 \text{ GeV/c}$ were very similar to each other in all cases, whereas the results with $p_{\text{e}} = p_{\text{e}} = 5 \text{ GeV/c}$ were sometimes significantly different. We therefore investigated the case $p_{\text{e}} = p_{\text{e}} = 10 \text{ GeV/c}$ in more detail.

Some of the results are collected in Table II and in Figs. 1, 2, 3 and 4. Unfortunately, the existing experimental data do not allow to determine the optimal value of the exponent α or 2β of the logarithmic term unambiguously. In order to see how compatible are the data with other possible asymptotic growth rates α or 2β we made a series of fits with α or 2β fixed at some given value. We found that the power could be chosen rather freely from 0.2, say, up to the Froissart limiting value 2 without any perceptible change of the χ^2 value. (The values of χ^2 are given in Table II.) Negative powers led, in general to considerably higher χ^2 values than the positive ones.

This situation suggests to consider these values of α and 2β as equally justified and, consequently, to introduce two infinite sets

$$\begin{aligned} \mathcal{F}_{\alpha} &= \{ F_{\alpha}(E) : \alpha \in (0.2, 2) \} \\ \mathcal{F}_{\beta} &= \{ F_{\beta}(E) : \beta \in (0.1, 1) \} \end{aligned} \tag{9}$$

of functions all of which describe the data equally well. Thus, we are in a position to confront predictions of infinitely many different parametrizations with each other.

Let us see now to what extent the elements of $\mathcal{F}_{\alpha}$ or $\mathcal{F}_{\beta}$, giving identical description of the existing data, differ in their predictions at higher energies. To this end, we have chosen three quantities which seem to be interesting at present: the value of σ and ρ at the NAL energy $E = 200 \text{ GeV}$, and the energy $E_{\rho=0}$ (if any) at which ρ changes its sign. (The same quantities have been chosen also for the crossed channel.) The values obtained are collected in Table III and give an idea of the experimental accuracy which would be necessary at the NAL energy 200 GeV to obtain some indication on the asymptotic growth rate. One thing deserving a special attention is that the value $E_{\rho=0}$ and $E_{\tilde{\rho}=0}$ for pion-proton scattering is almost independent of the growth rate, staying within narrow limits (52 GeV, 80 GeV) and (60 GeV, 73 GeV), respectively, for both of our parametrizations and independently of the growth rate:

$$\begin{aligned} 0.2 &\leq \alpha \leq 2 \\ 0.1 &\leq \beta \leq 1 \end{aligned} \quad (10)$$

However, if the exponent approaches zero, $E_{\rho=0}$ and $E_{\bar{\rho}=0}$ run very quickly towards infinity.

Being valid for both of our sets F_α , F_β of parametrizing functions, this effect may be rather independent of parametrization in general. We propose therefore to use it as criterion deciding whether the growth rate is zero or small or whether it is greater than 0.2. Furthermore, as seen from Table III, the predicted values of ρ and $\bar{\rho}$ at the NAL energy 200 GeV also appear to represent a good criterion of this kind. Measurements of ρ and $\bar{\rho}$ above 50 GeV will shed light on this question.

The fixed position of the zero in ρ , $\bar{\rho}$ in the case of pion-proton scattering is interesting also from the general point of view. Indeed, all functions $\text{Im} F_\alpha(E)$, $\alpha \in (0.2, 2)$ and $\text{Im} F_\beta(E)$, $\beta \in (0.1, 1)$ differ significantly from each other only at very high energies, $E > 200$ GeV. Yielding the same position of the zero in $\text{Re} F_\alpha(E)$, $\text{Re} F_\beta(E)$, these functions are conform with the "nearby singularities" philosophy, according to which distant singularities have only small influence. If, however, α or 2β becomes smaller than 0.1 and approaches zero, the influence of asymptotic energies on the 70 GeV region shows up suddenly.

It is worth mentioning that our values of $E_{\rho=0}$ (see Table III) are different from those following from the model of Cheng, Walker and Wu⁵⁾. According to this model, $E_{\rho=0}$ is expected to be around 100, 150 and 80 GeV for K^+p , π^+p and π^-p scattering respectively.

4. CONCLUDING REMARKS

The impossibility of a unique determination of the asymptotic growth rate of the scattering amplitude from existing experimental data was also met, in a slightly weaker form, in the proton-proton forward scattering dealt with in Ref. 8) and has to be understood as a warning against too hasty conclusions concerning asymptotic behaviour. It was pointed out in 8) that the logarithmic term in $F_\beta(E)$ [and in $F_\alpha(E)$ too] could be expanded into powers of the logarithm and safely approximated by the linear term in the whole measurable region for a large interval of the exponent α or β :

$$(1 + a_2 L_a)^{\alpha} = 1 + \alpha a_2 L_a + \dots \quad (9)$$

$$(1 + b_2 L_b^2)^{\beta} = 1 + \beta b_2 L_b^2 + \dots \quad (10)$$

αa_2 and βb_2 being fairly constant within large intervals of α and β respectively. The same is true both for the Kp and for the π p scattering. [Note, however, that for too small values of β , b_2 is forced to exceed the bound given by relation (8) and an additional singularity appears on the physical sheet.]

In the α parametrization, there is also another source of ambiguity of α . Indeed, $F_{\alpha}(E)$ represents the total cross-section in the form of the sum of a decreasing term [second term on the right-hand side of (2)] and an increasing term [first term on the right-hand side of (2)]. Since, however, the high energy growth both in Kp scattering and in π p scattering is not significant, the growth-producing term is fitted from its behaviour at energies at which it is negligibly small! This is probably the reason why fits based on very different asymptotic behaviours give equally good agreement with the data available on $\pi^{\pm}$ p and $K^{\pm}$ p scattering.

One can say, in conclusion, that the experimental data on pion-proton and kaon-proton scattering are even less significant for the determination of the asymptotic growth rate than it was in the case of proton-proton scattering. This is connected, evidently, with the lack of data above 70 GeV of laboratory energy. Measurements of the total cross-section and of the forward real part in the NAL energy region will certainly help clarifying questions concerning the actual high-energy behaviour of the kaon-proton and pion-proton scattering amplitude. It seems to us, nevertheless, that any model-independent investigation of this problem can be successful only if large classes of different parametrizations are considered simultaneously and only common features are considered as reliable.

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TABLE CAPTIONS

Table 1:

Values of the minimal momenta p_{ϵ} , $p_{\bar{\epsilon}}$, p_{ρ} , $p_{\bar{\rho}}$, above which all experimental points [see Refs. 10), 11)] have been taken in different fits.

Table 2:

Optimal values of parameters obtained by fit No. 2 (see Table 1).

Table 3:

Predictions of ϵ , $\bar{\epsilon}$, ρ and $\bar{\rho}$ at the NAL energy 200 GeV and of a zero in ρ , $\bar{\rho}$ for Kp and π p scattering. The values presented were obtained by the fit No. 2.

TABLE I

Fit No.	Kp scattering					π p scattering				
	p_{π^-}	$p_{\bar{\pi}^0}$	p_{ρ^-}	$p_{\bar{\rho}^0}$	GeV/c	p_{π^-}	$p_{\bar{\pi}^0}$	p_{ρ^-}	$p_{\bar{\rho}^0}$	GeV/c
1	5	5	4	4		5	5	5	5	
2	10	10	4	4		10	10	5	5	
3	20	20	4	4		20	20	5	5	

Table II

Kp - function F_α									
α	a_1 [mb]	a_2	a_3 [GeV ⁻¹]	a_4 [mbGeV ^{1/2}]	a_5 [mbGeV ^{1/2}]	a_6 [mbGeV]	a_7 [mb]	χ^2	No of points
.02	17.	580.	.0022	.0001	16.	-3.2	-2.3	95.8	54
.2	17.	.55	.023	.61	16.	-.43	-2.3	57.6	54
1.0	17.	.09	.03	.41	16.	-.81	-2.3	57.6	54
2.0	17.	.04	.048	2.1	16.	2.5	-2.3	57.3	54

Kp - function F_β									
2β	b_1 [mb]	b_2	b_3	b_4 [GeV ⁻¹]	b_5 [mbGeV ^{1/2}]	b_6 [mbGeV]	b_7 [mb]	χ^2	N.O.P.
.02	18.	.41	.001	.063	16.	7.8	-4.0	108.3	54
.2	18.	.13	.001	.074	16.	11.	-3.8	56.9	54
1.0	18.	.035	.001	.085	16.	12.	-3.7	56.9	54
2.0	18.	.019	.001	.087	16.	12.	-3.6	56.9	54

πp - function F_α									
α	a_1 [mb]	a_2	a_3 [GeV ⁻¹]	a_4 [mbGeV ^{1/2}]	a_5 [mbGeV ^{1/2}]	a_6 [mbGeV]	a_7 [mb]	χ^2	N.O.P.
.02	22.	19.	.0025	9.1	6.9	-3.4	.31	161.7	82
.2	22.	.62	.017	13.	7.1	6.6	.35	99.9	82
1.0	22.	.087	.027	13.	7.1	6.9	.35	101.8	82
2.0	22.	.041	.030	14.	7.1	7.4	.36	102.3	82

πp - function F_β									
2β	b_1 [mb]	b_2	b_3	b_4 [GeV ⁻¹]	b_5 [mbGeV ^{1/2}]	b_6 [mbGeV]	b_7 [mb]	χ^2	N.O.P.
.02	24.	.37	.001	.023	7.0	-31.	.62	357.6	82
.2	24.	.15	.001	.023	7.1	-19.	.59	99.0	82
1.0	24.	.048	.001	.023	7.1	-7.4	.62	101.1	82
2.0	24.	.028	.001	.023	7.1	-3.5	.63	103.2	82

Table III

Kp - Function F_α						
α	$\sigma(200)$ [mb]	$\bar{\sigma}(200)$ [mb]	$\rho(200)\%$	$\bar{\rho}(200)\%$	$E_{\rho=0}$ [GeV]	$E_{\bar{\rho}=0}$ [GeV]
.02	18.5	19.6	-15.8	15.9	∞	∞
.2	19.7	20.8	-9.9	20.2	∞	∞
1.0	20.0	21.1	-6.4	23.2	13500.	∞
2.0	19.8	20.9	-7.8	22.8	11000.	∞

Kp - Function F_β						
2β	$\sigma(200)$ [mb]	$\bar{\sigma}(200)$ [mb]	$\rho(200)\%$	$\bar{\rho}(200)\%$	$E_{\rho=0}$ [GeV]	$E_{\bar{\rho}=0}$ [GeV]
.02	18.2	19.3	-26.8	21.6	∞	∞
.2	19.3	20.4	-19.1	24.3	∞	∞
1.0	20.1	21.2	-10.6	29.8	3200.	∞
2.0	20.2	21.4	-7.8	32.0	580.	∞

πp - Function F_α						
α	$\sigma(200)$ [mb]	$\bar{\sigma}(200)$ [mb]	$\rho(200)\%$	$\bar{\rho}(200)\%$	$E_{\rho=0}$ [GeV]	$E_{\bar{\rho}=0}$ [GeV]
.02	23.2	23.7	-1.2	-1.8	400.	∞
.2	24.3	24.8	3.8	3.0	80.	73.
1.0	24.4	24.9	5.8	5.0	73.	66.
2.0	24.4	24.9	6.0	5.1	66.	61.

πp - Function F_β						
2β	$\sigma(200)$ [mb]	$\bar{\sigma}(200)$ [mb]	$\rho(200)\%$	$\bar{\rho}(200)\%$	$E_{\rho=0}$ [GeV]	$E_{\bar{\rho}=0}$ [GeV]
.02	23.9	24.4	1.0	-2.1	115.	∞
.2	24.8	25.3	6.3	3.4	61.	72.
1.0	24.9	25.4	12.0	8.8	54.	62.
2.0	24.9	25.4	13.8	10.5	52.	60.

FIGURE CAPTIONS

Figure 1 :
Total K^+p and K^-p cross-sections following from $F_\alpha(E)$ and $F_\beta(E)$ obtained by the fit No. 2 [see Tables 1, 2 and 3 and Refs. 10), 11)]. The extreme values of α and β given by (10) are indicated in the graph.

Figure 2 :
Ratio of the real to the imaginary part following from the fit shown in Fig. 1.

Figure 3 :
Total π^+p and π^-p cross-sections following from $F_\alpha(E)$ and $F_\beta(E)$ obtained by the fit No. 2 [see Tables 1, 2 and 3 and Refs. 10), 11)]. The extreme values of α and β given by (10) are indicated in the graph.

Figure 4 :
Ratio of the real to the imaginary part following from the fit shown in Fig. 3.

Fig. 1

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Fig. 2

Fig. 3

Fig. 4