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CONSTITUENT QUARKS AND CURRENT QUARKS +)

J. Weyers
CERN - Geneva

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Quarks have been used in hadronic physics both as a constituent model for the hadrons and as a model for their electromagnetic and weak currents.

Conceptually, these "constituent" and "current" quarks do not have to be identical. In fact, we know from experiment that although a nucleon is reasonably well described as a state made up of three constituent quarks, it looks considerably less simple when probed with currents. Indeed, in a current quark description which follows from the results of deep inelastic lepton scattering, the nucleon seems to contain in addition to three (current) quarks, many quark-antiquark pairs and perhaps some neutral "glue" as well.

Recently, H.J. Melosh has shown that, at least in the free quark world, a transformation relating constituent and current quarks can be explicitly constructed. As it turns out, this (Melosh) transformation has remarkably simple algebraic properties. Abstracting from the free quark model, several authors have then suggested that these algebraic properties might remain valid in the "real world". Many testable predictions follow from this assumption, in particular for electromagnetic and pionic transitions between hadronic states.

The purpose of these lectures is to review the main ideas underlying these recent developments in "resonance physics". Lectures 1 and 2 will be devoted, respectively, to the constituent and current quark pictures and their differences; lectures 3 and 4 will deal with the Melosh transformation in the free quark model, its possible extensions and its phenomenological consequences. Throughout these lectures exact $SU(3)$ invariance will be assumed.

. CONSTITUENT QUARKS

Hadron states at rest appear to be well described by a constituent quark model with orbital excitations ¹⁾. Most successes of the model follow from the assumption that the low lying hadrons act as if they were made up of a quark-antiquark pair ($q\bar{q}$) in the meson case or of three quarks (qqq) in the baryon case. More complex configurations such as $q\bar{q} q\bar{q}$ for mesons or $qqqq\bar{q}$ for baryons are usually called exotic: they do occur, of course, in multiparticle states but there is some experimental evidence ²⁾ that they may also have resonances corresponding to them.

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The symmetry associated with such a classification scheme is easily identified. Clearly the quark (resp. antiquark) content of any hadronic system can be described in terms of representations of a $U(6)_Q$ [resp. $U(6)_{\bar{Q}}$] algebra generated by

$$\sum_q \frac{\lambda_i}{2}$$

$$\sum_{\bar{q}} \frac{\lambda'_i}{2}$$

resp.

$$\sum_q \lambda_i \frac{\sigma_j}{2}$$

$$\sum_{\bar{q}} \lambda'_i \frac{\sigma_j}{2}$$

$$(i=0,1,\dots,8; \quad j=1,2,3)$$

where λ_i (λ'_i) are the usual $U(3)$ matrices and σ_j the Pauli spin matrices.

The quark-antiquark content of a hadron thus builds up representations of a $U(6)_Q \otimes U(6)_{\bar{Q}}$ symmetry group with

$$q \in (\underline{6}, \underline{1})$$

$$\bar{q} \in (\underline{1}, \underline{\bar{6}})$$

so that, for example, a qqq type baryon can be classified in one (or more) of the representations $(\underline{56}, \underline{1})$, $(\underline{70}, \underline{1})$ or $(\underline{20}, \underline{1})$ of $U(6)_Q \otimes U(6)_{\bar{Q}}$.

If, in addition, one imagines that the quarks and/or antiquarks can have some relative orbital momentum one is led to a classification scheme of the hadronic spectrum based on $U(6)_Q \otimes U(6)_{\bar{Q}} \otimes O(3)$ which will be referred to as "the rest symmetry". As is well known such a scheme works amazingly well for the low lying hadrons ³⁾.

For the mesons, for example, one expects as lowest representation of a $q\bar{q}$ system

$$(\underline{6}, \underline{\bar{6}}); L=0$$

which, under $SU(2)_S \otimes SU(3)$ (where S refers to the quark spin) decomposes as follows:

$$(\underline{6}, \underline{\bar{6}}); L=0 \supset \begin{cases} S=0 & \underline{1} \oplus \underline{8} & J^{PC} = 0^{-+} \\ S=1 & \underline{1} \oplus \underline{8} & J^{PC} = 1^{--} \end{cases}$$

with $\vec{J} = \vec{L} + \vec{S}$, $P = (-)^{L+1}$ and $C = (-)^{L+S}$. These nine pseudoscalars and nine vector mesons have all been observed and no other negative parity meson resonance has been found below 1200 MeV ⁴⁾. From the observed masses of these mesons, it is clear, however, that $U(6)_Q \otimes U(6)_{\bar{Q}} \otimes O(3)$ is a badly broken symmetry since e.g., $m_{\pi} = 140$ MeV while $m_{\eta'} = 960$ MeV!

The next level of the meson spectrum seems to be $(\underline{6}, \underline{\bar{6}}); L=1$. This representation of the rest symmetry contains the following nonets of resonances

$$(\underline{6}, \underline{\bar{6}}); L=1 \supset \begin{cases} S=0 & \underline{1} \oplus \underline{8} & J^{PC} = 1^{+-} \\ S=1 & \begin{aligned} &\underline{1} \oplus \underline{8} & J^{PC} = 2^{++} \\ &\underline{1} \oplus \underline{8} & J^{PC} = 1^{++} \\ &\underline{1} \oplus \underline{8} & J^{PC} = 0^{++} \end{aligned} \end{cases}$$

Here, the agreement between "theory" and experiment is much less spectacular ⁵⁾ especially for the axial vector mesons. An optimistic summary of the experimental situation ⁴⁾ is given in Table I. At least, experimental entries in the 1000 MeV-1500 MeV mass region are not incompatible with a simple $q\bar{q}$ picture with orbital excitations. The same remark holds for the $L=2$ and $L=3$ energy bands where experimental information is of course much scarcer.

Let me mention, as a final remark on the meson spectrum, that there is not, as yet, any evidence for exotic ⁶⁾ resonances.

For baryons, the successes of the rest symmetry are more impressive. The lowest configuration of baryons fits beautifully into a $(\underline{56}, \underline{1}); L=0$.

This representation decomposes as follows into multiplets of $SU(2)_S \otimes SU(3)$:

$$(\underline{56}, \underline{1}); L=0 \supset \begin{cases} S = \frac{1}{2} & \underline{8} : J^P = \frac{1}{2}^+ \\ S = \frac{3}{2} & \underline{10} : J^P = \frac{3}{2}^+ \end{cases}$$

The next level which includes all observed ⁴⁾ $Y=1,0$ baryon resonances with negative parity and mass below 2000 MeV fits as beautifully ^{3),7)} into a $(\underline{70}, \underline{1}); L=1$ representation of the rest symmetry, which has the following $SU(2)_S \otimes SU(3)$ content

$$(\underline{70}, \underline{1}); L=1 \supset \begin{cases} S = \frac{1}{2} & \underline{1}, \underline{8}, \underline{10} : J^P = \frac{1}{2}^-, \frac{3}{2}^- \\ S = \frac{3}{2} & \underline{8} : J^P = \frac{1}{2}^-, \frac{3}{2}^-, \frac{5}{2}^- \end{cases}$$

At the third energy level, there is strong evidence ³⁾ for the existence of the following multiplets

$$(\underline{56}, \underline{1}); L=2 \quad (\underline{56}, \underline{1}); L=0$$

and

$$(\underline{70}, \underline{1}); L=2 \quad (\underline{70}, \underline{1}); L=0$$

Likewise, at the fourth level, there are indications for a $(\underline{70}, \underline{1}); L=3$ multiplet but other multiplets may of course turn out to be also present.

A subgroup of the rest symmetry which is still popular is $SU(6)_S \otimes O(3)$. Let me recall that in a constituent quark picture this symmetry is generated by

$$\begin{aligned} \Lambda_i &= \sum_j \frac{\lambda_i}{2} + \sum_j \frac{\lambda'_i}{2} \\ S_j &= \sum_i \frac{\sigma_j}{2} + \sum_i \frac{\sigma'_j}{2} \\ \Lambda_i^j &= \sum_k \frac{\lambda_i \sigma_j}{2} + \sum_k \frac{\lambda'_i \sigma'_j}{2} \end{aligned} \quad \text{and} \quad \vec{L} = (L_x, L_y, L_z)$$

where $i=1, \dots, 8$ and $j=1, 2, 3$.

Irreducible representations of the rest symmetry decompose simply under $SU(6)_S \otimes O(3)$ according to the formula $(\underline{A}, \underline{B}); L \rightarrow \underline{A} \otimes \underline{B}; L$. For example, $(\underline{6}, \underline{1}) \rightarrow \underline{6}$; $(\underline{1}, \underline{\bar{6}}) \rightarrow \underline{\bar{6}}$; $(\underline{6}, \underline{\bar{6}}) \rightarrow \underline{6} \otimes \underline{\bar{6}} = \underline{35} \oplus \underline{1}$ etc. Hence, $SU(6)_S \otimes O(3)$ is essentially equivalent, for baryons, to the full rest symmetry while among the 36 mesons built up from a $q\bar{q}$ pair in a given orbital momentum, it allows for an extra splitting between the $SU(3)$ singlet, $S=0$ meson and the others.

All in all, the superficial discussion of the particle spectrum given here leads to the conclusion that there is fairly strong experimental evidence in favour of a broken $U(6)_Q \otimes U(6)_{\bar{Q}} \otimes O(3)$ [or $SU(6)_S \otimes O(3)$] symmetry describing hadronic states at rest.

Clearly the successes of these symmetries can be interpreted in purely algebraic terms: quarks are then viewed only as a convenient device from which the "correct" algebraic structure can be abstracted. From this point of view, quarks do not have to be real particles and hence one does not have to worry too much about their unusual properties (such as their charges or their statistics). This will be more or less the point of view adopted in these lectures.

Alternatively one may take quarks more seriously and try to explain the observed particle spectrum from quark dynamics. In the most popular of these models, quarks are bound inside hadrons by harmonic oscillator type forces⁸⁾. This has led to some remarkable successes such as linearly rising Regge trajectories and a baryon spectrum which is predicted to go as follows:

$$\begin{array}{llll}
 \underline{56} & L^P = 0^+ & & \\
 \underline{70} & L^P = 1^- & & \\
 \underline{56} & L^P = 2^+, 0^+ & \underline{70} & L^P = 2^+, 0^+ \quad \underline{20} \quad L^P = 1^+ \\
 - & - & - & -
 \end{array}$$

Except for the $\underline{20}$ which has not yet been observed, this is precisely the spectrum which seems to be present in nature. Furthermore, by adding spin-spin and spin orbit forces and eventually breaking $SU(3)$, one obtains in this scheme a quite reasonable fit to the observed particle masses⁹⁾.

It should be mentioned, however, that despite these "successes", no physically acceptable dynamical quark model exists as yet: lack of Lorentz invariance or of conservation of probability are some of the most common diseases of all models suggested so far.

Anyway, whatever the merits or flaws of these alternatives, the rest symmetry and its $SU(6)_S \otimes O(3)$ subgroup become totally useless for decay processes. They both forbid decays such as $\rho \rightarrow \pi \pi$ or $\Delta \rightarrow N \pi$; e.g.,

$$\begin{array}{ccccc} (56, \frac{1}{2}) & \otimes & (6, \frac{1}{2}) & \not\rightarrow & (56, \frac{1}{2}) \\ S = \frac{1}{2} & \otimes & S = 0 & \not\rightarrow & S = \frac{3}{2} \end{array}$$

Such disastrous results are not too surprising since decay amplitudes necessarily involve moving particles for which the rest classification should not be expected to hold.

Clearly, these bad "predictions" of the rest symmetry will hold in any scheme which conserves "intrinsic spin". Hence, if one looks for a symmetry which might be consistently extended to moving particles, the internal "angular degrees of freedom" must be described by a different group than $SU(2)_S$. It was pointed out by Lipkin and Meshkov¹⁰⁾ that the reflection and rotation invariance properties of collinear processes (such as two-body decays) lead to invariance under "W spin" or $SU(2)_W$ rather than S spin or $SU(2)_S$.

Let me consider for example a spin $\frac{1}{2}$ system moving along the z axis and let $\vec{J} = \vec{L} + (\vec{\sigma}/2)$ be its total angular momentum. The momentum p_z of such a system is leftinvariant under the following transformations

$$R_z = \exp i \theta J_z = \exp i \theta \frac{\sigma_z}{2}$$

$$R_x = \mathcal{P} \exp i \pi J_x = \mathcal{P}_{int} \exp i \pi \frac{\sigma_x}{2} = \mathcal{P}_{int} i \sigma_x = \exp i \pi \mathcal{P}_{int} \frac{\sigma_x}{2}$$

$$R_y = \mathcal{P} \exp i \pi J_y = \exp i \pi \mathcal{P}_{int} \frac{\sigma_y}{2}$$

where $\mathcal{P}$ is the parity operator and $\mathcal{P}_{\text{int}}$ the intrinsic parity of the system. $[\mathcal{P} = \mathcal{P}_{\text{int}}(-)^L]$. If the group which describes the internal angular degrees of freedom is to be meaningful when the system moves along the z axis, it should contain the transformations R_x , R_y and R_z since they leave the momentum invariant. These operations can be considered as generated by

$$W_x = \mathcal{P}_{\text{int}} \frac{\sigma_x}{2} \quad W_y = \mathcal{P}_{\text{int}} \frac{\sigma_y}{2} \quad W_z = \frac{\sigma_z}{2}$$

which are called the W spin generators. They obviously generate an $SU(2)$ algebra which allows one to describe the "internal degrees of freedom" of a moving particle with spin $\frac{1}{2}$. This property has earned W spin the name of "relativistic spin".

Clearly these arguments can be extended to any system with spin $\vec{S}$, built up from spin $\frac{1}{2}$ objects. The W spin generators for such a system are then given by

$$W_x = \mathcal{P}_{\text{int}} S_x \quad W_y = \mathcal{P}_{\text{int}} S_y \quad W_z = S_z$$

where, again, $\mathcal{P}_{\text{int}}$ is the intrinsic parity.

For a single quark moving in the z direction one thus writes the W spin generators as $W_x = \beta(\sigma_x/2)$, $W_y = \beta(\sigma_y/2)$; $W_z = (\sigma_z/2)$. Combining $SU(2)_W$ with $SU(3)$, leads to $SU(6)_W$ which, in a constituent quark model, would be generated by

$$W^i = \sum_{q, \bar{q}} \frac{\lambda_i}{2} + \frac{\lambda'_i}{2} \quad (i = 1, 2, \dots, 8)$$

$$W_x^i = \sum_{q, \bar{q}} \frac{\lambda_i}{2} \beta \sigma_x + \frac{\lambda'_i}{2} \beta \sigma_x \quad (i = 0, 1, 2, \dots, 8)$$

$$W_y^i = \sum_{q, \bar{q}} \frac{\lambda_i}{2} \beta \sigma_y + \frac{\lambda'_i}{2} \beta \sigma_y \quad (i = 0, 1, 2, \dots, 8)$$

$$W_z^i = \sum_{q, \bar{q}} \frac{\lambda_i}{2} \sigma_z + \frac{\lambda'_i}{2} \sigma_z \quad (i = 0, 1, 2, \dots, 8)$$

These expressions manifestly commute with $\sum_{q,\bar{q}} \alpha_z$, which for free quarks, is the generator of Lorentz boosts along the z axis. This property of the free constituent quark model suggests that $SU(6)_W$ is the appropriate subgroup of the rest symmetry which can be extended to describe particles moving along the z axis, and, hence, perhaps, two-body decays or general collinear processes.

These last assumptions, however, are not really too well motivated. There is indeed no compelling reason, first of all, to believe that the W_α^i 's, which we abstract from the quark model and postulate to exist in the real world, have the same Lorentz transformation properties as the naïve expressions given above. Secondly, one should keep in mind that even if $SU(6)_W$ turns out to be an approximately valid symmetry for the classification of one-particle moving states this certainly does not imply that it has to be an approximate symmetry of the whole Hilbert space of states.

In the rest frame $\vec{S}$ spin and $\vec{W}$ spin are both well defined since $SU(6)_W$ is a subgroup of the rest symmetry. Since $\beta q = +q$ while $\beta \bar{q} = -\bar{q}$, one finds, as is well known, that $\vec{W}_q = \vec{S}_q$ for quarks while for antiquarks $W_z = S_z$ but $W_{x,y} = -S_{x,y}$.

With $\vec{W} = \vec{S}$ for quark states, nothing will change in the baryon classification when going from $SU(6)_S$ to $SU(6)_W$: the nucleon octet and Δ decuplet belong to a $\underline{56}$ of $SU(6)_W$ as well as of $SU(6)_S$ and so on. The fact that $W_{x,y} = -S_{x,y}$ for antiquarks leads to completely different assignments of mesonic states in $SU(6)_S$ and $SU(6)_W$. Quarks and antiquarks form doublets both under S spin and W spin and hence a $q\bar{q}$ system decomposes as a triplet and a singlet in both $SU(2)$'s. It is then a simple matter of Clebsch-Gordon coefficients and phases to show that one has the so-called W - S flip relations, i.e.,

$$|S = 1 ; S_z = +1\rangle \rightarrow |W = 1 ; W_z = +1\rangle$$

$$|S = 1 ; S_z = 0\rangle \rightarrow -|W = 0 ; W_z = 0\rangle$$

$$|S = 1 ; S_z = -1\rangle \rightarrow -|W = 1 ; W_z = -1\rangle$$

$$|S = 0 ; S_z = 0\rangle \rightarrow -|W = 1 ; W_z = 0\rangle$$

A π meson which is an $S=0$ state is a $W=1$ object while the helicity 0 ρ meson becomes a W spin singlet. Table II gives the explicit classification ¹¹⁾ of the lowest lying mesons in $SU(6)_S$ and $SU(6)_W$. As can be seen from this table, the same $(6, \bar{6})$ multiplet of the rest symmetry gets thus reshuffled differently into $1 + 35$ representations of $SU(6)_S$ and $SU(6)_W$.

If, as mentioned earlier, one now assumes that, in the limit when they are conserved, the $SU(6)_W$ generators commute with Lorentz boosts in the $\hat{z}$ direction, it is trivial to classify moving states knowing their rest frame classification. Of course, under boosts in the $\hat{z}$ direction, L does not remain a good quantum number and only L_z continues to be conserved. The resulting symmetry is usually called $SU(6)_W \otimes O(2)_{L_z}$ although the conservation of W_z necessarily implies the conservation of L_z since $J_z = S_z + L_z = W_z + L_z$.

For the lowest lying mesons and baryons, $SU(6)_W$ thus appears to be a reasonable approximate symmetry in describing one-particle (moving) states, as reasonable, certainly, as the rest symmetry for states at rest. What such a statement means is that the generators of $SU(6)_W$ seem to approximately commute with the strong interaction Hamiltonian, at least when sandwiched between one-particle moving states. An interesting question is then whether this property can be extended to other sectors of the Hilbert space of (multiparticle) states moving along the $\hat{z}$ axis. That such an extension is not a trivial property is best illustrated by the case of the hydrogen atom ¹²⁾. The energy levels of the H atom possess an extra degeneracy beyond that of rotational invariance and it is easy to show that the generators of an $O(4)$ symmetry commute with the Hamiltonian of the H atom. However, the scattering of bound states does not possess this symmetry and hence $O(4)$ which is certainly useful for analyzing the spectrum is of no use for the dynamics of particle interactions.

Let me thus start by assuming that $SU(6)_W$ is a symmetry for collinear processes such as three point functions and try to determine from experiment whether such an assumption is approximately valid.

First of all the decays of $L=0$ mesons or baryons are not a problem anymore since, e.g., $\Delta \rightarrow N \pi$ and $\rho \rightarrow \pi \pi$ are now allowed. Indeed, since the π is a $W=1$ object and the longitudinal ρ a $W=0$ object, W spin conservation allows both

$$\Delta (W = \frac{3}{2}) \rightarrow N (W = \frac{1}{2}) + \pi (W = 1)$$

and

$$\rho (W = 0) \rightarrow \pi (W = 1) + \pi (W = 1)$$

Neglecting symmetry breaking corrections due to masses one obtains, from standard SU(6) Clebsch Gordon coefficients, e.g.,

$$\Gamma(\Delta \rightarrow N \pi) = \frac{12}{25} \frac{g_{N\bar{N}\pi}^2}{4\pi} \frac{p^3}{M_\Delta^2} \approx 60 \text{ Mev.}$$

which is low by about a factor of 2 but may be considered as a "reasonable" prediction in view of the approximations made on the masses.

Consider next the decays of $L \neq 0$ hadrons into two $L=0$ multiplets. Since the final state has $L_z=0$ (the z axis is chosen to be the decay axis) so must the initial state. The helicity amplitude for the decay $A(L_z=0; \lambda) \rightarrow B(\lambda) + C$, where C is a pseudoscalar meson will then be written as follows

$$M_\lambda(A \rightarrow BC) = \sum_i \left\{ \begin{array}{c} \underline{\underline{B}} \\ (\underline{\underline{b}}, 2W_B+1) \end{array} \quad \begin{array}{c} \underline{\underline{35}} \\ (2, 3) \end{array} \left| \begin{array}{c} \underline{\underline{A}} \\ (\underline{\underline{a}}, 2W_A+1) \end{array} \right\}_i \cdot \left\{ \begin{array}{c} \underline{\underline{b}} \quad \underline{\underline{g}} \quad \underline{\underline{a}} \\ I_b, Y_b \quad 1, 0 \quad I_a, Y_a \end{array} \right\}_{[i]}$$

$$\times (W_B \lambda; 10 | W_A \lambda) (S_A \lambda; L 0 | J_A \lambda) R_i [A \rightarrow B C]$$

Eq. (a)

The first two terms are respectively an "SU(3) scalar" factor ¹³⁾ and an isoscalar factor ¹⁴⁾; $\underline{\underline{A}}, \underline{\underline{B}}$ denote $SU(6)_W$ multiplets and $(\underline{\underline{a}}, 2W_A+1), (\underline{\underline{b}}, 2W_B+1)$ label the $SU(3) \otimes SU(2)_W$ multiplets. The sum over i and $[i]$ corresponds to symmetric or antisymmetric couplings when only octets are involved. The next two terms in Eq. (a) express, respectively, the W spin coupling in the decay and the spin orbit coupling building up the initial state of angular momentum J_A . Finally, the reduced matrix elements $R_i(\underline{\underline{A}} \rightarrow \underline{\underline{B}}, \underline{\underline{C}})$ depend only on the initial and final multiplets. For example the decays of the $\underline{\underline{70}} L=1$ or $\underline{\underline{56}} L=2$ baryons into a $\underline{\underline{56}} L=0$ baryon and a pseudoscalar meson are each described by a single reduced matrix element of $SU(6)_W \otimes O(2)_{L_z}$. For decays into specific charge

channels M_λ still has to be multiplied by the appropriate isospin Clebsch-Gordon coefficient. Partial widths of resonances are related to the helicity decay amplitudes by the standard formula

$$\Gamma(A \rightarrow BC) = \frac{1}{2J_A+1} \frac{p}{M_A^2} \sum_\lambda |M_\lambda|^2$$

where p is the magnitude of the three-momentum in the CM.

In the decays of $L=1$ mesons and baryons, the allowed partial waves in the final state are $\ell=0$ (S wave) or $\ell=2$ (D wave). $SU(6)_W \otimes O(2)_{L_z}$ relates these two partial waves and, as it turns out, it is precisely this relation which appears to be very badly broken in nature.

No detailed calculations are necessary to show that $SU(6)_W$ forbids the decays $A_1 \rightarrow \rho(\lambda=0) + \pi$ and $B \rightarrow \omega(\lambda=1) + \pi$. This last prediction seems to be very badly violated indeed: experiment suggests that the $\lambda=1$ decay amplitude of the B, far from being 0, is in fact dominant¹⁵⁾. This disagreement can be expressed in a different way by going to the partial wave couplings¹⁶⁾. With the formula

$$M_\lambda = \sum_L \sqrt{2L+1} (L 0 S_B \lambda | J_A \lambda) g_L$$

one obtains

$$M_1(B \rightarrow \omega \pi) = g_S + \frac{1}{\sqrt{2}} g_D$$

$$M_0(B \rightarrow \omega \pi) = g_S - \sqrt{2} g_D$$

$SU(6)_W$ predicts $g_S = (-1/\sqrt{2})g_D$ while experimentally $g_S \approx \sqrt{2} g_D$.

Similar troubles occur in the baryon case as well¹⁶⁾. For example, at the $L=1$ level, the $D_{33}(1690)$ is allowed to decay into $\Delta\pi$ only in the helicity $\lambda = \frac{1}{2}$ state. The suppression of the $\lambda = \frac{3}{2}$ decay requires a definite interference among S and D waves which again is not seen. The same argument would hold at the $L=2$ level where the $F_{15}(1690)$ is predicted, from W_z conservation, to decay into $\Delta\pi$ only in the $\lambda = \frac{1}{2}$ state. The experimental situation is, however, less clear cut than for the $L=1$ case.

On the other hand, $SU(6)_W$ also predicts

$$\frac{\Gamma(A_2 \rightarrow \Sigma \pi)}{\Gamma(A_2 \rightarrow K \bar{K})} = 6$$

and this ratio agrees with experiment ⁴⁾. However, both decays $A_2 \rightarrow \Sigma \pi$ and $A_2 \rightarrow K \bar{K}$ occur in a D wave only! Similarly, a detailed study of $L=1$ ⁷⁾ and $L=2$ ¹⁷⁾ baryon decays shows that the $SU(6)_W$ predictions among single partial waves are in satisfactory agreement with experiment while predictions relating different partial waves fail badly.

The conclusion one has to draw is thus clear: $SU(6)_W \otimes O(2)_{L_z}$ is not, even approximately, a symmetry of hadronic three-point functions although its generators appear to approximately commute with the strong interaction Hamiltonian when sandwiched between one-particle (moving) states.

Given this, phenomenological models were proposed a few years ago to explicitly break $SU(6)_W$ symmetry for vertices. One of these models, called $\mathcal{L}$ broken $SU(6)_W$ ^{7), 18)}, simply breaks, by fiat, the link imposed by $SU(6)_W$ between the various partial waves while maintaining the $SU(6)_W$ relations among each specific partial wave channel. The motivation for such a model comes essentially from the fact that different partial waves have to develop different kinematical factors at threshold. These factors depend sensitively on the masses involved and hence it does not seem too unreasonable to break the link between different final state partial waves since the mass breakings are huge. Another model is the so-called 3P_0 model ¹⁹⁾. Carlitz and Kislinger ²⁰⁾ have shown that one can construct $SU(6)_W$ invariant three-point functions by working with quark spin states and allowing a quark-antiquark pair to be produced with the quantum numbers of the vacuum [i.e., $SU(3)$ singlet, $L_z=0$ $P=C=+1$]. The produced pair then shares its quark and antiquark among the two decay particles. This result led Colglazier and Rosner ¹⁹⁾ to suggest a phenomenological breaking of $SU(6)_W$ (called the 3P_0 model) by allowing the produced pair to have $L_z = \pm 1$ as well as $L_z = 0$. A similar suggestion was made somewhat earlier by Micu ¹⁹⁾.

Neither of these models is theoretically well motivated. In fact, these models, although not identical, turn out to be equivalent in many practical applications and have met with some remarkable phenomenological successes. A good example is given by the work of Faiman and Plane ⁷⁾ where the decays $\bar{70} \quad L=1 \rightarrow \bar{56} \quad L=0 + \text{pseudoscalar}$ are discussed and compared

with experiment. In the last lecture, I will briefly come back to the differences between these phenomenological models.

As a last comment on constituent quarks, let me mention that several subgroups of $SU(6)_W$ have been applied to resonance decays, in particular, coplanar and chiral $U(3) \otimes U(3)$. Coplanar $U(3) \otimes U(3)$, first introduced by Dashen and Gell-Mann ²¹⁾ is generated either by W^i and W_x^i $[U(3) \otimes U(3) \rho \sigma_y]$ or by W^i and W_y^i $[U(3) \otimes U(3) \rho \sigma_x]$. If one assumes that the $SU(6)_W$ generators have the simple Lorentz transformation properties suggested by the naïve free quark model expressions, it is clear that W^i and W_x^i (resp. W^i and W_y^i) commute with Lorentz boosts along the $\hat{z}$ axis and along the $\hat{y}$ axis (resp. $\hat{x}$ axis). These algebraic structures are thus invariant for any "quark motion" either in the "y-z" plane or in the "x-z" plane. They were applied to resonance decays by Rosner ¹⁶⁾ who found that they are compatible with but weaker than the 3P_0 model. Chiral $U(3) \otimes U(3)$, on the other hand, is generated by W^i and W_z^i and will be discussed in more detail later on.

To conclude this elementary discussion of constituent quarks, let me insist once again on the following points:

- 1) there is fairly compelling experimental evidence for an approximate $SU(6)_W$ symmetry for one-particle hadronic states although $SU(6)_W$ does not seem to be an approximate symmetry of the full S matrix;
- 2) except for their commutation relations, very little is known about the $SU(6)_W$ generators and, in particular, about their Lorentz transformation properties: they may or may not be identical to those suggested by the naïve free quark expressions of these generators;
- 3) if quarks exist, they cannot be ordinary fermions but must obey unusual statistics, since the lowest lying baryon representation is a (symmetric) $\underline{56}$ of $SU(6)_W$. The standard answer is to consider quarks as "parafermions of rank three" ²²⁾. An equivalent procedure, recently suggested by Gell-Mann ²³⁾, is to introduce three triplets of quarks, distinguished by "color", and to require that hadrons transform as "color" singlets!
- 4) whether one uses the quark language or the more abstract algebraic language of $SU(6)_W$ there is no reason to expect that exotic resonances will remain absent or nearly absent in higher mass regions of the particle spectrum;
- 5) at the level of hadronic three-point functions, $SU(6)_W$ breaking is fairly well described by phenomenological models such as "1 broken $SU(6)_W$ or the 3P_0 model.

2. CURRENT QUARKS

A "current quark" algebraic structure was first suggested by Gell-Mann ²⁴⁾ for the vector and axial vector charges

$$F^i(t) = \int d^3x \mathcal{F}_0^i(x) \quad F^{i5}(t) = \int d^3x \mathcal{F}_0^{i5}(x) \quad (i=1,\dots,8)$$

where $\mathcal{F}_\mu^i(x)$ and $\mathcal{F}_\mu^{i5}(x)$ are octets of local current densities which can, in principle, be measured in weak and electromagnetic transitions. In a "current quark" model, one would write these densities as follows:

$$\mathcal{F}_\mu^i(x) = \bar{q}(x) \gamma_\mu \frac{\lambda_i}{2} q(x) \quad ; \quad \mathcal{F}_\mu^{i5}(x) = \bar{q}(x) \gamma_\mu \gamma_5 \frac{\lambda_i}{2} q(x)$$

With canonical equal-time anticommutation relations for quark fields, one easily derives from these expressions:

$$[F^i(t), F^j(t)] = i f_{ijk} F^k(t)$$

$$[F^i(t), F^{j5}(t)] = i f_{ijk} F^{k5}(t)$$

$$[F^{i5}(t), F^{j5}(t)] = i f_{ijk} F^k(t)$$

i.e., an $SU(3) \otimes SU(3)$ algebraic structure. Abstracting from the current quark model, one then assumes that this algebraic structure holds for the physical vector and axial vector charges independently of any underlying quark field. It is important to realize that the algebraic structure abstracted in this way has at least the chance of being rigorously exact in the real world. This is to be contrasted with the constituent quark approach where, at best, one only hopes for approximate algebraic properties of the particle spectrum.

Strictly speaking the $SU(3) \otimes SU(3)$ algebra of vector and axial vector charges has not yet been tested directly by experiment: it leads to an exact sum rule, first derived by Adler ²⁵⁾, relating the axial nuclear β decay constant to the strangeness conserving forward inelastic ν and $\bar{\nu}$ total differential cross-sections. Present data do not yet quite allow an experimental test of this sum rule.

With the help of PCAC ²⁶⁾, however, the Adler sum rule can be rewritten in terms of (off-mass shell) π N cross-sections giving the celebrated Adler-Weisberger ²⁷⁾ relation, which is remarkably successful.

Encouraged by this success, one may try to enlarge the $SU(3) \otimes SU(3)$ algebra. In a current quark model with canonical equal time anticommutation relations one is led naturally to a $U(12)$ algebra: its 144 generators are defined as integrals over the local densities:

$$\bar{q}(x) \Gamma \frac{\lambda_i}{2} q(x) \quad (i = 0, 1, \dots, 8)$$

with Γ one of the 16 Dirac covariants. In principle all these current densities are directly or indirectly measurable: for example, although nothing in nature seems to be directly coupled to a tensor current $\mathcal{F}_{\mu\nu}^i(x)$, which in the quark model would be expressed as $\bar{q}(x) \sigma_{\mu\nu} (\lambda_i/2) q(x)$, this current appears nevertheless on the right-hand side of commutators such as $[\mathcal{F}_{\mu}^i(x), \partial^{\nu} \mathcal{F}_{\nu}^{i5}(y)]_{x^0=y^0}$. In this sense, all 144 current densities could be meaningful physical operators provided, of course, that abstraction of algebraic relations from a quark model can be carried that far.

Let me insist again on the fact that in the current quark model one is dealing from the start with well defined operators: they are integrals over local current densities with known Lorentz transformation properties. Algebraic structures abstracted from the model could be exact no matter how badly broken, say, $SU(3)$ symmetry is in the particle spectrum.

The commutation relations abstracted from the quark model are best used by sandwiching them between infinite momentum states ^{28), 29)}. A more modern version of this procedure is to use the so-called light like charges ^{30), 31)}. A few comments on both of these languages are perhaps appropriate.

Going to the $P_z = \infty$ frame, many of the $U(12)$ operators have vanishing matrix elements between single particle states. Let us call "good" operators those charges whose matrix elements do not vanish when taken between finite mass states and "bad" operators the others. It is easy to verify ²⁹⁾ that in a current quark model one has 35 independent good operators whose expressions are given by:

$$F^i(t) = \int d^3x \tilde{F}_0^i(x) = \int d^3x q^\dagger(x) \frac{\lambda_i}{2} q(x) \sim \int d^3x q^\dagger(x) \alpha_z \frac{\lambda_i}{2} q(x) = \int d^3x \tilde{F}_3^i(x)$$

$$F_x^i(t) = \int d^3x \tilde{F}_{23}^i(x) = \int d^3x q^\dagger(x) \beta \alpha_x \frac{\lambda_i}{2} q(x) \sim \int d^3x q^\dagger(x) \beta \alpha_y \frac{\lambda_i}{2} q(x) = \int d^3x \tilde{F}_{20}^i(x)$$

$$F_y^i(t) = \int d^3x \tilde{F}_{31}^i(x) = \int d^3x q^\dagger(x) \beta \alpha_y \frac{\lambda_i}{2} q(x) \sim \int d^3x q^\dagger(x) \beta \alpha_x \frac{\lambda_i}{2} q(x) = \int d^3x \tilde{F}_{01}^i(x)$$

$$F_z^i(t) = \int d^3x \tilde{F}_3^{i5}(x) = \int d^3x q^\dagger(x) \sigma_z \frac{\lambda_i}{2} q(x) \sim \int d^3x q^\dagger(x) \sigma_5 \frac{\lambda_i}{2} q(x) = \int d^3x \tilde{F}_0^{i5}(x)$$

for $F^i(t) : i=1,2,\dots,8$; for $F_\alpha^i(t), \alpha=x,y,z : i=0,1,\dots,8$.

All other components are "bad" operators. At infinite momentum the "good" charges thus generate an $SU(6)$ algebra ³²⁾ which will be called $SU(6)_W$; currents to distinguish it from the $SU(6)_W$ algebra introduced in Section 1 and which will be referred to as $SU(6)_W$; strong.

Let me recall that one of the main reasons for going to the $P_z = \infty$ frame is that complicated multiparticle states corresponding to disconnected or semi-disconnected diagrams do not contribute to the saturation of sum rules derived from "good-good" commutation relations. This property can be made mathematically more precise in terms of light-like charges ³⁰⁾ (which roughly correspond to infinitely boosted space-like charges).

In the light-cone formulation of the free quark model ³¹⁾ one quantizes on the light-plane $x^+ = (x^0 + x^3)/\sqrt{2} = 0$ instead of the usual equal time quantization. For any four-component object (vector, δ matrix) let me define

$$V^\pm = \frac{V^0 \pm V^3}{\sqrt{2}}$$

$$\vec{V}_\perp = (V^1, V^2)$$

Introducing, in the free quark model, the projection operators $P_\pm = \frac{1}{2}(1 \pm \alpha_z)$ one defines the fields $q_\pm(x) = P_\pm q(x)$. On the light-plane $x^+ = 0$, $q_+(x)$ gives the only two independent components of the quark field and for these components, canonical anticommutation relations hold:

$$\{q_+^\dagger(x), q_+(y)\}_{x^+=y^+} = \frac{1}{\sqrt{2}} P_+ \delta(x^- - y^-) \delta^2(\vec{x}_\perp - \vec{y}_\perp)$$

$$\{q_+(x), q_+(y)\}_{x^+=y^+} = 0 \quad \text{etc ...}$$

The non-independent components $q_-(x)$ are related to $q_+(x)$ by a constraint equation ^{30), 31)} which depends on the dynamics of the problem. It can be solved in the free quark model yielding

$$q_-(x) = -\frac{i}{4} \int d\xi \varepsilon(x^- - \xi) (i\vec{\gamma}_\perp \cdot \vec{\gamma}_\perp + m) \gamma^+ q_+(x^+, \xi, \vec{x}_\perp)$$

In the light-plane formulation, the $SU(6)_{W; \text{current}}$ generators are then defined as follows:

$$\tilde{F}^i = \sqrt{2} \int d^4x \delta(x^+) q_+^\dagger(x) \frac{\lambda_i}{2} q_+(x)$$

$$\tilde{F}_x^i = \sqrt{2} \int d^4x \delta(x^+) q_+^\dagger(x) \gamma_x \gamma_5 \frac{\lambda_i}{2} q_+(x)$$

$$\tilde{F}_y^i = \sqrt{2} \int d^4x \delta(x^+) q_+^\dagger(x) \gamma_y \gamma_5 \frac{\lambda_i}{2} q_+(x)$$

$$\tilde{F}_z^i = \sqrt{2} \int d^4x \delta(x^+) q_+^\dagger(x) \sigma_z \frac{\lambda_i}{2} q_+(x)$$

In this language, the "good" classification of operators at $P_z = \infty$ corresponds to the fact that they are built up only from the canonical components $q_+(x)$ of the quark field. Bad operators in the $P_z \rightarrow \infty$ frame are now separated into "bad" and "terrible" operators depending on whether they contain one or two dependent fields $q_-(x)$.

An obvious property of the "good" light-like charges is that, when applied to a state of momentum p^μ they leave $p^0 + p^3$ and $\vec{p}_\perp$ invariant. Hence, in the absence of zero mass hadronic states, light-like charges annihilate the vacuum whether they are conserved or not ³³⁾

$$\tilde{F}^i |0\rangle = \tilde{F}_\alpha^i |0\rangle \equiv 0$$

In the $m_\pi \rightarrow 0$ limit, this property is of course lost for the light-like axial charges ³⁴⁾.

In the $P_z \rightarrow \infty$ frame, the Lorentz covariance properties of current density matrix elements are expressed by the complicated "angular condition" of Dashen and Gell-Mann ³⁵⁾. Similarly, the Lorentz transformation properties of the $\tilde{F}^i$ are not simple ³⁶⁾. They commute with boosts along the z axis, e.g., $[\tilde{F}^i, K_3] = 0$ but not with transverse boosts: a state with given helicity and transverse momentum $\vec{p}_\perp$ will have a different $SU(6)_{W; \text{current}}$ classification than a state with the same helicity but a different $\vec{p}_\perp$. One can partly avoid this difficulty with the help of "transverse" light-cone boosts defined by

$$E^1 = \frac{K_1 + J_2}{p^0 + p^3} \quad E^2 = \frac{K_2 - J_1}{p^0 + p^3}$$

Since the $\tilde{F}^i$ can be shown to commute with $\vec{E}_\perp$, states with transverse momentum $\vec{p}_\perp$ generated through $\vec{E}_\perp$ will then be classified in the same $SU(6)_{W; \text{currents}}$ representation, independently of $\vec{p}_\perp$,

$$|\vec{p}_\perp\rangle = e^{-i\vec{p}_\perp \cdot \vec{E}_\perp} |\vec{0}_\perp\rangle$$

(but $|\vec{p}_\perp\rangle$ will in general be a $\vec{p}_\perp$ dependent mixture of helicity states).

In the infinite momentum limit or in the light-cone language, it is clear that charge conjugation remains a good quantum number but not parity. Instead of parity, one can use the "natural parity" operator $\mathcal{N} = \mathcal{P} \exp(-i\pi J_y)$ (where $\mathcal{P}$ is the parity operator) under which the $\tilde{F}^i$ are eigenvectors (e.g., $\mathcal{N}^{-1} \tilde{F}_z^i \mathcal{N} = -\tilde{F}_z^i$), while in the infinite momentum frame $\mathcal{N}$ will lead to the standard constraints on helicity couplings ³⁷⁾.

The upshot of this discussion of the $P_z \rightarrow \infty$ frame and of the light-like charges is that, in the free quark model, one is led to an $SU(6)_{W; \text{currents}}$ algebraic structure for good operators. Abstracting such an algebraic structure for good operators is certainly not unreasonable (but not necessarily correct) since these operators involve only the canonical fields $q_+(x)$ and hence their algebraic relations may be preserved in interacting theories. For bad operators, it is clear that a similar abstraction from the free quark model is much more speculative (but not

necessarily wrong) since the "dependent" fields $q_-(x)$ are related to $q_+(x)$ by constraint equations which explicitly depend on the dynamics of the problem.

The free constituent quark model led us to postulate the existence of an approximate $SU(6)_{W;strong}$ symmetry, at least for one particle moving hadronic states while the free current quark model leads to an $SU(6)_{W;currents}$ algebraic structure among physically measurable operators in weak and electromagnetic transitions. The obvious question is of course whether there is any relation among the generators of these algebras ³⁸⁾.

As a first guess, one is of course tempted to identify the "constituent" and "current" algebraic structures. This is strongly suggested by the successes of the conserved vector current (CVC) idea in weak interactions ³⁹⁾. Let me recall that CVC identifies the F^i , which are integrals of the time component of the vector currents, with the W^i , which are the generators of ordinary $SU(3)$ symmetry in strong interactions:

$$F^i \equiv W^i \quad (CVC)$$

With this hint from CVC and the similarity in the algebraic structures it seems natural to identify all F^i 's with the corresponding W^i 's [either at the $SU(6)$ level or at the $SU(3) \otimes SU(3)$ level]. However, this does not seem to be the case in nature.

To discuss this point in some detail let me use the infinite momentum frame language ($P_z \rightarrow \infty$) and sketch how particles can be classified in $SU(6)_W$ and $SU(3) \otimes SU(3)$ multiplets ⁴⁰⁾. Since neither algebra uniquely specifies a hadronic state, at least one additional quantum number has to be introduced. A possible choice is the total "spin" in the $\hat{z}$ direction, i.e., the eigenvalue of J_z . A more useful choice follows from noting that W_z^0 and F_z^0 act like constituent and current quark "spin" operators. Both obviously commute with J_z and hence one can define a constituent or current orbital angular momentum by

$$L_z(W) = J_z - W_z^0 \quad \text{and} \quad L_z(F) = J_z - F_z^0$$

This procedure is perfectly general and independent of quarks⁴¹⁾. With the help of these L_Z 's one can now classify states, at infinite momentum, in terms of their $SU(6)_W$ quantum numbers and angular momentum. For example, the $SU(6)_W \otimes O(2)_{\text{strong}}$ classification of the low lying mesons and baryons would be, at $P_Z = \infty$, $\underline{35} \oplus \underline{1} L_Z(W) = 0$ and $\underline{56} L_Z(W) = 0$; the next level would correspond to $\underline{35} \oplus \underline{1} L_Z(W) = +1, 0, -1$ and $\underline{70} L_Z(W) = +1, 0, -1$ etc...

The $SU(3) \otimes SU(3) \otimes O(2)$ classification of hadronic states at $P_Z = \infty$, on the other hand, is easily obtained from the following trivial assignments for quark states: $q_\uparrow$, a spin up quark state with zero orbital momentum transforms as $[(\underline{3}, \underline{1})_{\frac{1}{2}}, 0]$ where the notation $[(\underline{A}, \underline{B})_{S_Z}, L_Z]$ means the following: $\underline{A}$ and $\underline{B}$ denote, say in the constituent picture, the representations of $\frac{1}{2}(W^i + W_Z^i)$ and $\frac{1}{2}(W^i - W_Z^i)$ respectively while S_Z is the eigenvalue of W_Z^0 and $L_Z = L_Z(W) = J_Z - W_Z^0$. Similarly

$$\begin{aligned} q_\downarrow &\sim [(\underline{1}, \underline{3})_{-\frac{1}{2}}, 0] \\ \bar{q}_\uparrow &\sim [(\underline{1}, \underline{\bar{3}})_{\frac{1}{2}}, 0] \\ \bar{q}_\downarrow &\sim [(\underline{\bar{3}}, \underline{1})_{-\frac{1}{2}}, 0] \end{aligned}$$

In the (symmetric) $\underline{56} L_Z(W) = 0$ representation of $SU(6)_W \otimes O(2)_{\text{strong}}$ only the following representation of $SU(3) \otimes SU(3) \otimes O(2)_{\text{strong}}$ can occur: $[(\underline{10}, \underline{1})_{\frac{3}{2}}, 0]$, $[(\underline{6}, \underline{3})_{\frac{1}{2}}, 0]$, $[(\underline{3}, \underline{6})_{-\frac{1}{2}}, 0]$ and $[(\underline{1}, \underline{10})_{-\frac{3}{2}}, 0]$ and these representations are easily associated to the nucleon octet and $\Delta(1238)$ decimet. In particular, the nucleon and Δ states with helicity $+\frac{1}{2}$ are seen to belong to the same $[(\underline{6}, \underline{3})_{\frac{1}{2}}, 0]$ representation of $SU(3) \otimes SU(3) \otimes O(2)_{\text{strong}}$.

If one identifies $SU(3) \otimes SU(3) \otimes O(2)_{\text{strong}}$ with $SU(3) \otimes SU(3) \otimes O(2)_{\text{currents}}$ $L_Z(F) \equiv L_Z(W)$ and the axial charges, which are generators of the algebra can only connect the nucleon octet to itself and to the Δ multiplet. Using PCAC this implies that no other resonance can decay into πN or $\pi \Delta$! This is clearly an absurd result. The same argument holds, of course, for $SU(6)_W \otimes O(2)$ as well.

Another argument, due to Dashen and Gell-Mann ⁴²⁾, does not depend on PCAC but concerns the dipole operator $\left[\int d^3x \, x J_0^{e.m.}(x) \right]$. This operator can be shown to transform according to the $\left[\left(\frac{3}{2}, 1 \right) \pm \left(1, \frac{3}{2} \right) \right]_{S_Z=0}$; $L_Z(F) = \pm 1$ representation of $SU(3) \otimes SU(3) \otimes O(2)$ currents. Identifying the constituent and current algebras then implies vanishing anomalous magnetic moments for the neutron and the proton as well as a vanishing transition moment between N and Δ , since all these states have $L_Z = 0$. Once again this result is unacceptable.

One must conclude from these arguments that physical baryons cannot be described as corresponding to simple irreducible representations of $SU(6)_W \otimes O(2)$ currents (or its chiral subgroup) and hence that the current and constituent algebras cannot be identified. This fact was recognized long ago and led to various "mixing" schemes ⁴³⁾ in an attempt to parametrize the impurity of physical states under transformations generated by $SU(6)_{W, \text{currents}}$ or its chiral subgroup.

In the picturesque language of quarks, one can thus contrast the images one develops for, say, a proton, in the "constituent" and "current" frameworks: in $SU(6)_{W, \text{strong}}$, a proton looks like an object made of just three constituent quarks in an S wave while in $SU(6)_{W, \text{currents}}$ it looks like a much more complex object with, perhaps, many "current" quarks and quark-antiquark pairs moving in various orbital momenta. Such an intuitive picture is certainly reinforced by the SLAC results on deep inelastic scattering ⁴⁴⁾.

The two $SU(6)_W$ algebras are thus distinct. Nevertheless, it is perhaps not unreasonable to assume that they are related by some unitary transformation ³⁵⁾, V, so that both $SU(6)_W$'s are equally appropriate for labelling states. Note that if the quantum numbers of either $SU(6)_W$ did uniquely specify a hadronic system, a unitary transformation relating both $SU(6)_W$'s would necessarily have to exist. Since this is not the case, the existence of V is an assumption. A necessary condition for the existence of V is that the structure of representations on physical states be identical for both $SU(6)_W$. In other words if we find a given representation of $SU(6)_{W, \text{strong}}$ in the spectrum, the same representation of $SU(6)_{W, \text{currents}}$ must occur in the Hilbert space of physical states.

Let me thus assume

$$W_{\beta}^i = V F_{\beta}^i V^{-1} \quad (\beta = -, x, y, z)$$

What can one then say about V ? Two obvious properties are that it must be invariant under rotations around the z axis i.e., $[\vec{J}_z, \vec{V}] = 0$ and that it must be an $SU(3)$ singlet i.e., $[\vec{F}^i, \vec{V}] = 0$ in order not to spoil CVC.

As discussed earlier, we have to impose that $[\vec{F}_z^i, \vec{V}] \neq 0$, $i = 1, \dots, 8$, as well as $[\vec{L}_z(F), \vec{V}] \neq 0$; since $[\vec{J}_z, \vec{V}] = 0$ and $\vec{J}_z = \vec{F}_z^0 + \vec{L}_z(F) = \vec{W}_z^0 + \vec{L}_z(W)$ one must also have $[\vec{F}_z^0, \vec{V}] \neq 0$. $\vec{F}_z^0$ and $\vec{W}_z^0$ are thus different operators and a state with a simple spin structure under $\vec{W}_z^0$ and simple orbital structure under $\vec{L}_z(W)$ will be classified as a mixture of different "current" spins ($\vec{F}_z^0$) and different orbital momenta $\vec{L}_z(F)$. The transformation V thus expresses in a compact and universal form the idea of the phenomenological mixing schemes⁴³⁾. With the help of V , all hadrons can be simultaneously described as simple "constituent" objects where strong interactions are concerned and as complicated "current" objects when weak and electromagnetic interactions are considered.

Universal mixing schemes have been "guessed" and developed phenomenologically by Buccella, Kleinert, Savoy and others⁴³⁾. The approach adopted in these lectures is more closely connected to the work of Melosh⁴⁵⁾, namely, it concentrates on general theoretical properties of the transformation V and, in particular, on its algebraic structure.

3. THE MELOSH TRANSFORMATION

In this Section I will follow almost literally the work of Melosh⁴⁵⁾. First, I will point out some further properties of V , and then such an operator will be explicitly constructed in the free quark model. This construction will be done in the standard equal-time quantization framework and briefly repeated in the light-cone version⁴⁶⁾ since there are some poorly understood differences⁴⁷⁾ between the two procedures. I will then try to indicate how V might be constructed in less trivial models as well as comment on which properties of V one may hopefully abstract from the free quark model and use in phenomenological applications.

To start with the simplest possible situation, let me consider the case where $SU(6)_{W; \text{strong}}$ is an "exact" symmetry for one-particle states. In this limit it is clear that V has to transform states belonging to irreducible representations of $SU(6)_{W; \text{currents}}$ into eigenstates of the Hamiltonian or, equivalently, V has to take us from a set of charges, F_β^i , defined as integrals over local densities, to a set of operators, W_β^i , which commute with the Hamiltonian.

Another important property of V is that it can only take "good" operators into "good" operators. This follows from the fact that the F_{β}^i 's are themselves "good" operators while the W_{β}^i 's, in the free quark model, also commute with boosts along the z axis. To ensure that, at infinite momentum, finite mass states mix only among themselves, this property of V must take even a stronger form: V itself has to be a good operator. As a result, V cannot be invariant under the complete spatial rotation group $O(3)$ since it cannot contain either a scalar density or the transverse components of vector densities, etc... This is of course not surprising: in defining $SU(6)_W$ algebras one has, from the beginning picked out a specific direction in space (the z axis)!

Let me now proceed to the construction of V in the free quark model. In such a model the fundamental operator of the theory is $q(x)$, a local relativistic field, which I first choose to quantize in the canonical equal-time formalism, i.e.,

$$\{q_{\alpha}(x), q_{\beta}^{\dagger}(y)\}_{x^0=y^0} = \delta_{\alpha\beta} \delta^3(\vec{x}-\vec{y}) \quad \dots$$

The Hamiltonian of the theory is given by:

$$H_{free} = \int d^3x \, q^{\dagger}(x) \{ -i \vec{\alpha} \cdot \vec{\partial} + \beta m \} q(x)$$

Local current densities can be constructed from the standard bilinears $q^{\dagger}(x) \Gamma(\lambda_i/2) q(x)$ and, in this model, as mentioned previously, one writes the generators F_{free}^i of $SU(6)_{W; currents}$ as follows

$$F_{free}^i = \int d^3x \, q^{\dagger}(x) \frac{\lambda_i}{2} q(x)$$

$$F_{x, free}^i = \int d^3x \, q^{\dagger}(x) \beta \sigma_x \frac{\lambda_i}{2} q(x)$$

$$F_{y, free}^i = \int d^3x \, q^{\dagger}(x) \beta \sigma_y \frac{\lambda_i}{2} q(x)$$

$$F_{z, free}^i = \int d^3x \, q^{\dagger}(x) \sigma_z \frac{\lambda_i}{2} q(x)$$

The $SU(6)_{W; strong}$ generators, on the other hand, can be partly defined by $[W_{\beta, free}^i, H_{free}] = 0$ and $W_{\beta, free}^i = V_{free} F_{\beta, free}^i V_{free}^{-1}$.

It is easily seen that even in this trivial free quark model the $SU(6)_{W; currents}$ generators do not commute with the Hamiltonian and hence cannot be identified with the symmetry generators $W_{\beta, free}^i$. V_{free} is thus not a trivial operator.

To construct a V_{free} explicitly it is convenient to work in a representation ⁴⁵⁾ where the $W_{\beta, \text{free}}^i$ have a simple form: let me define the "W representation" where operators have the form $O_W = V_{\text{free}}^{-1} O V_{\text{free}}$. In this representation, the $SU(6)_{W; \text{strong}}$ generators have the form of the F_{free}^i i.e., $(W_{\beta, \text{free}}^i)_W = F_{\beta, \text{free}}^i$ while $(F_{\beta, \text{free}}^i)_W = V_{\text{free}}^{-1} F_{\beta, \text{free}}^i V_{\text{free}}$ and $(H_{\text{free}})_W = V_{\text{free}}^{-1} H_{\text{free}} V_{\text{free}}$. The advantage of this trick is that one can tell at a glance whether the $(W_{\beta, \text{free}}^i)_W$ commute with $(H_{\text{free}})_W$: this will be the case only if $(H_{\text{free}})_W$ contains no other Dirac matrix but $1, \beta, \alpha_z$ and $\beta \alpha_z$. V_{free} must thus transform away the $\vec{\alpha}_\perp$ parts of the Hamiltonian. This is very similar to a Foldy-Wouthuysen transformation ⁴⁸⁾ where one transforms away all the "small components" of the Hamiltonian (i.e., those containing $\vec{\alpha}$'s). This analogy allows one to write at once a possible form of the unitary operator V_{free} (Melosh transformation):

$$V_{\text{free}} = \exp i Y_{\text{free}}$$

$$Y_{\text{free}} = \frac{1}{2} \int d^3x \, q^\dagger(x) \arctan\left(\frac{\vec{\alpha}_\perp \cdot \vec{\alpha}_\perp}{m}\right) q(x)$$

such that $(H_{\text{free}})_W = V_{\text{free}}^{-1} H_{\text{free}} V_{\text{free}} = \int d^3x \, q^\dagger(x) \left\{ -i \alpha_z \partial_z + \beta \sqrt{m^2 + (\vec{\alpha}_\perp \cdot \vec{\alpha}_\perp)} \right\} q(x)$ which obviously commutes with $(W_{\beta, \text{free}}^i)_W$.

Note that Y_{free} would be identical to the Foldy-Wouthuysen transformations if $\vec{\gamma} \cdot \vec{\sigma}$ were substituted for $\vec{\alpha}_\perp \cdot \vec{\alpha}_\perp$. The difference is, however, crucial since the additional term $\gamma_z \partial_z$ corresponds to a bad operator. Because of the absence of this "bad" term, V_{free} is a good operator and is not invariant under the full rotation group but only under rotations around the z axis: $[\vec{J}_z, V_{\text{free}}] = 0$. It is easily checked that V_{free} possesses all the other properties which were required earlier $[SU(3) \text{ singlet, } C=+1, \text{ etc.}]$.

In the free quark model, one has thus a set of exactly conserved operators which generate $SU(6)_{W; \text{strong}}$. In terms of the (local) quark field $q(x)$ they can be written as:

$$W_{\text{free}}^i = F_{\text{free}}^i$$

$$W_{x, \text{free}}^i = F_{x, \text{free}}^i + \int d^3x \, q^\dagger(x) \frac{1}{x} \left\{ \frac{1}{1+\delta} \frac{\vec{\alpha}_\perp \cdot \vec{\alpha}_\perp}{m} - i \right\} \gamma_5 \frac{\partial_x}{m} \frac{1}{2} q(x)$$

$$W_{y, free}^i = F_{y, free}^i + \int d^3x \, q^\dagger(x) \frac{1}{x} \left\{ \frac{1}{1+x} \frac{\vec{\sigma}_\perp \cdot \vec{\partial}_\perp}{m} - i \right\} \gamma_5 \frac{\partial_y}{m} \frac{\lambda_i}{2} q(x)$$

$$W_{z, free}^i = F_{z, free}^i + \int d^3x \, q^\dagger(x) \frac{1}{x} \left\{ \frac{1}{1+x} \frac{\vec{\sigma}_\perp \cdot \vec{\partial}_\perp}{m} - i \right\} \sigma_z \frac{\vec{\sigma}_\perp \cdot \vec{\partial}_\perp}{m} \frac{\lambda_i}{2} q(x)$$

with

$$x = \left[1 + \left(\frac{\vec{\sigma}_\perp \cdot \vec{\partial}_\perp}{m} \right)^2 \right]^{1/2}$$

To understand somewhat the complexity of these operators it is useful to expand the quark field $q(x)$ in terms of creation and annihilation operators

$$q(x) = \frac{1}{\sqrt{V}} \sum_{\vec{k}, r} \left\{ a^r(\vec{k}) u^r(\vec{k}) e^{-ik \cdot x} + b^{+r}(\vec{k}) v^r(\vec{k}) e^{ik \cdot x} \right\}$$

where r runs over $SU(3)$ as well as Dirac indices. One then obtains the following expressions:

$$W_{free}^i = \sum_{\vec{k}, r, s} \left\{ a^{+r}(\vec{k}) a^s(\vec{k}) u^{+r}(0) \frac{\lambda_i}{2} u^s(0) - b^{+s}(-\vec{k}) b^r(-\vec{k}) v^{+r}(0) \frac{\lambda_i}{2} v^s(0) \right\}$$

$$W_{x, free}^i = \sum_{\vec{k}, r, s} \left\{ a^{+r}(\vec{k}) a^s(\vec{k}) [u^{+r}(0) R^{-1} \sigma_x \frac{\lambda_i}{2} R u^s(0)] + b^{+s}(-\vec{k}) b^r(-\vec{k}) [v^{+r}(0) R^{-1} \sigma_x \frac{\lambda_i}{2} R v^s(0)] \right\}$$

$$W_{y, free}^i = \sum_{\vec{k}, r, s} \left\{ a^{+r}(\vec{k}) a^s(\vec{k}) [u^{+r}(0) R^{-1} \sigma_y \frac{\lambda_i}{2} R u^s(0)] + b^{+s}(-\vec{k}) b^r(-\vec{k}) [v^{+r}(0) R^{-1} \sigma_y \frac{\lambda_i}{2} R v^s(0)] \right\}$$

$$W_{z, free}^i = \sum_{\vec{k}, r, s} \left\{ a^{+r}(\vec{k}) a^s(\vec{k}) [u^{+r}(0) R^{-1} \sigma_z \frac{\lambda_i}{2} R u^s(0)] - b^{+s}(-\vec{k}) b^r(-\vec{k}) [v^{+r}(0) R^{-1} \sigma_z \frac{\lambda_i}{2} R v^s(0)] \right\}$$

with R representing a rotation and given by the following 4×4 matrix

$$R = \left[\frac{(\omega + k^0)(\omega + m)}{2\omega(k^0 + m)} \right]^{\frac{1}{2}} \left\{ 1 - \frac{i k_z (\vec{k} \times \vec{\sigma})_z}{(\omega + k^0)(\omega + m)} \right\}$$

where

$$\omega = \sqrt{k_z^2 + m^2} \quad \text{and} \quad k^0 = \sqrt{k_z^2 + m^2}$$

These expressions for the $W_{\beta, \text{free}}^i$ clearly annihilate the vacuum as required. For $\vec{k}_\perp = 0$ or $k_z = 0$, $R \equiv 1$. Consider then a single quark state moving, say, along the $\hat{x}$ axis with spin up along the $\hat{z}$ axis. On such a state, W_z is equal to $+\frac{1}{2}$. Next, boost this state along the $\hat{z}$ axis: one no longer sees a "spin up" state because of the Wigner rotation of the spins which will mix spin up and spin down components. R is precisely the inverse of the Wigner rotation so as to guarantee that the mixture of spin components induced by the boosts remains a $W_z = +\frac{1}{2}$ eigenstate of W_z^0 . Hence the complexity of the $W_{\beta, \text{free}}^i$ arises from the need for invariance of the $SU(6)_{W; \text{strong}}$ charges under boosts in the $\hat{z}$ direction. Noting that $R \neq 1$ as $k_z \rightarrow \infty$ it is clear that at infinite momentum an eigenstate of $W_{z, \text{free}}^0$ will correspond to mixtures of representations of $SU(6)_{W; \text{currents}}$ since $F_{z, \text{free}}^0$ simply counts the number of spin up and spin down quarks irrespective of their transverse momenta while $W_{z, \text{free}}^0$ eigenstates contain a specific mixture of up and down quarks.

Let me now analyze the structure of V_{free} in more detail. The most obvious property is that it is non local in the transverse directions. In particular, a quark field transforms under V_{free} as follows:

$$V_{\text{free}} q(x) V_{\text{free}}^{-1} = \frac{(\not{x} + 1) - i \frac{\vec{\sigma}_\perp \cdot \vec{\partial}_\perp}{m}}{\sqrt{2x(x+1)}} q(x)$$

The exact $SU(6)_{W; \text{strong}}$ which has been constructed is thus generated by non-local operators which annihilate the vacuum. Although we have an exact symmetry the $\hat{z}$ axis still plays a special role in the theory: since the $W_{\beta, \text{free}}^i$ do not commute with transverse boosts their eigenvalues depend on the value of transverse momentum. This leads thus to a rather unconventional "symmetry". It should be emphasized, however, that there is no fundamental reason to believe that symmetries of the strong interactions have to be generated by integrals over local densities: in the case of $SU(6)_W$, for example, the only thing we really have been able to infer from experiment is the existence of an algebraic structure which holds (approximately) for the

integrated charges in the one-particle sector of the Hilbert space. As pointed out, in particular by F. Gürsey and co-workers ⁴⁹⁾, such types of non-local transformations may turn out to be an essential ingredient if internal and space-time symmetries are ever to be combined in a consistent manner.

Next, in the free quark model, one can introduce the quark spin ($\Sigma_{m,free}$) and orbital angular momentum ($L_{m,free}$) operators. They are given by the standard formulae:

$$\Sigma_{m,free} = \int d^3x \, q^\dagger(x) \frac{\sigma_m}{2} q(x)$$

$$L_{m,free} = i \epsilon_{mnp} \int d^3x \, x^n q^\dagger(x) \partial^p q(x)$$

The sum of these operators is the total angular momentum $J_{m,free}$ and one should note that $L_{z,free}$ and $\Sigma_{z,free}$ correspond to the operators $L_z(F)$ and F_z^0 which were discussed in the previous section.

With these operators and the F_β^i 's it is easily seen that Y_{free} corresponds to the $\Delta J_{z,free}=0$, $\Delta \Sigma_{z,free}=\pm 1$ member of a $(\underline{3}, \underline{\bar{3}}) \pm (\underline{\bar{3}}, \underline{3})$ representation of $SU(3) \otimes SU(3) \otimes O(2)$ currents or to the corresponding member of a $\underline{35}$ of $SU(6)_W \otimes O(2)$ currents. While $L_{z,free}$ is only changed by ± 1 it is clear that, due to the power series in $(\vec{\sigma}_1 \cdot \vec{\sigma}_1)/m$, Y_{free} has a complex tensor structure which includes operators with $\Delta j=0,1,2,\dots$. Y_{free} can thus mix states with any total angular momentum as well as states with different quark spin $\Sigma_{z,free}$.

V_{free} thus possesses many of the qualitative properties expected of the real transformation V . A particularly interesting property is that V_{free} "connects" the vacuum to states with exotic quantum number, e.g., $J_{PC}^{free} = (\text{odd})^{-+}$.

In the light-cone formulation, the expression corresponding to V_{free} is easily seen to become $\tilde{V}_{free} = \exp i \tilde{Y}_{free}$ with

$$\tilde{Y}_{free} = \frac{1}{\sqrt{2}} \int d^4x \, \delta(x^+) \, q_+^\dagger(x) \arctan\left(\frac{\vec{\sigma}_+ \cdot \vec{\sigma}_+}{m}\right) q_+(x).$$

and the algebraic structure of $\tilde{Y}_{free}$ is again obvious: it belongs to a $(\underline{3}, \underline{\bar{3}}) \pm (\underline{\bar{3}}, \underline{3})$, $L_z(\tilde{F}) = \pm 1$ representation of $SU(3) \otimes SU(3) \otimes O(2)$ currents and the $\tilde{W}_\beta^i$ defined by $\tilde{W}_\beta^i; free = \tilde{V}_{free} \tilde{F}_\beta^i \tilde{V}_{free}^{-1}$ are again conserved since $[\tilde{H}_{free} - P_z, \tilde{W}_\beta^i] = 0$.

It turns out, however, that in the free quark model, the light-like $SU(6)_{W,currents}$ generators are also conserved ⁴⁶⁾. This is somewhat puzzling since the lack of conservation of the (space-like) charges $F_{\beta,free}^i$ was precisely the criterion by which a V_{free} was constructed. Several authors ^{47),50)} have interpreted this result as indicating the $\tilde{V}_{free}$ cannot be well defined on a light-like plane. This may be true but presumably $\tilde{V}_{free}$ should do more than just ensure $SU(6)_{W,strong}$ conservation. As discussed in the infinite momentum approach, the rotations induced by V_{free} may be necessary to guarantee, e.g., that multiquark states have a definite total spin. In particular, let me recall that the "angular condition" of Dashen and Gell-Mann ³⁵⁾, which ensures the correct angular properties of matrix elements of current densities at infinite momentum, is known ²⁹⁾ to force, by itself, mixing of $SU(6)_{W,current}$ representations even in the free quark model. Hence, it is possible that some spin criterion may lead to a unique definition of $\tilde{V}$. Clearly a better understanding of this question may be of great importance particularly if one wants to extend the Melosh transformation to the broken symmetry case.

As a last remark on $\tilde{Y}_{free}$ let me point out that the postulated large algebra of good light-like operators of Fritzsche and Gell-Mann ⁵¹⁾ includes the operators which make up Y_{free} . This could eventually lead to a definition of $\tilde{Y}_{free}$ in terms of observable operators.

Having discussed the construction of V_{free} (or $\tilde{V}_{free}$) as well as its algebraic properties, how can one use, extend or generalize the free quark Melosh transformation in more realistic models? The main problem is that in a realistic situation $SU(6)_{W,strong}$ is not an exact symmetry anymore even at the one-particle level: it is not obvious how its generators and hence the Melosh transformation itself, should be defined. One possibility is, of course, to ignore the problem and to take a purely pragmatic point of view: one simply abstracts algebraic properties from the free quark model and directly applies them to the computation of matrix elements of the axial vector charges and of moments of the vector and axial vector current densities. Comparison with experimental data will then, hopefully, indicate whether or not the abstracted structure is approximately correct. This is probably the safest approach to the problem. Nevertheless I would like to present some arguments, mostly due to Melosh ⁴⁵⁾, which make this abstraction procedure from the free quark model somewhat more plausible.

As emphasized in Section 1, the intuitive picture one develops for hadrons from the successes of the $SU(6)_W$ classification scheme is that of

baryons made of $3q$ and mesons of a $q\bar{q}$ pair, i.e., essentially a "free (constituent) quark like structure". On the other hand, for local charges and currents, one finds again a structure similar to that of "free current quarks". Both of these facts are somewhat surprising, but if a transformation relating constituent to current quarks exists and is similar to V_{free} they are strongly correlated and this would leave only one mystery to explain!

One can imagine also that the "free quark like behaviour" in the hadron spectrum occurs through the following mechanism: consider first, in a "quark theory" the chain:

$$H_{\text{free}} \rightarrow H_{\text{SU}(6)_W} \rightarrow H_{\text{strong}}$$

where H_{free} is the free quark Hamiltonian, $H_{\text{SU}(6)_W}$ is, at the one-particle level, an $\text{SU}(6)_W$ invariant Hamiltonian closely related to the strong interaction Hamiltonian H_{strong} [still assumed to be invariant under $\text{SU}(3)$]. Approximate free quark like behaviour, say, for the baryons, can then be taken to mean that there is a correspondance between the "three quark" eigenstates of H_{strong} and some three quark eigenstates of H_{free} . For example, to an approximate $\underline{56}$ multiplet of H_{strong} one can associate an exact $\underline{56}$ of $H_{\text{SU}(6)_W}$ and further, these correspond to $\underline{56}$ like eigenstates of H_{free} . In other words one seems to be able to associate to the Hilbert space of physical states, which are eigenstates of H_{strong} , a subspace of eigenstates of H_{free} (obviously containing $3q$ and $q\bar{q}$ states as well, perhaps, as some exotic configurations but presumably excluding single quark states). Let us call U the operator which describes this association, i.e.,

$$| \text{physical state} \rangle = U | \text{free constituent quark state} \rangle$$

One can then obviously use V_{free} to describe the $\text{SU}(6)_W$;currents content of free constituent quark states. If one imagines, for example, some kind of gluons binding quarks together, as long as they are $\text{SU}(6)_W$ singlets, the generators of $\text{SU}(6)_W$;currents will be identical to the free quark model expressions and the gluons do not alter their structure.

If the operator U can be defined, it is clear that the Melosh transformation will be given by $V = UV_{\text{free}}$ and that

$$W_{\beta}^i = V F_{\beta}^i V^{\dagger} = U W_{\beta; \text{free}}^i U^{\dagger}$$

This form of the Melosh transformation satisfies all the general constraints which we previously derived, provided U transforms good operators into good operators. It should be noted once more that even with exactly degenerate $SU(6)_W$ multiplets in the hadron spectrum, which seems to be a reasonable approximation in many practical calculations, one is certainly not forced to restrict oneself to exactly conserved W^i 's. As discussed in Section 1, hadron spectroscopy suggests $[W^i_\beta, H_{\text{strong}}] = 0$ but only within the subspace of one-particle states: it does not force one to assume a full symmetry of the S matrix. To interpret this "one-particle symmetry", the simplest possibility is to require that the matrix elements of U be large between states which have the same $SU(6)_W$;strong classification, i.e., the dominant terms in U are $SU(6)_W$ singlet terms. Admittedly these arguments are not too convincing, but they do provide some rationale for abstracting algebraic properties from V_{free} in computing trix elements of current-like operators between constituent states. Clearly the preceding arguments can be restricted to $SU(3) \otimes SU(3)$.

In order to indicate how the algebraic structure of V actually affects physical transitions, let me consider the forward matrix element of a current type operator between constituent states, i.e.,

$$\langle H'; \text{strong} | \mathcal{F}_A^i | H; \text{strong} \rangle \quad (a)$$

where $|H; \text{strong}\rangle$ and $|H'; \text{strong}\rangle$ are single particle states with definite $SU(6)_W$;strong or $SU(3) \otimes SU(3)$;strong transformation properties, and, say, with zero transverse momentum; $\mathcal{F}_A^i$ is the Fourier transform (at zero transverse momentum transfer) of some local operator specified by the index A and which has known transformation properties under $SU(6)_W$;currents or its chiral subgroup.

As repeatedly emphasized, $|H; \text{strong}\rangle$ does not lie in a single irreducible representation of $SU(6)_W$;currents. Let me write $|H; \text{strong}\rangle = a_{H; \text{strong}}^+ |0\rangle$; the commutator $[W^i_\beta, a_{H; \text{strong}}^+] = C_{HH}^i a_{H; \text{strong}}^+$ then determines the $SU(6)_W$;strong classification of $|H; \text{strong}\rangle$ since $W^i_\beta |0\rangle = 0$. The operator $V^{-1} a_{H; \text{strong}}^+ V$ has obviously the same commutation relations with F^i_β as $a_{H; \text{strong}}^+$ with W^i_β . Hence, since the F^i_β annihilate the "transformed" vacuum $V^{-1} |0\rangle$, the state $|H; \text{currents}\rangle = V^{-1} a_{H; \text{strong}}^+ V V^{-1} |0\rangle = V^{-1} |H; \text{strong}\rangle$ has the same transformation properties under the F^i_β as $|H; \text{strong}\rangle$ under the W^i_β . For example if $|H; \text{strong}\rangle$ is a member of a $\underline{35}$ $L_Z(W) = 0$ of $SU(6)_W \otimes O(2)_{\text{strong}}$, $V^{-1} |H; \text{strong}\rangle$ will be the corresponding member of a $\underline{35}$ $L_Z(F) = 0$ of $SU(6)_W \otimes O(2)_{\text{currents}}$.

One can thus write the matrix element (a) as

$$\langle H'; \text{strong} | \mathcal{F}_A^i | H; \text{strong} \rangle = \langle H'; \text{currents} | V^{-1} \mathcal{F}_A^i V | H; \text{currents} \rangle$$

and what remains to be done is to find the $SU(6)_W \otimes O(2)$ currents structure of $V^{-1} \mathcal{F}_A^i V$. This is easily done in the free quark model where $V_{\text{free}}^{-1} \mathcal{F}_{A;\text{free}}^i V_{\text{free}}$ can be explicitly evaluated. One then abstracts the algebraic structure from the free quark model by assigning to $V^{-1} \mathcal{F}_A^i V$ the same algebraic properties as those established for $V_{\text{free}}^{-1} \mathcal{F}_{A;\text{free}}^i V_{\text{free}}$. For non-forward matrix elements a similar procedure applies provided some care is taken in defining transversely boosted states. These formulae will be used systematically in the next section which is devoted to phenomenological applications of the Melosh transformation.

Concerning further theoretical developments, on the transformation between constituent and current quarks, it is clear that a lot of work still needs to be done. To mention but a few of the problems worth investigating, a better understanding of the rather puzzling difference between V_{free} and $\tilde{V}_{\text{free}}$ would obviously be very helpful; if some spin criterion determines the structure of $\tilde{V}_{\text{free}}$, can one use a similar criterion to define V more precisely in some interacting theories? Using the bilocal light-cone algebra of Fritzsche and Gell-Mann⁵¹⁾ can one actually test the presence of the operators which formally appear in V ? Since V_{free} explicitly introduces spin-parity exotics in the theory, can its structure be used to saturate local current algebra⁵²⁾?

4. APPLICATIONS

In this last section I will briefly discuss some of the predictions which follow from the algebraic structure of the Melosh transformation. This model will be referred to as the "constituent-current quark model" and some of the results will be presented at the chiral $SU(3) \otimes SU(3)$ level as well as at the $SU(6)_W$ level. Further details on the derivations of these results can be found in Refs. 45), 53) and 54). The motivation for restricting oneself to the chiral subgroup of $SU(6)_W$ is twofold: first of all tensor currents may not exist in nature and the correspondence between current and constituent quarks would then be meaningful only at the $SU(3) \otimes SU(3)$ level; secondly, it is quite possible that $SU(3) \otimes SU(3)$ may turn out to be a significantly "better" approximation than full $SU(6)_W$.

Some support for this point of view will emerge mainly when I will discuss the charge radii of the baryons.

Let me start with some additional remarks on the classification of particles in $SU(3) \otimes SU(3) \otimes O(2)$ multiplets and, for simplicity, let me use the infinite momentum language. As discussed in Section 2, the $\underline{56}$ $L_z = 0$ baryons are easily classified in $SU(3) \otimes SU(3) \otimes O(2)_{\text{strong}}$ multiplets. For higher "L's" one may use the standard S-L coupling for states at rest and then boost to infinite momentum. At the $L=2$ level, for example, this leads to the following assignments for positive helicity states in the $\underline{56}$ $L=2$ multiplet

$${}^2_8 \frac{5}{2} \left\{ \begin{array}{l} J_z = \frac{5}{2} : [(\underline{6}, \underline{3})_{\frac{1}{2}}, 2] \\ J_z = \frac{3}{2} : \frac{1}{\sqrt{5}} [(\underline{3}, \underline{6})_{-\frac{1}{2}}, 2] + \frac{2}{\sqrt{5}} [(\underline{6}, \underline{3})_{\frac{1}{2}}, 1] \\ J_z = \frac{1}{2} : \sqrt{\frac{2}{5}} [(\underline{3}, \underline{6})_{-\frac{1}{2}}, 1] + \sqrt{\frac{3}{5}} [(\underline{6}, \underline{3})_{\frac{1}{2}}, 0] \end{array} \right.$$

$${}^2_8 \frac{3}{2} \left\{ \begin{array}{l} J_z = \frac{3}{2} : -\frac{2}{\sqrt{5}} [(\underline{3}, \underline{6})_{-\frac{1}{2}}, 2] + \frac{1}{\sqrt{5}} [(\underline{6}, \underline{3})_{\frac{1}{2}}, 1] \\ J_z = \frac{1}{2} : -\sqrt{\frac{3}{5}} [(\underline{3}, \underline{6})_{-\frac{1}{2}}, 1] + \sqrt{\frac{2}{5}} [(\underline{6}, \underline{3})_{\frac{1}{2}}, 0] \end{array} \right.$$

$${}^4_{10} \frac{7}{2} \left\{ \begin{array}{l} J_z = \frac{7}{2} : [(\underline{10}, \underline{1})_{\frac{3}{2}}, 2] \\ J_z = \frac{5}{2} : \frac{2}{\sqrt{7}} [(\underline{10}, \underline{1})_{\frac{3}{2}}, 1] + \sqrt{\frac{3}{7}} [(\underline{6}, \underline{3})_{\frac{1}{2}}, 2] \\ J_z = \frac{3}{2} : \sqrt{\frac{2}{7}} [(\underline{10}, \underline{1})_{\frac{3}{2}}, 0] + \frac{2}{\sqrt{7}} [(\underline{6}, \underline{3})_{\frac{1}{2}}, 1] + \frac{1}{\sqrt{7}} [(\underline{3}, \underline{6})_{-\frac{1}{2}}, 2] \\ J_z = \frac{1}{2} : \frac{2}{\sqrt{35}} [(\underline{10}, \underline{1})_{\frac{3}{2}}, -1] + \sqrt{\frac{18}{35}} [(\underline{6}, \underline{3})_{\frac{1}{2}}, 0] + \\ + \sqrt{\frac{12}{35}} [(\underline{3}, \underline{6})_{-\frac{1}{2}}, 1] + \frac{1}{\sqrt{35}} [(\underline{1}, \underline{10})_{-\frac{3}{2}}, 2] \end{array} \right.$$

$${}^4_{\underline{10}} \begin{cases} J_z = \frac{5}{2} : \sqrt{\frac{3}{7}} [(\underline{10}, \underline{1})_{3/2}, 1] - \frac{2}{\sqrt{7}} [(\underline{6}, \underline{3})_{1/2}, 2] \\ J_z = \frac{3}{2} : \sqrt{\frac{18}{35}} [(\underline{10}, \underline{1})_{3/2}, 0] - \frac{1}{\sqrt{35}} [(\underline{6}, \underline{3})_{1/2}, 1] - \frac{4}{\sqrt{35}} [(\underline{3}, \underline{6})_{-1/2}, 2] \\ J_z = \frac{1}{2} : \frac{3\sqrt{3}}{\sqrt{70}} [(\underline{10}, \underline{1})_{3/2}, -1] + \sqrt{\frac{3}{35}} [(\underline{6}, \underline{3})_{1/2}, 0] \\ \quad - \sqrt{\frac{5}{14}} [(\underline{3}, \underline{6})_{-1/2}, 1] - \sqrt{\frac{6}{35}} [(\underline{1}, \underline{10})_{-3/2}, 2] \end{cases}$$

$${}^4_{\underline{10}} \begin{cases} J_z = \frac{3}{2} : \frac{1}{\sqrt{5}} [(\underline{10}, \underline{1})_{3/2}, 0] - \sqrt{\frac{2}{5}} [(\underline{6}, \underline{3})_{1/2}, 1] + \sqrt{\frac{2}{5}} [(\underline{3}, \underline{6})_{-1/2}, 2] \\ J_z = \frac{1}{2} : \sqrt{\frac{2}{5}} [(\underline{10}, \underline{1})_{3/2}, -1] + \frac{1}{\sqrt{5}} [(\underline{6}, \underline{3})_{1/2}, 0] + \sqrt{\frac{2}{5}} [(\underline{1}, \underline{10})_{-3/2}, 2] \end{cases}$$

$${}^4_{\underline{10}} J_z = \frac{1}{2} : \frac{1}{\sqrt{10}} [(\underline{10}, \underline{1})_{3/2}, -1] - \frac{1}{\sqrt{5}} [(\underline{6}, \underline{3})_{1/2}, 0] + \sqrt{\frac{3}{10}} [(\underline{3}, \underline{6})_{-1/2}, 1] \\ - \sqrt{\frac{2}{5}} [(\underline{1}, \underline{10})_{-3/2}, 2]$$

The negative helicity states can be constructed similarly.

For a ${}^7_0 L=0$ multiplet the situation is slightly more complicated: clearly singlets and decuplets with helicity $\frac{1}{2}$ have to be assigned to $[(\underline{\bar{3}}, \underline{3})_{\frac{1}{2}}, 0]$ and $[(\underline{6}, \underline{\bar{3}})_{\frac{1}{2}}, 0]$ representations of $SU(3) \otimes SU(3)$ but both of these representations contain octets. One expects thus, in general

$$\begin{array}{lll} {}^4_{\underline{8}} \frac{3}{2} & J_z = \frac{3}{2} & : (\underline{8}, \underline{1})_{3/2} \\ & J_z = \frac{1}{2} & : \alpha (\underline{6}, \underline{3})_{1/2} + \beta (\underline{\bar{3}}, \underline{\bar{3}})_{1/2} \\ {}^2_{\underline{1}} \frac{1}{2} & J_z = \frac{1}{2} & : (\underline{\bar{3}}, \underline{3})_{1/2} \\ {}^2_{\underline{10}} \frac{1}{2} & J_z = \frac{1}{2} & : (\underline{6}, \underline{\bar{3}})_{1/2} \\ {}^2_{\underline{8}} \frac{1}{2} & J_z = \frac{1}{2} & : \gamma (\underline{6}, \underline{3})_{1/2} + \delta (\underline{\bar{3}}, \underline{\bar{3}})_{1/2} \end{array}$$

To determine $\alpha, \beta, \gamma, \delta$ one has to use the explicit "70" like wave functions. One obtains from them $\alpha = \beta = \gamma = -\delta = 1/\sqrt{2}$. For the L excited levels one then proceeds as for the ⁵⁶.

For mesonic states, on the other hand, some care with phases is needed. One has indeed to take into account charge conjugation invariance as well as natural parity constraints ³⁷⁾. Under these operations, one has ⁴⁰⁾

$$\begin{aligned} [(A, B)_{S_2}, L_2] &\xrightarrow{\mathcal{N}} \eta_{AB}^N [(\bar{B}, \bar{A})_{-S_2}, -L_2] \\ [(A, B)_{S_2}, L_2] &\xrightarrow{\mathcal{C}} \eta_{AB}^C [(\bar{B}, \bar{A})_{S_2}, L_2] \end{aligned}$$

with phase factors which do depend on the orbital as well as the W spin structure of the state. For example, at the "L=0" level one may choose $\mathcal{C} |[(8, 1)_0, 0] \rangle = - |[(1, 8)_0, 0] \rangle$ while at the "L=1" level $\mathcal{C} |[(8, 1)_0, 0] \rangle = + |[(1, 8)_0, 0] \rangle$ ⁵⁵⁾ (the tilde indicates that although the labels are identical, the representations occurring at the L=0 and L=1 levels are distinct). With this caveat in mind the L=0 and L=1 meson octets, for example, are classified as follows in $SU(3) \otimes SU(3) \otimes O(2)$ strong in the infinite momentum frame

L=0 mesonic octets

$$|8, J^{PC} = 0^{-+}; J_2 = 0 \rangle = \frac{1}{\sqrt{2}} |[(8, 1)_0, 0] \rangle - \frac{1}{\sqrt{2}} |[(1, 8)_0, 0] \rangle$$

$$|8, J^{PC} = 1^{--}; J_2 = +1 \rangle = |[(3, \bar{3})_1, 0] \rangle$$

$$|8, J^{PC} = 1^{--}; J_2 = 0 \rangle = \frac{1}{\sqrt{2}} |[(8, 1)_0, 0] \rangle + \frac{1}{\sqrt{2}} |[(1, 8)_0, 0] \rangle$$

$$|8, J^{PC} = 1^{--}; J_2 = -1 \rangle = |[(\bar{3}, 3)_{-1}, 0] \rangle$$

L=1 mesonic octets

$$|8, J^{PC} = 2^{++}; J_2 = 2 \rangle = |[(3, \bar{3})_1, 1] \rangle$$

$$|8, J^{PC} = 2^{++}; J_2 = 1 \rangle = \frac{1}{\sqrt{2}} |[(3, \bar{3})_1, 0] \rangle + \frac{1}{2} |[(8, 1)_0 + (1, 8)_0, 1] \rangle$$

$$\begin{aligned} |8, J^{PC} = 2^{++}; J_2 = 0 \rangle &= \frac{1}{\sqrt{6}} |[(3, \bar{3})_1, -1] + [(\bar{3}, 3)_1, 1] \rangle \\ &+ \frac{1}{\sqrt{3}} |[(8, 1)_0 + (1, 8)_0, 0] \rangle \end{aligned}$$

$$|8, J^{PC} = 2^{++}; J_2 = -1\rangle = \frac{1}{\sqrt{2}} |[(\bar{3}, 3)_{-1}, 0]\rangle + \frac{1}{2} |[(8, 1)_0 + (1, 8)_0, -1]\rangle$$

$$|8, J^{PC} = 2^{++}; J_2 = -2\rangle = |[(\bar{3}, 3)_{-1}, -1]\rangle$$

$$|8, J^{PC} = 1^{++}; J_2 = +1\rangle = \frac{1}{\sqrt{2}} |[(\bar{3}, \bar{3})_1, 0]\rangle - \frac{1}{2} |[(8, 1)_0 + (1, 8)_0, 1]\rangle$$

$$|8, J^{PC} = 1^{++}; J_2 = 0\rangle = \frac{1}{\sqrt{2}} |[(\bar{3}, \bar{3})_1, -1] - [(\bar{3}, 3)_{-1}, 1]\rangle$$

$$|8, J^{PC} = 1^{++}; J_2 = -1\rangle = -\frac{1}{\sqrt{2}} |[(\bar{3}, 3)_{-1}, 0]\rangle + \frac{1}{2} |[(8, 1)_0, -1 + (1, 8)_0, -1]\rangle$$

$$|8, J^{PC} = 0^{++}; J_2 = 0\rangle = \frac{1}{\sqrt{3}} |[(\bar{3}, \bar{3})_1, -1] + [(\bar{3}, 3)_{-1}, 1]\rangle - \frac{1}{\sqrt{6}} |[(8, 1)_0 + (1, 8)_0, 0]\rangle$$

$$|8, J^{PC} = 1^{+-}; J_2 = 1\rangle = \frac{1}{\sqrt{2}} |[(8, 1)_0 - (1, 8)_0, 1]\rangle$$

$$|8, J^{PC} = 1^{+-}; J_2 = 0\rangle = \frac{1}{\sqrt{2}} |[(8, 1)_0 - (1, 8)_0, 0]\rangle$$

$$|8, J^{PC} = 1^{+-}; J_2 = -1\rangle = \frac{1}{\sqrt{2}} |[(8, 1)_0 - (1, 8)_0, -1]\rangle$$

while in $SU(6)_{W; \text{strong}}$ one has, for example, at the $L=0$ level,

$$|8, J^{PC} = 0^{-+}; J_2 = 0\rangle \rightarrow |35; (8, W=1) W_2=0; J_2=0\rangle$$

$$|8, J^{PC} = 1^{--}; J_2 = \pm 1\rangle \rightarrow |35; (8, W=1) W_2=\pm 1; J_2=\pm 1\rangle$$

$$|8, J^{PC} = 1^{--}; J_2 = 0\rangle \rightarrow |35; (8, W=0) W_2=0; J_2=0\rangle$$

$$|1, J^{PC} = 0^{-+}; J_2 = 0\rangle \rightarrow |35; (1, W=1) W_2=0; J_2=0\rangle$$

$$|1, J^{PC} = 1^{--}; J_2 = \pm 1\rangle \rightarrow |35; (1, W=1) W_2=\pm 1; J_2=\pm 1\rangle$$

$$|1, J^{PC} = 1^{--}; J_2 = 0\rangle \rightarrow |1; (1, W=0) W_2=0; J_2=0\rangle$$

Having classified hadronic states, one can now proceed to compute current like matrix elements.

A. The axial charge

Matrix elements of the axial charge, F_Z^i (at $P_Z = \infty$), are related to those of the pion field by the PCAC assumption ²⁶). In the narrow resonance approximation, this leads to the following formula for the decay width of the process $A \rightarrow B + \pi$

$$\Gamma^{(0)} = \frac{c}{2J_A+1} p_\pi^{(0)} \frac{(M_A^2 - M_B^2)^2}{M_A^2} \sum_\lambda |\langle A, \lambda | F_Z^i | B, \lambda \rangle|^2$$

where c is a constant (depending on isospin), $p_\pi^{(0)}$ the pion (three) momentum in the CM frame and the sum extends over all helicities. This formula is exact in the limit of PCAC i.e., essentially when $m_\pi = 0$. To correct for $m_\pi \neq 0$ one has no "universal" prescription but the following is perhaps not unreasonable ⁵⁶): one writes

$$\Gamma^{(0)} = \sum_L (2L+1) \Gamma_L^{(0)}$$

where $\Gamma_L^{(0)}$ is the width (still for $m_\pi = 0$) in the L 'th final partial wave. Simple kinematics then suggests

$$\Gamma(m_\pi \neq 0) = \sum_L (2L+1) \Gamma_L \quad \text{and} \quad \Gamma_L = \Gamma_L^{(0)} \left(\frac{p_\pi}{p_\pi^{(0)}} \right)^{2L+1}$$

where $p_\pi^{(0)}$ and p_π are the "pions" three momenta for $m_\pi = 0$ and $m_\pi = \text{physical pion mass}$. Although p_π and $p_\pi^{(0)}$ will typically differ by $\lesssim 10\%$ in the decays considered, this introduces a significant correction factor for higher partial waves.

As discussed earlier, one has

$$\langle A; \text{strong} | F_Z^i | B; \text{strong} \rangle = \langle A; \text{current} | V^{-1} F_Z^i V | B; \text{current} \rangle$$

In the free quark model, one finds easily that

$$V_{\text{free}}^{-1} F_{Z, \text{free}}^i V_{\text{free}} \equiv V_{\text{free}}^{-1} F_{\text{free}}^{i5} V_{\text{free}} = \int d^3x q^\dagger(x) \frac{1}{\gamma_5} \left(1 - i \frac{\vec{\gamma}_1 \cdot \vec{\gamma}_2}{m} \right) \gamma_5 \frac{\lambda_i}{2} q(x)$$

hence the transformed axial charge consists of two terms ⁴⁵): the first one transforms as an $[(8,1)_0 - (1,8)_0; L_Z(F)=0]$ representation of $SU(3) \otimes SU(3) \otimes O(2)$ currents while the second is classified in $[(\bar{3}, \bar{3})_1, -1] - [(\bar{3}, 3)_{-1}, 1]$. The factor γ_5 implies that the transformed axial charge contains all angular momenta. In $SU(6)_W$, the two terms correspond to $\underline{35}$'s with $W=1$, $W_Z=0$ $L_Z=0$ for the first one and $W=1$ $W_Z=\pm 1$, $L_Z=\mp 1$ for the second.

In general, for interacting models one should expect that the operator U will introduce additional terms. The arguments presented in Section 3 suggest however that these extra terms may be small compared to the free quark terms. Throughout this section, I will assume that these extra terms can be neglected. Let me now discuss briefly the implications of these algebraic properties of the axial charge for various pion couplings.

a) $\underline{56} \ L=0 \text{ baryons} \rightarrow \underline{56} \ L=0 \text{ baryons} + \pi$

For these transitions, which are relevant for the "ground state" $\underline{56}$ as well as for the $\underline{56} \ L=0$ which presumably contains the Roper resonance $N(1470)$, only the first term in the transformed axial charge can contribute since ΔL_z must be zero. Since this term has precisely the same algebraic structure as F_z^1 itself, one gets back the old $SU(6)_W$ results with, however, one important correction: the first term is not a generator anymore, hence its reduced matrix element is not determined by the symmetry. One finds, in the $SU(6)_W$ limit ⁴⁵⁾

$$\frac{G_A}{G_V} = -\eta \frac{5}{3} \quad G^* = -\eta \frac{4}{3} \quad \alpha = \left(\frac{F}{D} \right)_{\text{axial}} = \frac{2}{3}$$

the F/D ratio is unchanged since the η factor (which is the unknown reduced matrix element) cancels out. It is interesting to calculate the predictions of the less restrictive symmetry $SU(3) \otimes SU(3)$ ⁵⁷⁾. One finds for any $\underline{56} \ L=0 \rightarrow \underline{56} \ L=0$ transitions

$$\begin{aligned} \frac{3}{4} g(\Delta' \rightarrow N\pi) &= \frac{1}{1+\alpha} g(N' \rightarrow N\pi) \\ g(N' \rightarrow \Delta\pi) &= -\sqrt{2} g(\Delta' \rightarrow N\pi) \\ \sqrt{2} g(N' \rightarrow \Delta\pi) + 2 g(N' \rightarrow N\pi) &= \frac{2}{\sqrt{5}} g_{\frac{1}{2}}(\Delta' \rightarrow \Delta\pi) \end{aligned}$$

where $g_{\frac{1}{2}}(\Delta' \rightarrow \Delta\pi)$ is the helicity $\frac{1}{2}$ coupling of a Δ' resonance to $\Delta\pi$. Within the fairly large uncertainties ⁴⁰⁾ on G^* and α these predictions are in agreement with experiment and there is no significant improvement when restricting oneself to $SU(3) \otimes SU(3)$.

b) $35 L=1$ mesons $\rightarrow 35 L=0$ mesons + π

In this case, the results are somewhat more spectacular. Both terms in the transformed axial charge can now contribute and one easily verifies that both decays $1^{+-} \rightarrow 1^{--} + 0^{-+}$ (e.g., $B \rightarrow \omega \pi$) and $1^{++} \rightarrow 1^{--} + 0^{-+}$ are allowed to occur in both helicity states ⁵⁷⁾. This is a remarkable improvement over the old $SU(6)_W$ results which, in contradiction with experiment ⁵⁸⁾, forbid the helicity one decay of the B meson (as well as the helicity 0 decay of the A_1).

Experiment suggests that the decay $B \rightarrow \omega \pi$ may be purely in helicity one. Taken literally this implies that the reduced matrix element of the $[(8,1)_0 + (1,8)_0, 0]$ term of the transformed axial charge vanishes, hence in the $SU(6)_W$ limit one is left with one single parameter. The resulting predictions for the various $L=1$ meson decays were computed, in the exact PCAC limit, by Gilman, Kugler and Meshkov ⁵⁹⁾ and are shown in Table III.

Note that in calculating these predictions the η meson has been taken as a pure octet state while ω , f and σ are taken as ideal mixtures of singlets and octets so as to be purely constituted of non-strange quarks. In addition the validity of "Zweig's rule" (i.e., $B \not\rightarrow \rho \pi$) has been assumed.

Restricting oneself to $SU(3) \otimes SU(3)$ instead of full $SU(6)_W$ leads to only the following predictions ⁵⁷⁾

$$\sqrt{2} g_0(\delta \rightarrow \eta \pi) + g_0(A_2 \rightarrow \eta \pi) = \frac{1}{\sqrt{2}} g_0(A_1 \rightarrow \rho \pi) \quad (a)$$

$$\sqrt{2} g_0(A_2 \rightarrow \eta \pi) - g_0(\delta \rightarrow \eta \pi) = g_0(B \rightarrow \omega \pi) \quad (b)$$

$$g_1(A_1 \rightarrow \rho \pi) - g_1(A_2 \rightarrow \rho \pi) = 2g_1(B \rightarrow \omega \pi) \quad (c)$$

where again ω has been assumed to be a pure non-strange quark state, η a pure octet state and Zweig's rule has been used. With $\Gamma(A_2 \rightarrow \eta \pi) \approx 20$ MeV and $\Gamma(\delta \rightarrow \eta \pi) \approx 30$ MeV, relation (b) leads to a helicity zero width of the B meson of ≈ 20 or 100 MeV depending on the relative signs of $g_0(\delta \rightarrow \eta \pi)$ and $g_0(A_2 \rightarrow \eta \pi)$. A 20 MeV helicity zero width would be perfectly compatible with the data ⁵⁸⁾.

To summarize the overall situation for $L=1 \rightarrow L=0 + \pi$ mesonic decays, predictions derived from coplanar $SU(3) \otimes SU(3)$ as well as from the present model in the $SU(3) \otimes SU(3)$ limit and in the $SU(6)_W$ limit [which in this case is algebraically equivalent to a broken $SU(6)_W$ and to the 3P_0 model] are assembled in Table IV⁵³⁾ for decays of the type " $K' \rightarrow K + \pi$ ".

c) Other meson decays

It is straightforward to work out the predictions of the "constituent-current quark model" for other decay processes. In general for an arbitrary decay $A(L) \rightarrow B(L') + \pi$

$$"K'" \rightarrow "K" + \pi$$

the helicity couplings can be parametrized as follows⁵³⁾

$$g_\lambda = \sum_{L_z} \left\{ \begin{matrix} \underline{35} \\ (\underline{8}, 2W_B+1) \end{matrix} \begin{matrix} \underline{35} \\ (\underline{8}, 3) \end{matrix} \middle| \begin{matrix} \underline{35} \\ (\underline{8}, 2W_A+1) \end{matrix} \right\}_{\alpha_1} \left\{ \begin{matrix} \underline{8} & \underline{8} & \underline{8} \\ B & \pi & A \end{matrix} \right\}_{\alpha_2} \times$$

$$\times \left\{ \alpha^{L_z, L_z} (W_B \lambda - L_z; 10 | W_A \lambda - L_z) (S_A \lambda - L_z; L L_z | J_A \lambda) (S_B \lambda - L_z; L' L_z | J_B \lambda) \right.$$

$$+ \alpha^{L_z+1, L_z} (W_B \lambda - L_z - 1; 11 | W_A \lambda - L_z) (S_A \lambda - L_z; L L_z | J_A \lambda) (S_B \lambda - L_z - 1; L' L_z + 1 | J_B \lambda)$$

$$\left. + \alpha^{L_z-1, L_z} (W_B \lambda - L_z + 1; 1-1 | W_A \lambda - L_z) (S_A \lambda - L_z; L L_z | J_A \lambda) (S_B \lambda - L_z + 1; L' L_z - 1 | J_B \lambda) \right\}$$

where the PCAC factor (including $m_A^2 - m_B^2$) has been absorbed in the reduced matrix elements $a_{L_z L_z}^{I_z I_z}$.

It is interesting to compare the structure of this "constituent-current quark model" parametrization of decay amplitudes with other phenomenological models. In particular it is easy to show that for pionic decays, the 3P_0 model^{18), 19)} is algebraically equivalent to the constituent-current quark model⁵³⁾. What is meant by this statement is a one to one correspondence between independent parameters of both models. Since the 3P_0 model is essentially a phenomenological prescription for breaking $SU(6)_W$; strong symmetry for vertices, any function of the external masses can always be included in the definition of its parameters. In the 3P_0 model one would have

$$\tilde{g}_\lambda = \sum_{L_z} \left\{ \begin{matrix} 35 \\ (\underline{2}, 2S_B+1) \end{matrix} \begin{matrix} 35 \\ (\underline{2}, 1) \end{matrix} \middle| \begin{matrix} 35 \\ (\underline{2}, 2S_A+1) \end{matrix} \right\}_{\beta_1} \left\{ \begin{matrix} 8 \\ B \end{matrix} \begin{matrix} 8 \\ \pi \end{matrix} \middle| \begin{matrix} 8 \\ A \end{matrix} \right\}_{\beta_2} \times$$

$$\times \left\{ \begin{aligned} & b^{L_z, L_z} (S_B \lambda - L_z; 10 | S_A \lambda - L_z) (S_A \lambda - L_z; L_z | J_A \lambda) (S_B \lambda - L_z; L' L_z | J_B \lambda) \\ & + b^{L_z+1, L_z} (S_B \lambda - L_z - 1; 11 | S_A \lambda - L_z) (S_A \lambda - L_z; L_z | J_A \lambda) (S_B \lambda - L_z - 1; L' L_z + 1 | J_B \lambda) \\ & + b^{L_z-1, L_z} (S_B \lambda - L_z + 1; 1-1 | S_A \lambda - L_z) (S_A \lambda - L_z; L_z | J_A \lambda) (S_B \lambda - L_z + 1; L' L_z - 1 | J_B \lambda) \end{aligned} \right\}$$

and with the correspondences

$$a^{L_z, L_z} \leftrightarrow b^{L_z, L_z}$$

$$a^{L_z \pm 1, L_z} \leftrightarrow -\frac{4}{3\sqrt{2}} b^{L_z \pm 1, L_z} \quad \text{for } L+L' \text{ even}$$

and

$$a^{L_z, L_z} \leftrightarrow b^{L_z, L_z}$$

$$a^{L_z \pm 1, L_z} \leftrightarrow -\frac{3\sqrt{2}}{4} b^{L_z \pm 1, L_z} \quad \text{for } L+L' \text{ odd}$$

one finds easily that $g_\lambda \equiv \tilde{g}_\lambda$ which proves the equivalence. In a similar way it is easy to verify that, in general, ℓ broken $SU(6)_W$ is a stronger symmetry (i.e., has less independent parameters) than the 3P_0 model except for $L'=L$ and L or $L'=0$ where they are equivalent ⁵³⁾.

d) $L=1$ and $L=2$ baryons $\rightarrow L=0$ baryons + π

Since for baryons $\vec{W}=\vec{S}$ the constituent-current quark model for pionic decays is clearly equivalent, algebraically, to the 3P_0 model.

Although the algebraic structures are identical, the actual predictions of these models for decay widths may differ quite a bit. In the 3P_0 or ℓ broken $SU(6)_W$ models, one usually introduces ⁷⁾ universal barrier factors $B_L(kR)$ (with R the radius of interaction) for each final partial wave and these are not quite equivalent to the $(p/p_0)^L$ factors which

one may assume to be the dominant corrections to the exact PCAC predictions. In a sense, $(p/p_0)^L$ is a barrier factor but with a different radius of interaction ($\approx 1/p_0$) for each resonance.

A "best fit" is clearly called for to test the constituent-current quark model but it does not exist as yet. The exact PCAC predictions have been derived by Gilman, Kugler and Meshkov^{54),59),60)} while the broken $SU(6)_W$ predictions have been fitted to experiment, for the 70 $L=1$ multiplet by Faiman and Plane⁷⁾.

Last but not least the constituent-current quark^{54),59),60)} model, as well as the 3P_0 model⁶¹⁾, leads to extremely interesting phase predictions for $N^* \rightarrow N \pi$ and $N^* \rightarrow \Delta \pi$ amplitudes.

Table V lists these predictions for $\pi N \rightarrow N^* \rightarrow \pi \Delta$ with N^* 's in the 70 $L=1$ and 56 $L=2$ multiplets. For simplicity, $S_{11}(1710)$ and $D_{13}(1700)$ are taken as pure quadruplet states and $S_{11}(1550)$ and $D_{13}(1520)$ as pure doublet states. The amplitudes in $\pi N \rightarrow \pi \Delta$ are denoted by $L L'_{2I2J}(M)$ where L refers to the πN partial wave and L' to the $\pi \Delta$ partial wave while I, J and M denote, as usual, the isospin, spin and mass of the resonance. I have also chosen my phase conventions to agree with those of Gilman, Kugler and Meshkov⁵⁴⁾.

Present data⁶²⁾ analysis of $\pi N \rightarrow \pi \Delta$ disagrees with these predictions for any combination of $\Delta L_z = 0$ and $\Delta L_z = \pm 1$ terms. If no other solution to the phase shifts can be found this would definitely kill the present model [in its $SU(6)_W$ form] as well as the 3P_0 model. One should check, but it has not been done yet, whether the less strong $SU(3) \otimes SU(3)$ phase predictions can be made to agree with experiment.

B. The electromagnetic current

In this subsection the light-cone language will be used systematically. Since the electromagnetic charge operator $\tilde{F}^{e.m.}$ commutes with $\tilde{V}_{free}$ its matrix elements are of course trivial. Matrix elements of various moments of the electromagnetic current, on the other hand, are much more interesting. Hence one wants to compute matrix elements of the form

$$\langle H', p' | \int d^4x \delta(x^+) \tilde{F}_{e.m.}^+(x) \exp\{i(\vec{p}'_1 - \vec{p}_1) \cdot \vec{x}_\perp\} | H, p \rangle$$

where H and H' are arbitrary hadronic states. Note that these matrix elements can depend only on $\vec{q}_\perp = \vec{p}'_\perp - \vec{p}_\perp$. If one generates transverse momentum with the $\vec{E}_\perp$ boosts, which commute with the light-like charges, one may rewrite these matrix elements as follows ⁴⁵⁾

$$\begin{aligned} & \langle H', \vec{p}'_\perp = 0 | \int d^4x \delta(x^+) \tilde{F}_{e.m.}^+(x) \exp\{i(\vec{p}'_\perp - \vec{p}_\perp) \cdot (\vec{x}_\perp + \vec{E}_\perp)\} | H, \vec{p}_\perp = 0 \rangle \\ & = \langle H', \vec{p}'_\perp = 0 | \tilde{F}_{e.m.}(\vec{q}_\perp) | H, \vec{p}_\perp = 0 \rangle \end{aligned}$$

It is then straightforward to compute the matrix elements of various moments. For spin $\frac{1}{2}$ particles, for example, the anomalous magnetic moment μ_A is defined by

$$\frac{\mu_A}{2M} = \frac{\partial}{\partial k_x} \langle H; \text{strong}; \text{rest}; \lambda = -\frac{1}{2} | \tilde{F}_{e.m.}(k_x) | H; \text{strong}; \text{rest}; \lambda = +\frac{1}{2} \rangle \Big|_{k_x=0}$$

which by direct differentiation leads to

$$\begin{aligned} \frac{\mu_A}{2M} &= i \langle H; \text{strong}; \text{rest}; \lambda = -\frac{1}{2} | \int d^4x \delta(x^+) (x + E^1) \tilde{F}_{e.m.}^+(x) | H; \text{strong}; \text{rest}; \lambda = +\frac{1}{2} \rangle \\ &= i \langle H; \text{strong}; \text{rest}; \lambda = -\frac{1}{2} | \int d^4x \delta(x^+) x \tilde{F}_{e.m.}^+(x) + Q E^1 | H; \text{strong}; \text{rest}; \lambda = +\frac{1}{2} \rangle \end{aligned}$$

Some care is needed in handling this expression since the "forward" matrix elements of both terms on the right-hand side are ill defined (each term individually can change $\vec{p}_\perp$). However, as pointed out originally by Melosh ⁴⁵⁾, one may choose to define these expressions in terms of symmetric wave packets. This removes all ambiguities and one then obtains

$$\langle \text{rest}; \lambda = -\frac{1}{2} | E^1 | \text{rest}; \lambda = +\frac{1}{2} \rangle = \frac{1}{M} \langle \text{rest} | J_y + K_x | \text{rest} \rangle = \frac{i}{2M}$$

since, for symmetric wave packets, the matrix element of K_x vanishes while

$$\langle \text{rest}; \lambda = -\frac{1}{2} | J_y | \text{rest}; \lambda = +\frac{1}{2} \rangle = \frac{i}{2}$$

One arrives thus at the expression

$$\frac{\mu_A}{2M} = -\frac{Q}{2M} + i \langle H; \text{strong}; \text{rest}; \lambda = -\frac{1}{2} | \int d^4x \delta(x^+) x \tilde{F}_{e.m.}^+(x) | H; \text{strong}; \text{rest}; \lambda = +\frac{1}{2} \rangle$$

The first term on the right-hand side is precisely the opposite of the Dirac moment, hence

$$\frac{\mu_T}{2M} = \frac{\mu_A}{2M} + \frac{Q}{2M} = i \langle H; \text{strong}; \text{rest}; \lambda = -\frac{1}{2} | \int d^4x \delta(x^+) x \mathbb{F}_{e.m}^+(x) | H; \text{strong}; \text{rest}; \lambda = +\frac{1}{2} \rangle$$

with $\mu_{T/2M}$ the total magnetic moment of the hadron H .

Using the Melosh transformation, one obtains as before

$$\frac{\mu_T}{2M} = i \langle H; \text{currents}; \text{rest}; \lambda = -\frac{1}{2} | \tilde{V}^{-1} \int d^4x \delta(x^+) x \mathbb{F}_{e.m}^+(x) \tilde{V} | H; \text{currents}; \text{rest}; \lambda = +\frac{1}{2} \rangle$$

and in the free quark model, one finds

$$\begin{aligned} \tilde{V}_{\text{free}}^{-1} \int d^4x \delta(x^+) x \mathbb{F}_{e.m}^+(x) \tilde{V}_{\text{free}} &= \int d^4x \delta(x^+) x \mathbb{F}_{e.m}^+(x) \\ &- i\sqrt{2} \int d^4x \delta(x^+) q_+^+(x) \frac{1}{2x^2} \left\{ \left(1 - \frac{1}{1+x} \frac{\partial_y \partial_y}{m^2}\right) \frac{\delta_x}{m} + \frac{1}{1+x} \frac{\partial_x \partial_y}{m^2} \frac{\delta_y}{m} + \frac{x}{1+x} \frac{\sigma_z \partial_y}{m^2} \right\} \frac{\lambda_{e.m}}{2} q_+(x) \end{aligned}$$

The first term is a $W=0$ $L_z=\pm 1$ member of a $\underline{35}$, and the last term corresponds to a $W=1$ $W_z=0$ $L_z=\pm 1$ member of a $\underline{35}$ while the remaining terms have the algebraic structure of $W=1$ $W_z=\pm 1$ $L_z=0, \mp 2$ (still in $\underline{35}$'s). Abstracting from the free quark model implies that for nucleons, only the terms $W=1$ $W_z=\pm 1$ $L_z=0$ can contribute and one finds easily

$$\frac{\mu_T(\text{proton})}{\mu_T(\text{neutron})} = -\frac{3}{2}$$

One thus recovers this well-known ratio. The new element in the derivation given here and which is due to Melosh⁴⁵⁾ is that the result is obtained from a good operator, provided one uses symmetric wave packets, as opposed to the standard derivations which use bad operators (like the transverse components of the electromagnetic current).

Similar calculations lead to the following predictions for the $M1(\mu^*)$ and $E2$ moments of the $p \rightarrow \Delta^+$ transition⁴⁵⁾:

$$\mu^* = \frac{2\sqrt{2}}{3} \mu_T(\text{proton}) \quad E2 = 0$$

which also agree fairly well with experiment.

One can clearly go on to higher moments, such as, for example, the charge radii for spin $\frac{1}{2}$ particles. The calculation is straightforward but lengthy. One finds in the $SU(6)_W$ limit that for the $\underline{56}$ $L=0$ baryons the charge radii are proportional to the charges. This implies, in particular $\langle r^2 \rangle_{\text{neutron}} = 0$ which is a surprisingly bad prediction when compared with the accuracy of the previous results. This may be the first indication of effects due to the operator U . It should be pointed out, however, that $\langle r^2 \rangle_{\text{neutron}} = 0$ is not predicted at the $SU(3) \otimes SU(3)$ level ⁶³⁾.

To calculate still higher moments, it should be possible to use systematically the "angular condition" ³⁵⁾ which correlates higher and lower moments. It is conceivable that, in this way, one may be able to derive results for the whole series such as, e.g., scaling of $G_E(q^2)$ and $G_M(q^2)$.

As a last application of the algebraic properties of the Melosh transformation, let me briefly discuss the helicity structure of photoproduction amplitudes at resonances ^{64), 65)}.

Resonance photoproduction off nucleons is in general described by two independent helicity couplings. The matrix elements one is concerned with correspond to

$$F_{\perp} \equiv \langle N^*; \text{strong}; p'; \lambda' | \int d^4x \delta(x^+) \vec{F}_{e.m.}^{\perp} \exp i k \cdot x | N; \text{strong}; p; \lambda \rangle$$

where $\vec{F}_{e.m.}^{\perp}(x)$ refers to the transverse components of the electromagnetic current i.e., to bad operators. One can, however, use current conservation to relate matrix elements of the transverse components to those of the dipole operator $\tilde{D}^{\perp}$

$$F_{\perp} = q^- \langle N^*; \text{strong}; p'; \lambda' | \tilde{D}^{\perp} | N; \text{strong}; p; \lambda \rangle \Big|_{q^+ = \vec{q}_{\perp} = 0}$$

where $q = p' - p$. In the nucleon rest frame, for example

$$q^- = \frac{M_{N^*}^2 - M_N^2}{\sqrt{2} M_N}$$

Let me recall that the dipole operator $\tilde{D}^{\perp}$ is a good operator defined by

$$\tilde{D}^{\perp} = \int d^4x \delta(x^+) \vec{x}_{\perp} \cdot \vec{F}_{e.m.}^{\perp}(x)$$

Absorbing mass factors in the definitions of amplitudes, one obtains finally

$$F_{\perp} = \langle N^*; \text{currents}; p+q; \lambda' | \tilde{V}^{-1} \tilde{D}^{\perp} \tilde{V} | N; \text{currents}; p; \lambda \rangle$$

with $q^+ = \vec{q}_{\perp} = 0$

As discussed earlier, one has, in the free quark model, the following algebraic structure

$$\tilde{V}_{\text{free}}^{-1} \tilde{D}_{\text{free}}^{\perp} \tilde{V}_{\text{free}} = A \{ \underline{35}; W=0, W_2=0; \Delta L_2 = \pm 1 \} + B \{ \underline{35}; W=1, W_2 = \pm 1, \Delta L_2 = 0 \} \\ + C \{ \underline{35}; W=1, W_2=0; \Delta L_2 = \pm 1 \} + D \{ \underline{35}; W=1, W_2 = \mp 1; \Delta L_2 = \pm 2 \}$$

Abstracting this structure from the free quark model one then derives easily the helicity structure of photoproduction amplitudes. The results are tabulated for resonances in the $\underline{70}$ $L=1$ and $\underline{56}$ $L=2$ multiplets ⁶⁵⁾, (Tables VI and VII).

As pointed out originally by Love and Manopoulos ⁶⁴⁾ the assumed algebraic structure for $V^{-1} \tilde{D} V$ preserves the well-known Moorhouse selection rule ⁶⁶⁾ which forbids photoexcitation, off protons, of quadruplet states in the $\underline{70}$ $L=1$ multiplet.

From Tables VI and VII one also deduces the following relations among helicity amplitudes at a given resonance:

$$\begin{aligned} \underline{70} \quad L=1 \quad & \left\{ \begin{array}{l} \underline{2} \underline{8} \underline{3/2} \\ \underline{4} \underline{8} \underline{5/2} \end{array} \right. \quad : \quad \begin{array}{l} F_+^P + 3 F_+^{\eta} = \sqrt{3} (F_-^P + 3 F_-^{\eta}) \\ F_+^{\eta} = -\sqrt{2} F_-^{\eta} \end{array} \\ \\ \underline{56} \quad L=2 \quad & \left\{ \begin{array}{l} \underline{2} \underline{8} \underline{5/2} \\ \underline{2} \underline{8} \underline{3/2} \\ \underline{4} \underline{10} \underline{7/2} \end{array} \right. \quad : \quad \begin{array}{l} F_+^P + \frac{3}{2} F_+^{\eta} = \sqrt{2} (F_-^P + \frac{3}{2} F_-^{\eta}) \\ \sqrt{3} (F_+^P + \frac{3}{2} F_+^{\eta}) = F_-^P + \frac{3}{2} F_-^{\eta} \\ F_+^{P,\eta} = -\sqrt{\frac{5}{3}} F_-^{P,\eta} \end{array} \end{aligned}$$

In addition one has, of course, many predictions relating amplitudes at different resonances ⁶⁴⁾ including phase relations ⁶⁷⁾, but I will only concern myself, here, with the helicity structure predictions. It is interesting to point out that in naïve quark models ^{68),69)} one usually parametrizes phototransitions by a quark spin flip and an orbital flip term only. Algebraically this is equivalent to $C=D=0$. It turns out that the 3P_0 model with vector dominance ⁷⁰⁾ is equivalent to $A=C$, $D=0$. Unfortunately the available data ⁷¹⁾ do not allow as yet to definitely exclude any model. It is interesting, however, that the equivalence between the 3P_0 model and the constituent-current quark model which was shown to hold for pionic transitions ⁵³⁾ is lost for vector transitions.

This makes it worthwhile to compute $N \rho$ and $\Delta \rho$ decay amplitudes of nucleon resonances (assuming some kind of vector dominance) but this calculation has yet to be done.

CONCLUSION

These lectures should have made clear that the constituent and current quark points of view are widely different. Obviously a lot more work needs to be done to fully understand the nature as well as the structure of the transformation relating constituent to current quarks. Hopefully this review may incite some of its readers to participate in the exciting theoretical and phenomenological developments which lie ahead.

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TABLE I : L = 1 MESONS

J^{PC}	$I = 1$	$I = \frac{1}{2}$	$I = 0$	$I = 0$
2^{++}	$A_2(1310)$	$K^{**}(1420)$	$f^0(1270)$	$f'(1514)$
1^{++}	$A_1(1100?)$	$K_A(1240)?$	$D(1285)$	?
0^{++}	$\delta(970)$	$K(1200-1400?)$	$\sigma(700?)$	$S(997)$
1^{++}	$B(1235)$	$K_C(1280)?$	??	??

TABLE II $\underline{\text{SU}(6)}_S$ and $\underline{\text{SU}(6)}_W$ FOR $L=0$ MESONS

$\underline{\text{SU}(6)}_S$	$\underline{\text{SU}(3)} \otimes \underline{\text{SU}(2)}_S$	S_Z		$\underline{\text{SU}(6)}_W$	$\underline{\text{SU}(3)} \otimes \underline{\text{SU}(2)}_W$	W_Z	
$35 \sim$	$\left. \begin{array}{l} 1 \oplus 8 : S=1 \\ 8 \\ S=0 \end{array} \right\}$	$S_Z = 1$	$8 \sim [\underline{8}] \oplus 1 \sim [\underline{\emptyset}]$	$35 \sim$	$1 \oplus 8 \quad W=1$ $8 \quad W=0$	$W_Z = 1$	$8 \sim [\underline{8}] + 1 \sim [\underline{\emptyset}]$
		$S_Z = 0$	$8 \sim [\underline{8}] \oplus 1 \sim [\underline{\emptyset}]$			$W_Z = 0$	$8 \sim [\underline{\pi}] + 1 \sim [\underline{\eta}']$
		$S_Z = -1$	$8 \sim [\underline{8}] + 1 \sim [\underline{\emptyset}]$			$W_Z = -1$	$8 \sim [\underline{8}] + 1 \sim [\underline{\emptyset}]$
		$S_Z = 0$	$8 \sim [\underline{\pi}]$				$8 \sim [\underline{8}]$
$1 \sim$	$1 \quad S=0$	$S_Z = 0$	$1 \sim [\underline{\eta}']$	$1 \sim$	$1 \quad W=0$		$1 \sim [\underline{\emptyset}]$

TABLE III Decay rates of $L=1$ mesons (taken from Gilman, Kugler, Meshkov) ⁵⁹⁾

Decay		Γ (predicted)(MeV)	Γ (experimental)(MeV) ⁵⁸⁾
$A_2(1310) \rightarrow \rho \pi$		77 (input)	77 ± 20
$B(1235) \rightarrow \omega \pi$	$\lambda = 0$	0 (input))	} dominantly $\lambda = 1$
	$\lambda = 1$	76	
$A_1(1070) \rightarrow \rho \pi$	$\lambda = 0$	52	100 ± 20
	$\lambda = 1$	26	?
$A_2(1310) \rightarrow \eta \pi$		17	16 ± 4
$\delta(975) \rightarrow \eta \pi$		37	~ 60 total width
$f(1260) \rightarrow \pi \pi$		118	125 ± 25
$\sigma(760?) \rightarrow \pi \pi$		234	Broad ?

TABLE IV Predictions for $L=1$ meson decays ($K' \rightarrow K + \pi$)

Coplanar $SU(3) \otimes SU(3)$

$$\begin{aligned} g_1(2^{++} \rightarrow 1^{--}) + g_1(1^{++} \rightarrow 1^{--}) &= 2 g_0(1^{+-} \rightarrow 1^{--}) \\ g_1(2^{++} \rightarrow 1^{--}) &= (\sqrt{3}/2) g_0(2^{++} \rightarrow 0^{-+}) \\ 2 g_1(1^{++} + 1^{--}) &= g_0(1^{++} \rightarrow 1^{--}) \\ 2 g_0(2^{++} \rightarrow 0^{-+}) - g_0(0^{++} \rightarrow 0^{-+}) &= \sqrt{3} g_0(1^{+-} \rightarrow 1^{--}) \end{aligned}$$

Chiral $SU(3) \otimes SU(3)$ $[\bar{V}_{\text{free}} F_{\text{free}}^{i5} V_{\text{free}}^{-1}]$ in $SU(3) \otimes SU(3)$

$$\begin{aligned} g_1(2^{++} \rightarrow 1^{--}) + \sqrt{2} g_1(1^{+-} \rightarrow 1^{--}) &= g_1(1^{++} \rightarrow 1^{--}) \\ g_0(2^{++} \rightarrow 0^{-+}) + \sqrt{2} g_0(0^{++} \rightarrow 0^{-+}) &= -\sqrt{3} g_0(1^{++} \rightarrow 1^{--}) \\ \sqrt{2} g_0(2^{++} \rightarrow 0^{-+}) - g_0(0^{++} \rightarrow 0^{-+}) &= \sqrt{3} g_0(1^{+-} \rightarrow 1^{--}) \end{aligned}$$

$V_{\text{free}} F_{\text{free}}^{i5} V_{\text{free}}^{-1}$ model in $SU(6)_W$ [equivalent to 3P_0 and 1D_2 broken $SU(6)_W$]

$$\begin{aligned} g_1(2^{++} \rightarrow 1^{--}) &= ((3/\sqrt{2})D) & g_0(1^{++} \rightarrow 1^{--}) &= \sqrt{2}(S-D) \\ g_0(2^{++} \rightarrow 0^{-+}) &= \sqrt{6}D & g_1(1^{++} \rightarrow 1^{--}) &= \sqrt{2}(S+D/2) \\ g_0(0^{++} \rightarrow 0^{-+}) &= -\sqrt{3}S & g_0(1^{++} \rightarrow 1^{--}) &= S+2D \\ & & g_0(1^{+-} \rightarrow 1^{--}) &= S-D \end{aligned}$$

S and D are conveniently normalized S and D wave amplitudes.

TABLE V Signs of resonant amplitudes in $\pi N \rightarrow \pi \Delta$

Amplitude	$\Delta L_z = 0$	$\Delta L_z = \pm 1$
$DD_{15}(1670)$	-	- (input)
$DD_{13}(1700)$	-	-
$DS_{13}(1700)$	-	+
$SD_{11}(1710)$	+	-
$70 \text{ } L=1 \text{ } SD_{11}(1550)$	+	-
$DD_{13}(1520)$	+	+
$DS_{13}(1520)$	-	+
$SD_{31}(1650)$	+	-
$DD_{33}(1690)$	+	+
$DS_{33}(1690)$	-	+
$FF_{37}(1950)$	+	+
$FF_{35}(1890)$	+	+
$FP_{35}(1890)$	+	-
$PF_{33}(?)$	-	+
$PP_{33}(?)$	-	-
$56 \text{ } L=2 \text{ } PP_{31}(1860)$	-	-
$FF_{15}(1688)$	-	-
$FP_{15}(1688)$	+	-
$PF_{13}(1850)$	-	+
$PP_{13}(1860)$	+	+

The second column in this Table gives the sign predicted from the $\Delta L_z = 0$ part of the transformed axial charge and the third column from the $\Delta L_z = \pm 1$ part.

TABLE VI Photoproduction helicity structure for $70 \text{ } L=1$ resonances

$2S+1 \text{ } [SU(3)]_J$	F_-^p	F_+^p	F_-^n	F_+^n
$4 \text{ } 8_{\frac{1}{2}}$	0	-	$\frac{1}{18}B - \frac{1}{18}C$	-
$4 \text{ } 8_{\frac{3}{2}}$	0	0	$\frac{1}{18\sqrt{5}}B - \frac{2}{9\sqrt{5}}C$	$\frac{1}{6\sqrt{5}}B - \frac{C}{3\sqrt{15}}$
$4 \text{ } 8_{5/2}$	0	0	$-\frac{1}{6\sqrt{5}}B - \frac{1}{6\sqrt{5}}C$	$\frac{\sqrt{2}}{6\sqrt{5}}B + \frac{\sqrt{2}}{6\sqrt{5}}C$
$2 \text{ } 8_{\frac{1}{2}}$	$\frac{1}{6}A + \frac{1}{6}B - \frac{1}{6}C$	-	$-\frac{1}{6}A - \frac{1}{18}B + \frac{1}{18}C$	
$2 \text{ } 8_{\frac{3}{2}}$	$\frac{1}{6\sqrt{2}}A - \frac{1}{3\sqrt{2}}B - \frac{1}{6\sqrt{2}}C$	$\frac{1}{2\sqrt{6}}A + \frac{1}{2\sqrt{6}}C$	$-\frac{1}{6\sqrt{2}}A + \frac{1}{9\sqrt{2}}B + \frac{1}{18\sqrt{2}}C$	$-\frac{1}{2\sqrt{6}}A - \frac{1}{6\sqrt{6}}C$
$2 \text{ } 10_{\frac{1}{2}}$	$\frac{1}{6}A - \frac{1}{18}B + \frac{1}{18}C$	-	$\equiv F_-^p$	$\equiv F_+^p$
$2 \text{ } 10_{\frac{3}{2}}$	$\frac{1}{6\sqrt{2}}A + \frac{1}{9\sqrt{2}}B + \frac{1}{18\sqrt{2}}C$	$\frac{1}{2\sqrt{6}}A - \frac{1}{6\sqrt{6}}C$	$\equiv F_-^p$	$\equiv F_+^p$

$F_{\pm}^{p,n}$ are the $\lambda = \frac{1}{2}$ to $\lambda = \frac{3}{2}$ transition amplitudes off proton (p) and neutron (n). $F_{\pm}^{p,n}$ are the $\lambda = \frac{1}{2}$ to $\lambda = -\frac{1}{2}$ amplitudes.]

TABLE VII Photoproduction helicity structure for $56 \text{ } L=2^+$ resonances

$2S+1 \text{ } [SU(3)]_J$	F_-^p	F_+^p	F_-^n	F_+^n
$4 \text{ } 10_{\frac{1}{2}}$	$\frac{4}{45\sqrt{2}}B' - \frac{4\sqrt{3}}{45\sqrt{2}}C' - \frac{4}{15\sqrt{35}}D'$		For 10 :	$F_{\pm}^n \equiv F_{\pm}^p$
$4 \text{ } 10_{\frac{3}{2}}$	$\frac{-2\sqrt{2}}{45}B' - \frac{4\sqrt{3}}{45}D'$	$-\frac{2\sqrt{2}}{15\sqrt{3}}B' + \frac{4\sqrt{2}}{45}C' + \frac{4}{45}D'$		
$4 \text{ } 10_{5/2}$	$\frac{-2\sqrt{6}}{45\sqrt{7}}B' + \frac{2\sqrt{2}}{9\sqrt{7}}C' - \frac{4}{15\sqrt{7}}D'$	$-\frac{4\sqrt{3}}{15\sqrt{7}}B' + \frac{4}{45\sqrt{7}}C' - \frac{8\sqrt{2}}{45\sqrt{7}}D'$		
$4 \text{ } 10_{7/2}$	$\frac{4}{15\sqrt{7}}B' + \frac{8\sqrt{3}}{45\sqrt{7}}C' - \frac{2\sqrt{6}}{45\sqrt{7}}D'$	$-\frac{4\sqrt{3}}{9\sqrt{35}}B' - \frac{8}{9\sqrt{35}}C' + \frac{2\sqrt{2}}{9\sqrt{35}}D'$		
$2 \text{ } 8_{3/2}$	$-\frac{2}{15\sqrt{2}}A' - \frac{2\sqrt{2}}{15}B' + \frac{2\sqrt{3}}{15\sqrt{2}}C'$	$-\frac{2}{15}A' - \frac{\sqrt{2}}{15}C' - \frac{4}{15}D'$	$\frac{4\sqrt{2}}{45}B' - \frac{4\sqrt{3}}{45\sqrt{2}}C'$	$\frac{2\sqrt{2}}{45}C' + \frac{8}{45}D'$
$2 \text{ } 8_{5/2}$	$-\frac{2}{15}A' + \frac{2\sqrt{3}}{15}B' + \frac{2}{15}C'$	$-\frac{2\sqrt{2}}{15}A' - \frac{2\sqrt{2}}{15}C' + \frac{2}{15}D'$	$-\frac{4\sqrt{3}}{45}B' - \frac{4}{45}C'$	$\frac{4\sqrt{2}}{45}C' - \frac{4}{45}D'$

REFERENCES AND FOOTNOTES

- 1) G. Zweig, CERN preprints TH.401 and 412 (1964), unpublished;
M. Geil-Mann, Phys.Letters 8, 214 (1964).
- 2) For a review of recent work on the Z^* 's see e.g.,
J.D. Dowell "The search for Z^* 's", Proceedings of the XVI International
Conference on High Energy Physics, Chicago-Batavia (1972).
- 3) See e.g., the reviews by
H. Harari, Proceedings of the XVI International Conference on High
Energy Physics, Vienna, edited by J. Prentki and J. Steinberger
(1968);
R.H. Dalitz in Symmetries and Quark Models, edited by Ramesh Chand,
New York, Gordon and Breach (1970).
- 4) Particle Data Group, Reviews of Modern Phys. Vol. 45, N. 2 (1973).
- 5) In addition to the absence of some states, one has well-known problems
with the scalar meson nonet: e.g., the broken $SU(3)$ mass formula
cannot work for any mixing angle, since $M_K > M_S$ and $M_K > M_{\phi}$. The
recent "disappearance" of the $A_1(1100)$ makes the situation at the
 $L=1$ level even worse.
- 6) By "exotic meson" I mean any meson with quantum numbers $[J]^{PC}$ as well
as $SU(3)$ not accessible to a $q\bar{q}$ system.
- 7) D. Faiman and D.E. Plane, Nuclear Phys. B50, 379 (1972) and references
therein.
- 8) O.W. Greenberg, Phys.Rev.Letters 13, 598 (1964).
- 9) See e.g.,
R.H. Dalitz, Les Houches Lectures (1965) (Gordon and Breach, N.Y., 1965);
O.N. Greenberg and M. Resnikoff, Phys.Rev. 163, 1844 (1967);
C.T. Chen Tsai, S.I. Chu and T.Y. Lee, Phys.Rev. D6, 2451, 2459 (1972);
R.R. Horgan, Oxford Ph.D. Thesis (1972), unpublished.
- 10) H.J. Lipkin and S. Meshkov, Phys.Rev.Letters 14, 670 (1965); see also
K.J. Barnes, P. Carruthers and F. von Hippel, Phys.Rev. Letters 14, 82
(1965).
- 11) $\eta - \eta'$ and $\omega - \phi$ mixings have been neglected.
- 12) See e.g.,
Y. Neeman, Algebraic Theory of Particle Physics, W.A. Benjamin Inc.,
New York (1967).

- 13) C.L. Cook and G. Murtaza, Nuovo Cimento 39, 531 (1965).
- 14) J.J. de Swart, Revs.Modern Phys. 35, 916 (1963).
- 15) G. Ascoli, H.B. Crawley, D.W. Mortara and A. Shapiro, Phys.Rev.Letters 21, 1411 (1968).
- 16) See e.g.,
J.L. Rosner, Phys.Rev. D6, 1781 (1972).
- 17) See e.g.,
D. Faiman, J.L. Rosner and J. Weyers, Nuclear Phys. B57, 45 (1973)
and references therein.
- 18) W.P. Petersen and J.L. Rosner, Phys.Rev. D6, 820 (1972); Phys.Rev. D7,
747 (1973).
- 19) L. Micu, Nuclear Phys. B10, 521 (1969);
E.W. Colglazier and J.L. Rosner, Nuclear Phys. B27, 349 (1971);
W.P. Petersen and J.L. Rosner, Ref. 18).
- 20) R. Carlitz and M. Kislinger, Phys.Rev. D2, 336 (1970).
- 21) R. Dashen and M. Gell-Mann, Phys.Letters 17, 142, 146 (1965).
- 22) O.W. Greenberg, Phys.Rev.Letters 13, 598 (1964)
- 23) M. Gell-Mann, "Lectures given at the Schladming Winter School, CERN
preprint TH.1543 (1972).
- 24) M. Gell-Mann, Phys.Rev. 125, 1067 (1962).
- 25) S.L. Adler, Phys.Rev. 140, B736 (1965).
- 26) M. Gell-Mann and M. Levy, Nuovo Cimento 16, 705 (1960);
Y. Nambu, Phys.Rev.Letters 4, 380 (1960).
- 27) S.L. Adler, Phys.Rev.Letters 14, 1051 (1965);
W.I. Weisberger, Phys.Rev.Letters 14, 1047 (1965).
- 28) See e.g.,
S. Fubini and G. Furlan, Physics 1, 229 (1965).
- 29) M. Gell-Mann in Strong and Weak Interactions - Present Problems,
Academic Press, Inc., New York, (1966).
K. Bardakçi and G. Segré, Phys.Rev. 153, 1263 (1967).
- 30) J. Jersak and J. Stern, Nuclear Phys. B7, 413 (1968);
K. Bardakçi and M.B. Halpern, Phys.Rev. 176, 1686 (1968);
H. Leutwyler, Springer Tracts in Modern Phys. 50, 29 (1969).
- 31) J.B. Kogut and D.E. Soper, Phys.Rev. D1, 2901 (1970).

- 32) R. Dashen and M. Gell-Mann, Phys.Letters 17, 142 (1965).
- 33) J. Jersak and J. Stern, Nuovo Cimento 59, 315 (1969). See also Ref. 30).
- 34) R. Carlitz, Lectures in these Proceedings.
- 35) R. Dashen and M. Gell-Mann, Phys.Rev.Letters 17, 340 (1966).
- 36) H. Osborn, Nuclear Phys. B38, 429 (1972).
- 37) M. Jacob and G.C. Wick, Ann.Phys. 7, 404 (1959).
- 38) $SU(6)_{W;currents}$ requires the existence of tensor currents for which there is, as yet, no experimental evidence. Even if one is unwilling to speculate on the existence of these currents, the $SU(3) \otimes SU(3)_{currents}$ algebra for the vector and axial vector charges can obviously be maintained and one can still ask whether this algebra is related to the (chiral) $SU(3) \otimes SU(3)_{strong}$ subalgebra of $SU(6)_{W;strong}$.
- 39) R.P. Feynman and M. Gell-Mann, Phys.Rev. 109, 193 (1958);
S.S. Gerstein and J.B. Zeldovich, JETP 29, 689 (1955).
- 40) See e.g.,
H. Harari in Spectroscopic and Group Theoretical methods in Physics (North-Holland, Amsterdam, 1968);
F.J. Gilman and H. Harari, Phys.Rev. 165, 1803 (1968); and references quoted therein.
- 41) M. Gell-Mann, Phys.Rev.Letters 14, 77 (1965).
- 42) R. Dashen and M. Gell-Mann, Symmetry Principles at High Energy (1966);
Coral Gables Conference, W.H. Freeman and Co.
- 43) See e.g.,
R. Gatto, L. Maiani and G. Preparata, Phys.Rev.Letters 10, 377 (1966);
N. Cabibbo and H. Ruegg, Phys.Letters 22, 85 (1966);
H. Harari, Phys.Rev.Letters 16, 964 (1968);
I.S. Gerstein and B.W. Lee, Phys.Rev.Letters 16, 1060 (1968);
D. Horn, Phys.Rev.Letters 17, 778 (1966);
F. Buccella, J. Kleinert, C.A. Savoy, E. Celeghini and E. Sorace,
Nuovo Cimento 59A, 133 (1970) and references cited therein.
- 44) See, e.g.,
H.W. Kendall, in Proc.Intern.Symp. on Electron and Photon Interactions at High Energies (1971), editor N.B. Mistry, Cornell University Press, Ithaca, N.Y. (1972).
- 45) H.J. Melosh, Cal.Tech.Ph.D. Thesis (1973), unpublished.

- 46) S.P. de Alwis, B55, 427 (1973).
- 47) E. Eichten, J. Willemssen and F. Feinberg, SLAC preprint (1973).
- 48) L.L. Foldy and S.A. Wouthuysen, Phys.Rev. 78, 29 (1950).
- 49) F. Gürsey, Phys.Rev.Letters 14, 330 (1965);
K.M. Bitar and F. Gürsey, Phys.Rev. 164, 1805 (1967).
- 50) S.P. de Alwis and J. Stern, CERN preprint TH.1679 (1973).
- 51) H. Fritzsch and M. Gell-Mann, Proceedings of the Tel-Aviv Conference on Duality and Symmetry (1971).
- 52) See Ref. 29), 35), as well as
M. Gell-Mann, D. Horn and J. Weyers, Proceedings of the Heidelberg Conference on elementary particles, North-Holland Publishing Co., (1968);
S.J. Chang, R. Dashen and L. O'Raiifeartaigh, Phys.Rev. 182, 1805, 1819 (1969);
K. Young, Phys.Letters 34B, 88 (1971).
- 53) A.J.G. Hey, J.L. Rosner, J. Weyers, CERN preprint TH.1859 (1973).
- 54) F.J. Gilman, M. Kugler and S. Meshkov, SLAC PUB 1286 (1973).
- 55) An even simpler example is given by the SU(3) singlets at the L=0 level: $\eta^{(1)}$ and $\phi^{(1)}$ both transform as $(1,1)$ representations of SU(3) $\otimes$ SU(3) but have opposite charge conjugation.
- 56) See e.g.,
S. Weinberg, Phys.Rev. 177, 2609 (1969).
- 57) F.J. Gilman and M. Kugler, Phys.Rev.Letters 30, 518 (1973);
A.J.G. Hey and J. Weyers, Phys.Letters 44B, 263 (1973).
- 58) See e.g.,
P. Söding et al., Phys.Letters 39B, 1 (1972).
- 59) F.J. Gilman, M. Kugler and S. Meshkov, SLAC PUB 1235 (1973).
- 60) F.J. Gilman, Invited talk presented at the Purdue Conference on baryon resonances, West Lafayette, Indiana (1973).
- 1) D. Faiman and J.L. Rosner, CERN preprint TH.1636.
- 62) D. Herndon, R. Longacre, L. Miller, A. Rosenfeld, G. Smadja, P. Söding, R. Cashmore and D. Leith, "A partial wave analysis of the reaction $\bar{\pi}N \rightarrow \pi \pi N$ in the C.M. energy range 1300-2000 MeV, LBL report 1065 (1972).

- 63) $SU(3) \otimes SU(3)$ does predict that $\mu_T(\text{proton}) = -\frac{5}{2}\mu_T(\text{neutron})$ and $\mu^* = 2\sqrt{2}/3 \mu_T(\text{proton})$ but does not predict $E_2 = 0$.
- 64) A. Love and D.V. Nanopoulos, University of Sussex preprint (1973).
- 65) A.J.G. Hey and J. Weyers, CERN preprint TH.1718 (1973).
- 66) R. Moorhouse, Phys.Rev.Letters 18, 772 (1966).
- 67) F.J. Gilman and I. Karliner, SLAC PUB 1271 (1973). These authors assume, however, that $C = 0$.
- 68) D. Faiman and A. Hendry, Phys.Rev. 173, 1720 (1968); 180, 1572 (1969);
L.A. Copley, G. Karl and E. Obryk, Phys.Letters 29B, 117 (1969);
Nuclear Phys. B13, 303 (1969);
R.P. Feynman, M. Kislinger and F. Ravndal, Phys.Rev. D3, 2706 (1971).
- 69) F.E. Close, and F.J. Gilman, Phys.Letters 38B, 541 (1972);
F.E. Close, F.J. Gilman and I. Karliner, Phys.Rev. D6, 2533 (1972).
- 70) J.L. Rosner and W.P. Petersen, Phys.Rev. D7, 747 (1973).
- 71) See e.g.,
R. Walker, Proceedings of the Fourth International Symposium on Electron and Photon Interactions at High Energies, Liverpool (1969);
A. Donnachie, Proceedings of the 1971 International Symposium on electron and Photon Interactions at High Energies, Cornell (1971);
R.G. Moorhouse and H. Oberlack, Phys.Letters 43B, 44 (1973).