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SPIN-DEPENDENT DEEP INELASTIC ELECTRON SCATTERING
IN AN ASYMPTOTICALLY FREE GAUGE THEORY

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A B S T R A C T

We compute the logarithmic deviations from Bjorken scaling for the spin dependent structure functions of deep inelastic electron scattering in an asymptotically free gauge theory. Continuations to the elastic limit and to the Regge region are also considered.

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Models in which the strong interactions are described by asymptotically free gauge theories (AFGT) provide a natural field-theoretic explanation of scaling ¹⁾. In such models logarithmic corrections to Bjorken scaling are predicted and the presence of these logarithmic terms is a crucial test of the AFGT explanation. The logarithmic corrections to e^+e^- annihilation have been analyzed by Applequist and Georgi and by Zee ²⁾. The corrections for spin averaged electroproduction and deep inelastic neutrino scattering have been analyzed by Gross and Wilczek, by Georgi and Politzer and by Bailin, Love and Nanopoulos ³⁾. It is the purpose of this note to extend the analysis to include spin-dependent deep inelastic electron scattering.

We work with the "standard" model for the strong interactions in which the strong interactions are described by a $SU(n)$ gauge group G acting on the colour indices of n quark multiplets. The strong group commutes with the charge operator.

In electron scattering we are interested in the product of two electromagnetic currents between polarized proton states. The forward Compton amplitude is given by

$$W_{\mu\nu}(p, q, s) = \frac{1}{2\pi} \int d^4x e^{iq \cdot x} \langle p, s | [J_\mu(x), J_\nu(0)] | p, s \rangle$$

$$= W_{\mu\nu}^{[S]} + i W_{\mu\nu}^{[A]} \quad (1)$$

where p , s and q are the target four-momentum, covariant polarization and virtual photon four-momentum, respectively. In addition to the structure functions W_1 and W_2 of unpolarized scattering contributing to the symmetric part $W_{\mu\nu}^{[S]}$ there are two structure functions G_1 and G_2 contributing to the antisymmetric part $W_{\mu\nu}^{[A]}$

$$W_{\mu\nu}^{[A]}(p, q, s) = i M \epsilon_{\mu\nu\lambda\sigma} q^\lambda s^\sigma G_1(p, q, q^2)$$

$$+ \frac{i}{M} \epsilon_{\mu\nu\lambda\sigma} q^\lambda (p \cdot q s^\sigma - q \cdot s p^\sigma) G_2(p, q, q^2) \quad (2)$$

where M is the target mass.

The part of the light cone expansion for the product of currents contributing to $W_{\mu\nu}^{[A]}$ is given by

$$\begin{aligned}
 J_\mu(x) J_\nu(0) = & -i \epsilon_{\mu\nu\lambda\sigma} \partial^\lambda \frac{1}{(x^2 - i\epsilon x_0)} \sum_{n=0}^{\infty} \sum_{\substack{\mu_1 \dots \mu_n \\ n \text{ even}}} \left\{ E_{i,1}^{(n)}(x^2 - i\epsilon x_0) R_1^{i\sigma\mu_1 \dots \mu_n}(0) x_{\mu_1} \dots x_{\mu_n} \right\} \\
 & - i (\epsilon_{\mu\rho\lambda\sigma} \partial_\nu \partial^\rho - \epsilon_{\nu\rho\lambda\sigma} \partial_\mu \partial^\rho - \epsilon_{\mu\nu\lambda\sigma} \square) \sum_{n=1}^{\infty} \sum_{\substack{\mu_1 \dots \mu_n \\ n \text{ odd}}} \left\{ E_{i,2}^{(n)}(x^2 - i\epsilon x_0) R_2^{i\lambda\sigma\mu_1 \dots \mu_n}(0) x_{\mu_1} \dots x_{\mu_n} \right\} \quad (3)
 \end{aligned}$$

where i denotes any internal quantum numbers. It was shown by Heimann ⁴⁾ that this is the most general form. The matrix elements of the local operators R_1 and R_2 , between nucleon states are defined by

$$\begin{aligned}
 \langle p, s | R_1^{i\sigma\mu_1 \dots \mu_n}(0) | p, s \rangle &= (i)^n a_n^i \{ s^\sigma p^{\mu_1} \dots p^{\mu_n} \} \quad (4) \\
 \langle p, s | R_2^{i\lambda\sigma\mu_1 \dots \mu_n}(0) | p, s \rangle &= (i)^{n+1} d_n^i \frac{1}{2} (p^\lambda s^\sigma - p^\sigma s^\lambda) p^{\mu_1} \dots p^{\mu_n}
 \end{aligned}$$

where $\{ \}$ denotes symmetrization of the indices, and division by $n+1$.

The Fourier transforms of the $E_{i,k}^{(n)}$ are determined by the moments of the structure functions

$$\begin{aligned}
 \int_1^\infty d\omega \omega^{-n-2} M(p, q) G_1(\omega, q^2) &= \sum_i a_n^i \tilde{E}_{i,1}^{(n)}(q^2) \quad n \text{ even} \\
 \int_1^\infty d\omega \omega^{-n-1} \frac{(q \cdot p)^2}{M} G_2(\omega, q^2) &= - \sum_i \left(a_{n+1}^i \tilde{E}_{i,1}^{(n+1)}(q^2) \right. \\
 &\quad \left. + \frac{1}{2} d_{n+2}^i \tilde{E}_{i,2}^{(n+2)}(q^2) \right) \quad n \text{ odd} \quad (5)
 \end{aligned}$$

where

$$\begin{aligned}
 \tilde{E}_{i,1}^{(n)}(q^2) &= -\frac{1}{8} (-q^2)^{n+1} \left(\frac{\partial}{\partial q^2} \right)^{n+1} \left(\frac{\partial}{\partial q^2} \right)^n \int d^4x e^{iq \cdot x} \frac{1}{(x^2 - i\epsilon)} E_{i,1}^{(n)}(x^2 - i\epsilon) \\
 \tilde{E}_{i,2}^{(n)}(q^2) &= -\frac{1}{8} (-q^2)^{n+2} \left(\frac{\partial}{\partial q^2} \right)^n \int d^4x e^{iq \cdot x} E_{i,2}^{(n)}(x^2 - i\epsilon)
 \end{aligned}$$

In the quark model (with interaction mediated by non-Abelian gauge gluons) the leading operators contributing to $R_1^{i\sigma\mu_1\cdots\mu_n}(0)$ are twist 2 and are

$$\begin{aligned} F R_1^{i\sigma\mu_1\cdots\mu_n}(0) &= (i)^n S \bar{\psi} \gamma_5 \gamma^\sigma D^{\mu_1} \cdots D^{\mu_n} \frac{\lambda_i}{2} \psi \\ F R_1^{o\sigma\mu_1\cdots\mu_n}(0) &= (i)^n S \bar{\psi} \gamma_5 \gamma^\sigma D^{\mu_1} \cdots D^{\mu_n} \psi \\ \nu R_1^{o\sigma\mu_1\cdots\mu_n}(0) &= (i)^n S \frac{1}{2} \epsilon^{\sigma\alpha\beta\gamma} \text{Tr} F_{\beta\gamma} D^{\mu_1} \cdots D^{\mu_n} F_{\alpha}^{\mu_n} \end{aligned} \quad (6)$$

with

$$F^{\mu\nu} = \left\{ \partial^\mu A^\nu - \partial^\nu A^\mu - g f_{abc} A_b^\mu A_c^\nu \right\} t^a$$

The λ_i , $i=1\ldots 8$ are the standard λ matrices of Gell-Mann and S denotes symmetrization over $\sigma\mu_1\cdots\mu_n$. A_μ^a , $a=1\ldots n^2-1$, are the vector gauge fields of the colour group $SU(n)$. The covariant derivatives D_μ are $\partial_\mu + ig \tau^a A_\mu^a$ acting on vectors and $\partial_\mu + ig \sigma^a A_\mu^a$ acting on fermions where τ^a and σ^a are the representation matrices of the vectors and fermions, respectively, for the colour group.

The leading operators contributing to $R_2^{i\lambda\sigma\mu_1\cdots\mu_n}(0)$ are twist 3 and are

$$\begin{aligned} F R_2^{i\lambda\sigma\mu_1\cdots\mu_n}(0) &= (i)^{n+1} S' \bar{\psi} \gamma_5 \gamma^\lambda D^\sigma D^{\mu_1} \cdots D^{\mu_n} \frac{\lambda_i}{2} \psi \\ F R_2^{o\lambda\sigma\mu_1\cdots\mu_n}(0) &= (i)^{n+1} S' \bar{\psi} \gamma_5 \gamma^\lambda D^\sigma D^{\mu_1} \cdots D^{\mu_n} \psi \\ \nu R_2^{o\lambda\sigma\mu_1\cdots\mu_n}(0) &= (i)^{n+1} S' \epsilon^{\sigma\alpha\beta\gamma} F_{\beta\gamma} D^{\mu_1} \cdots D^{\mu_n} F_{\alpha}^{\mu_n} \end{aligned} \quad (7)$$

where S' denotes symmetrization on the indices $\sigma\mu_1\cdots\mu_n$ and anti-symmetrization on the indices $\lambda\sigma$.

In order to analyze the large momentum behaviour of these operators we apply the renormalization group techniques to the operator product expansion of Eq. (3). The Wilson coefficients $E_{i,k}^{(n)}$ will obey the renormalization group equation ³⁾

$$\left(\mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g}\right) E_{i,k}^{(n)} = \sum_j \gamma_{jk}^{(n)}(g) E_{j,k}^{(n)} \quad (8)$$

Here g is the strong gauge group coupling constant and μ is the subtraction point introduced to perform the required renormalization. The anomalous dimensions $\gamma_{jk}^{(n)}$ form a matrix defined by

$$\left(\gamma_{jk}^{(n)}(g)\right)_j^i = \left(\gamma_{jk}^{(n)} \frac{\partial}{\partial \mu} \gamma_{kj}^{(n)}\right)_{ij} \Big|_{g_0, \Lambda \text{ fixed}} \quad (9)$$

where g_0 is the unrenormalized strong coupling constant and Λ is the cut-off. $\gamma_{jk}^{(n)}$ is the matrix of wave function renormalization constants of the operators $R_k^{i\mu_1 \dots \mu_n}$ appearing in Eq. (3). $\gamma_{jk}^{(n)}$ may be determined in lowest order perturbation theory by computing the one-gluon exchange corrections to these operators as described in Ref. 3). The calculation will be described in detail elsewhere, here we quote the results. Note they were obtained ignoring the contribution of ghost fields ⁵⁾.

For the operator R_1 , in the $SU(3)$ singlet case

$$\begin{aligned} \gamma_{FF}^{(n)} &= \frac{g^2 C_2(R)}{8\pi^2} \left\{ 1 - \frac{2}{(n+1)(n+2)} + \sum_{j=2}^{n+1} \frac{4}{j} \right\} \\ \gamma_{VV}^{(n)} &= - \frac{g^2 T(R)}{8\pi^2} \left\{ \frac{4n}{(n+1)(n+2)} \right\} \\ \gamma_{FF}^{(n)} &= - \frac{g^2 C_2(R)}{8\pi^2} \left\{ \frac{2(n+3)}{(n+1)(n+2)} \right\} \\ \gamma_{VV}^{(n)} &= \frac{g^2}{8\pi^2} \left[C_2(G) \left\{ \frac{4}{3} + \frac{2}{(n+1)} + \sum_{j=0}^{n-2} \left(\frac{2}{j+3} + \frac{2}{j+2} \right) \right\} + \frac{4}{3} T(R) \right] \end{aligned} \quad (10)$$

For the operator R_2 , in the $SU(3)$ singlet case

$$\begin{aligned}
 {}^n_2 \gamma_{FF}^F &= \frac{g^2 C_2(R)}{8\pi^2} n \left\{ \frac{1}{(n+2)} + \frac{1}{(n+1)} \sum_{j=2}^{n+2} \frac{4}{j} \right\} \\
 {}^n_2 \gamma_{VV}^F &= 0 \\
 {}^n_2 \gamma_{FF}^V &= \frac{g^2 C_2(R)}{8\pi^2} \left\{ \frac{4}{n+2} \right\} \\
 {}^n_2 \gamma_{VV}^V &= \frac{g^2}{8\pi^2} \left[C_2(G) \left\{ \frac{1}{3} + \frac{2(2n+5)}{(n+2)(n+3)} + \sum_{j=0}^{n-1} \left(\frac{2}{j+3} + \frac{2}{j+2} \right) \right\} + \frac{4}{3} T(R) \right]
 \end{aligned} \tag{11}$$

Here $C_2(G)$ and $C_2(R)$ are the quadratic Casimir operators of the gauge group evaluated for the adjoint representation and the fermion representation R , respectively. $T(R)$ is the square of a matrix in the Lie algebra of the representation R .

The solution of Eq. (8) for the Fourier transform of $E_{i,k}^n$ is

$$\begin{aligned}
 \tilde{E}_{i,k}^n \left(\frac{q^2}{\mu^2}, g \right) &= \sum_j \left\{ T \exp - \int_0^t {}^n_k \gamma(\bar{g}(x, g)) dx \right\}_{ij} \\
 &\quad \tilde{E}_{j,k}^{(n)}(1, \bar{g}(t, g))
 \end{aligned} \tag{12}$$

where $t = \frac{1}{2} \ln(-q^2/\mu^2)$ and $\bar{g}(t, g)$ is the effective coupling constant defined by

$$\frac{d\bar{g}^2}{dt} = -b_0 \bar{g}^4 \quad ; \quad \bar{g}(0, g) = g \tag{13}$$

T means the exponential is to be time-ordered. In lowest order ¹⁾

$$b_0 = \frac{1}{8\pi^2} \left[\frac{11}{3} C_2(G) - \frac{4}{3} T(R) \right] \tag{14}$$

For SU(3) singlet operators the leading behaviour of the $\tilde{E}^n$ is found by diagonalizing $\frac{n}{k}\chi$ and picking the lowest eigenvalue $\frac{n}{k}\chi$. For SU(3) octet operators only $\frac{n}{\gamma_{FF}}F$ contributes to $\frac{n}{k}\chi$ and is as given in Eqs. (10) and (11).

For both singlet and octet operators $\frac{0}{1}\gamma = 0$ corresponding to conserved axial vector currents. For $n \neq 0$ the off diagonal elements vanish like $1/n$ and we may, to a good approximation, set them equal to zero. Then for both singlet and octet operators we have

$$\frac{n}{k}\gamma = \lim \left\{ \frac{n}{k}\gamma_{FF}^F, \frac{n}{k}\gamma_{VV}^V \right\} \quad (15)$$

Using Eqs. (12), (13) and (14) we find for the leading behaviour of $\tilde{E}_{i,j}^n$

$$\tilde{E}_{i,j}^n \xrightarrow{-q^2 \rightarrow \infty} C_j^n (1 + g^2 b_0 t)^{-\frac{n}{j}\gamma/2b_0 g^2} \quad (16)$$

where the C_j^n are constants.

Since $\frac{n-1}{2}\gamma < \frac{n}{1}\gamma$ we see from Eq. (5) that the leading behaviour of the moments of G_2 are given by $\tilde{E}_{1,2}^n$. Then from Eqs. (5) and (16) we have

$$\int_1^\infty d\omega \omega^{-n-2} M \cdot q G_1(\omega, q^2) \underset{-q^2 \rightarrow \infty}{\sim} (1 + g^2 b_0 t)^{A_n} \quad (17)$$

where

$$A_n \approx \frac{3C_2(R)}{22C_2(G) - 8\pi(R)} \left\{ -0.69 + 4 \ln(n+2) \right\}$$

$$\int_1^\infty d\omega \omega^{-n-1} \frac{(q \cdot p)^2}{M} G_2(\omega, q^2) \underset{-q^2 \rightarrow \infty}{\sim} (1 + g^2 b_0 t)^{B_n} \quad (18)$$

where $B_n \approx \frac{3C_2(R)}{22C_2(G) - 8\pi(R)} \left\{ -0.69 + \frac{4n}{n+1} \ln(n+3) \right\}$

The interpolation used here for $\frac{n}{k} \gamma$ is valid to $O(1/n)$.

In the red, white and blue quark model $[G = SU(3)]$

$$\begin{aligned} A_n &\approx 0.296 \ln(n+2) - 0.051 \\ B_n &\approx 0.296 \left(\frac{n}{n+1} \right) \ln(n+3) - 0.051 \end{aligned} \quad (19)$$

It may be seen from Eqs. (17) and (18) that, in general, sum rules for the structure functions G_1 and G_2 pick up logarithmic suppressions. However, since both a_0 and d_{-1} do not exist,

$$\int_1^\infty d\omega \omega^{-2} \frac{(q \cdot p)^2}{M} G_2(\omega, q^2) = 0 + O\left(\frac{1}{\ln q^2}\right)$$

$q^2 \rightarrow \infty$

as in the free quark model ^{4),6)}. Furthermore, since the axial vector current has no anomalous dimension, the integral

$$\int_1^\infty d\omega \omega^{-2} M(p \cdot q) G_1(\omega, q^2)$$

if convergent, must scale. Similarly the Bjorken sum rule

$$\int_1^\infty d\omega^2 \omega^{-2} M(p \cdot q) [G_1^p(\omega, q^2) - G_1^n(\omega, q^2)] = \frac{1}{6} \left(\frac{G_A}{G_V} \right) \quad (20)$$

has no anomalous dimensions.

The implications of Eqs. (17) and (18) for the structure functions G_1 and G_2 themselves require a knowledge of the constants of proportionality in these equations. This means an analysis of the type suggested by Parisi and by Gross ⁷⁾ is necessary to relate the structure functions at one value of q^2 to another. However, some things may immediately be said for small and large ω .

Near $\omega=1$ the high n moments dominate the structure functions ⁸⁾. For large n the forms given by Eqs. (17), (18) and (19) are good approximations for $G = SU(3)$. If $F(\omega, t')$ [where $F \equiv (q \cdot p)G_1$ or $(q \cdot p)^2 G_2$] has a power behaviour near threshold

$$F(\omega, t') \sim (\omega-1)^d, \quad \omega \rightarrow 1 \quad (21)$$

Then 8) at $t > t'$

$$\frac{F(\omega, t)}{F(\omega, t')} \xrightarrow{\omega \rightarrow 1} \left(\frac{t}{t'}\right)^{0.051} \frac{\Gamma'(d+1)}{\Gamma'(d+1+P)} (\ln \omega)^P \quad (22)$$

where $P = 0.296 \ln t/t'$.

On a more speculative note we may use this result to investigate the behaviour of the proton form factor. If we assume Bloom-Gilman type duality, the structure function near $\omega = 1$ is related to the proton form factor as described in Ref. 8). This leads to the prediction that the form factors fall faster than any power of q^2 for large q^2 . In the resonance region the higher order terms in $\bar{g}^2$ are important 9) and this result must be viewed with caution. However, in the unpolarized case a direct evaluation of these terms showed them to be small in the RWB model with $10 < Q^2 < 50$ if $\omega \geq 1.3$ 9).

For large ω the nearest singularity in n dominates the structure functions 10). These occur at $n = -1$ in $\frac{n}{1}\chi$ and $n = -2$ in $\frac{n}{2}\chi$ [see Eqs. (10) and (11)].

If one assumes that the C_j^n 's have no singularities in $\text{Re } n$ to the right of those in $\frac{n}{k}\chi$ then the following bounds apply for large W

$$\begin{aligned} G_1(\omega, q^2) & \underset{\substack{\omega \rightarrow \infty \\ q^2 \text{ fixed}}}{<} \omega^{-1} \\ G_2(\omega, q^2) & \underset{\substack{\omega \rightarrow \infty \\ q^2 \text{ fixed}}}{<} \omega^{-2} \end{aligned} \quad (23)$$

Regge lore would suggest G_1 fall off slightly faster than ω^{-1} and G_2 like some inverse power of $\log \omega$. This would suggest the singularity at $n = -1$ in χ_1^n could dominate. We assume C_1^n has this singularity in the form

$$C_1^n = M(n+1) K^{\frac{a}{n+1}} \quad (24)$$

where

$$\chi_1^n \xrightarrow{n \rightarrow -1} -\frac{a}{n+1} + b$$

and K is a constant > 1 . $M(n+1)$ is regular at $n = -1$. Then

$$G_1 \xrightarrow{\omega \rightarrow \infty} \frac{1}{2\sqrt{\pi}} \lambda^{-b} M\left(\sqrt{\frac{z}{\log \omega}}\right) \left(\frac{z}{\log^3 \omega}\right)^{\frac{1}{4}} e^{2\sqrt{z \log \omega}} \quad (25)$$

where

$$\lambda = \frac{\log q^2/\mu^2}{\log q_0^2/\mu^2}$$

and $z = a \log(K\lambda)$ and q_0^2 denotes the onset of scaling.

The terms of higher order in $\bar{g}^2$ are expected also to have the singularity at $n = -1$ and the corrections will become important unless ⁹⁾

$$\frac{\bar{g}^2}{4\pi^2} \left(\frac{\ln \omega}{a \ln K \lambda} \right)^{3/2} \ll 1$$

Clearly this condition cannot be satisfied for very large ω , and the form of Eq. (24) cannot there be justified. However, there will be a range of ω , depending on the unknown constant K , for which the form should be a good approximation.

For $G_2(\omega, q^2)$ we expect the Regge singularity to dominate. This would have to correspond to an $n=0$ singularity in C_1^n or an $n=1$ singularity in C_2^n . If we assume this is present, we can predict the q^2 dependence of $G_2(\omega, q^2)$. Since $\frac{n}{2}\gamma=0$ the leading q^2 dependence will be zero

$$G_2(\omega, q^2) \underset{\omega \rightarrow \infty}{\sim} \frac{1}{(\ln \omega)^\alpha}$$

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