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MULTI-PION PRODUCTION IN A CASCADE MODEL
FOR e^+e^- ANNIHILATION

N.S. Craigie

II. Institut für Theoretische Physik
der Universität Hamburg

and

K.D. Rothe ^{*)}

CERN -- Geneva

ABSTRACT

We consider a generalization of the vector dominance model for e^+e^- annihilation, in which multi-pion production is the result of a vector meson cascade involving the ρ and ω mesons. The corresponding coupled set of integral equations for the total and n particle cross-sections are solved in the limit of large c.m. energy. Depending on the asymptotic behaviour of the $\rho\omega\pi$ form factor, the solutions are able to accommodate a pion multiplicity increasing like the logarithm or some power α (including $\alpha=0$) of the total c.m. energy, as well as a total cross-section falling slower than point-like.

*) On leave from the "Institut für Theoretische Physik der Universität Heidelberg".

Although the parton model ¹⁾ has been very successful in describing deep inelastic electroproduction, it exhibits a number of unsatisfactory features when applied to the electron-positron annihilation into hadrons. In its simplest form, it supposes that two uncorrelated jets of very energetic particles will dominate the production of hadrons at high energies, thus leading naturally to a point-like behaviour of the total e^+e^- cross-section. However, within the framework of conventional field theories ²⁾ such a picture appears to be realized only in a softened ³⁾ or super-renormalizable ⁴⁾ theory. More embarrassing yet, the jets would be expected to carry quark-like quantum numbers.

It has been argued ^{5),6)} that the e^+e^- annihilation may exhibit a plateau in rapidity space analogous to the one observed in hadronic production processes. The uncorrelated jet model evidently implies a constant mean multiplicity, and hence no plateau. If the centre of the rapidity plot is to be filled somehow, the two partons coupling to the virtual photon will eventually have to interact ⁵⁾. However, as it was shown in Ref. 7), conventional field theories may not provide a dynamical framework for generating such a plateau.

Experiment, on the other hand, reveals a surprisingly large cross-section which may not tend to a point-like limit ⁸⁾, and a mean multiplicity which may actually increase like the logarithm or, some power of the total c.m. energy ⁹⁾. Data on $\bar{p}p$ annihilation furthermore suggest that the hadrons might be produced with rather large relative transverse momenta.

It appears to be difficult to realize all of the above features within a dynamical framework of the parton model ^{6),7)}. We therefore depart here from the parton point of view and consider a generalization of the vector dominance model which allows for multipion production, and has the virtue of being explicitly soluble at high energies. It therefore provides a convenient dynamical framework for gaining some insight as to what might be expected.

We suppose the production of pions to take place via a kind of "bremsstrahlung" process in a vector meson cascade involving only ρ and ω mesons, as shown in Fig. 1 for the $I=1$ channel. We take the Gell-Mann - Sharp - Wagner model ¹⁰⁾ for the ω decay into three pions. The cascade corresponds to rainbow-type graphs in the cross-section, which may be summed up in the form of a coupled set of Volterra integral equations (see Fig. 2) for the $I=1$ and $I=0$ total cross-section :

$$\sigma^{(1)}(s) = \sigma_0(s) + \int_{16}^{(\sqrt{s}-1)^2} \frac{ds'}{s} K^{(g)}(s, s') \sigma^{(0)}(s') \quad (1.a)$$

$$\sigma^{(0)}(s) = \int_9^{(\sqrt{s}-1)^2} \frac{ds'}{s} K^{(v)}(s, s') \sigma^{(1)}(s') \quad (1.b)$$

where the pion mass has been set equal to unity. The kernel of the integral equations has the form,

$$K^{(v)}(s, s') = \frac{\pi}{6} \left(\frac{s'}{s} \right)^2 \frac{[(s-s'+1)^2 - 4s]^{3/2}}{(s - m_v^2)^2} |F(s, s')|^2 \quad (2)$$

$F(q^2, (q-k)^2)$ being the off-shell $\rho \omega \pi$ form factor with the conventional normalization $F(m_\rho^2, m_\omega^2) = f_{\rho\omega\pi}$, $(f_{\rho\omega\pi}^2)/4\pi = \frac{1}{2}$. The n particle cross-section evidently satisfies the corresponding integral equation,

$$\sigma_n(s) = \int_{(n+2)^2}^{(\sqrt{s}-1)^2} \frac{ds'}{s} K^{(v)}(s, s') \sigma_{n-1}(s') \quad (3)$$

where n even (odd) refers to the $I=1$ ($I=0$) channel and $v = \rho(\omega)$.

We observe that the inhomogeneous (or driving) term in Eq. (1.a) is just the cross-section for 2π production, and is entirely determined by the electromagnetic form factor of the pion. It therefore gives a contribution of the order of $O(s^{-3})$ to $\sigma(s)$, due to the well-known s^{-1} fall-off of the form factor. A point-like behaviour of the total cross-section would thus have to be the result of adding the infinite set of rainbow graphs shown in Fig. 1.

An attractive feature of the model is that it describes well the low energy data (driving term plus first iteration), by construction. On the other hand, the high energy behaviour of the cross-section is uniquely determined by the two-pion production, provided we know the behaviour of the $\rho \omega \pi$ form factor $F(s, s')$ as either one, or both of the vector meson masses s, s' grow large. Since this is not known, we can only classify the solutions according to the asymptotic behaviour of $F(s, s')$.

We shall suppose that the limit

$$F(s, \eta s) \xrightarrow{s \rightarrow \infty} \sqrt{\lambda} h(\eta) s^{\beta - \frac{1}{2}} \quad (4)$$

is reached uniformly with respect to η , λ being a dimensionless "coupling constant". We shall furthermore consider a minimal ansatz for $h(\eta)$, consistent with this requirement and the symmetry of $F(s, s')$ for s/m_g^2 , $s'/m_\omega^2 \gg 1$:

$$h(\eta) = (\sqrt{\eta})^{\beta - \frac{1}{2}}. \quad (5)$$

For a point-like $\rho\omega\pi$ vertex ($\beta = \frac{1}{2}$), an extrapolation to the ρ and ω mass shell would give $\lambda = 2\pi$ as an order of magnitude estimate.

It turns out that the character of the solutions to the integral equations (1) and (3) depends crucially on whether $\beta < 0$, $\beta = 0$ or $\beta > 0$. The detailed features of each of these solutions depend on the subdomains corresponding to $\beta < -\frac{1}{2}$, $\beta = -\frac{1}{2}$, $-\frac{1}{2} < \beta < 0$; and $0 < \beta < \frac{1}{2}$, $\beta = \frac{1}{2}$, $\beta > \frac{1}{2}$.

In this letter we merely like to stress the physically interesting aspects of the model. Hence, we shall concentrate here on the three representative cases $\beta = -\frac{1}{2}$ (jet type), $\beta = 0$ (multiperipheral type) and $\beta = \frac{1}{2}$ (thermodynamic type). A more detailed account for arbitrary β , including a numerical comparison with the experimental data will be given elsewhere ¹¹.

The case $\beta = -\frac{1}{2}$

The n particle cross-section is found to have the same asymptotic behaviour as the two-pion cross-section, i.e.,

$$\sigma_n(s) \xrightarrow{s \rightarrow \infty} \frac{2C_n(\pi\lambda)^n}{(n+2)!(n+1)!n!(n-1)!} s^{-3} \quad (6)$$

$$C_n = \int_4^\infty ds s^{2-n} \sigma_0(s)$$

and, correspondingly,

$$\sigma(s) = O(s^{-3}).$$

Summation over the infinite number of cascades thus merely leads to a numerical enhancement of the 2π contribution to $\sigma(s)$. The average particle multiplicity evidently tends to a constant, so that the secondaries carry an average energy proportional to the c.m. energy of the incoming electron. It is in fact easy to see that the solution corresponds to the formation of two energetic jets moving in opposite directions in the c.m. system, one of the jets

containing only a single pion. However, the experimentally observed trend of $\sigma(s)$ for large s ⁸⁾ very likely rules out this type of solution.

The case $\beta = +\frac{1}{2}$

In this case the solution to the integral equation (3) exhibits the energy dependence typical of a pure phase space distribution (except for the $\log s$ factor) :

$$\sigma_n(s) \xrightarrow{s \rightarrow \infty} 2 \sigma_0(s) \frac{(\pi \lambda s)^n \log s}{(n+2)!(n+1)!n!(n-1)!} . \quad (7)$$

This would lead to an exponentially rising total cross-section ("run-away" solution) and hence must be discarded as a possible solution. We may, however, reinterpret the cascade as a model for the probability $P_n(s) = \sigma_n(s)/\sigma(s)$ for producing n pions. We find,

$$P_n(s) \xrightarrow{s \rightarrow \infty} 2(2\pi)^{\frac{3}{2}}(\pi \lambda s)^{\frac{3}{2}} \frac{(\pi \lambda s)^n e^{-4(\pi \lambda s)^{\frac{1}{4}}}}{(n+2)!(n+1)!n!(n-1)!} . \quad (8)$$

The result resembles the behaviour found in the thermodynamical ¹²⁾ and statistical bootstrap models ¹³⁾ with an essential difference, however; in our case, the exponential fall-off is much slower - hence no critical temperature - and the average energy of the pions is correspondingly much larger. In fact,

$$\langle n \rangle \xrightarrow{s \rightarrow \infty} (\pi \lambda s)^{\frac{1}{4}} \quad (9)$$

corresponding to an average energy per secondary of $\langle E_1 \rangle \sim \sqrt{E}$. The much slower increase in the mean particle multiplicity, as compared to the linear rise found in statistical bootstrap models, is readily traced to the fact that the angular momentum barrier associated with the spin of the virtual photon keeps the pions from "leaking out" in too large numbers. As for the angular distribution, the pions are produced with large relative transverse momenta, a situation one may well find to be a characteristic feature of e^+e^- annihilation. In fact, a power-like rise of $\langle n \rangle$ with the total c.m. energy appears to be quite compatible with experiment ⁹⁾.

The case $\beta = 0$

This is probably the most interesting of all cases. The n particle cross-section is found to behave like

$$\sigma_n(s) \sim \frac{\alpha^2 \pi \lambda}{s} s^{-3/2} \frac{(\frac{\pi \lambda}{6} \log s)^{n-1}}{(n-1)!} \quad (10)$$

up to a numerical multiplicative constant. For the total cross-section we find ¹⁴⁾

$$\sigma(s) \sim \frac{\alpha^2}{s} b(\lambda) s^{\alpha(\lambda) - \frac{3}{2}} \quad (11)$$

where $b(\lambda)$ is a calculable function of the coupling constant λ . The "trajectory" $\alpha(\lambda)$ is determined by the eigenvalue equation

$$\frac{\pi \lambda}{6} \int_0^1 d\eta \eta^{\alpha(\lambda) - \frac{1}{2}} (1-\eta)^3 |h(\eta)|^2 = 1 \quad (12)$$

and is a monotonically increasing function of λ , with $\alpha(0) = 0$. Hence, for a sufficiently strong $\rho \omega \pi$ coupling constant, $\sigma(s)$ could exhibit a point-like behaviour ($\alpha(\lambda) = \frac{3}{2}$) or could increase even faster than s^{-1} , as the CEA data seem to indicate ⁸⁾. The power $s^{-\frac{3}{2}}$ in Eq. (11) corresponds to the asymptotic behaviour of the 3π contribution arising from the first iteration of Eq. (1.a). Hence, the remaining infinite number of cascades combine to enhance this contribution by a power $s^{\alpha(\lambda)}$.

From Eq. (11) we find an average pion multiplicity increasing logarithmically with the c.m. energy $\sqrt{s}$

$$\langle n \rangle \xrightarrow{s \rightarrow \infty} \left(\lambda \frac{\partial}{\partial \lambda} \alpha(\lambda) \right) \ln s$$

corresponding to a mean energy $\langle E_i \rangle = E \ln^{-1} E$ per secondary. Our model is thus able to accommodate a plateau in rapidity space analogous to the one found in hadronic production processes. The logarithmic increase in multiplicity can be traced to an AFS type enhancement mechanism similar to the one found in the multiperipheral model ¹⁵⁾, even though, in the cascade decay in question, the mass of the vector mesons grows like some power of the c.m. energy. The most energetic particles tend to cluster along some direction - the angular spread being determined by the energy of the secondary - while one of the pions carries away a good fraction of the available c.m. energy in the opposite direction.

One may obtain a more detailed picture concerning the kinematics of the secondaries by studying the inclusive e^+e^- annihilation. This can be readily done within the context of the cascade model. The behaviour of the structure functions in the Bjorken limit, as well as a more detailed treatment of the general case, β arbitrary, will be given elsewhere ¹¹⁾.

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FIGURE CAPTIONS

Figure 1 : Vector meson cascade contributing to $\sigma_n^{(1)}(s)$. The ρ , ω and π mesons are denoted by a single, double and dotted line, respectively.

Figure 2 : Integral equation for the :
a) $I = 1$ and
b) $I = 0$ total cross-section (with the e^+e^- vertex stripped off).
The pions are understood to be on the mass shell.

FIG.1

(a)

(b)

FIG.2