Accelerator Cavities as a Probe of Millicharged Particles

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PACS. 14.80.-j – Other particles (including hypothetical).

Abstract. – We investigate Schwinger pair production of millicharged fermions in the strong electric field of cavities used for particle accelerators. Even without a direct detection mechanism at hand, millicharged particles, if they exist, contribute to the energy loss of the cavity and thus leave an imprint on the cavity’s quality factor. Already conservative estimates substantially constrain the electric charge of these hypothetical particles; the resulting bounds are competitive with the currently best laboratory bounds which arise from experiments based on polarized laser light propagating in a magnetic field. We propose an experimental setup for measuring the electric current comprised of the millicharged particles produced in the cavity.

Strong electromagnetic fields offer a new window to particle physics. Experiments involving strong fields have a new-physics discovery potential which is partly complementary to accelerator experiments. For instance, experiments such as BFR T\textsuperscript{[1]}, CAST\textsuperscript{[2]}, or PVLAS\textsuperscript{[3]}, using strong magnetic fields, are involved in the search for light, weakly coupled particles such as axions.

Usually, particle physics effects in strong fields result from the the macroscopic spatial extent of the fields which can support coherent phenomena. If the mass of the new particles is sufficiently low, yet another set of mechanisms opens up new phenomenological possibilities: processes can become non-perturbative in the external field, leading to significant enhancements or increase of phase space. In a recent work\textsuperscript{[4]}, we have shown that the search for birefringence and dichroism of polarized laser light propagating in a strongly magnetized vacuum\textsuperscript{[1, 3]} gives the currently best laboratory bounds on the charge of millicharged particles. For these bounds, the nonperturbative account for the magnetic field is crucial.

We would like to stress that improved constraints on millicharged particles are very welcome. For one thing, the apparently much stronger astrophysical and cosmological bounds\textsuperscript{[5–10]} (for a recent review, see Ref.\textsuperscript{[11]}) have recently been shown to be quite model dependent\textsuperscript{[12]}. On the other hand, millicharged particles arise naturally in a large class of standard model extensions\textsuperscript{[13–17]}, most notably in a bottom-up approach to the string embedding of the standard model\textsuperscript{[15, 18, 19]}. Therefore, searches for millicharged particles are a powerful tool to probe fundamental physics.
The above mentioned laser experiments exploit strong magnetic fields. One may wonder what can be learned from experiments using strong electric fields. There is indeed a paradigm for a nonperturbative mechanism in strong fields: quantum electrodynamics predicts that electron-positron pairs are produced from vacuum in strong electric fields \[20–22\]. A sizeable rate for spontaneous \( e^+e^- \) pair production requires extraordinary strong electric field strengths \( E \) of order or above the critical value \( \frac{\hbar}{c} \equiv \frac{m_e^2}{e} \)

\[
E_c \equiv \frac{m_e^2}{e} \simeq 1.3 \times 10^{18} \text{ V/m},
\]

for which the work of the field on a unit charge \( e \) over the Compton wavelength of the electron, \( \lambda_c = 1/m_e \), equals the electron’s rest mass \( m_e \). The process can be viewed as quantum tunneling, giving rise to an exponential field dependence, \( \propto \exp(-\pi E_c/E) \), which exhibits the nonperturbative structure in \( eE \).

Currently, it seems inconceivable to produce macroscopic fields with electric field strengths of order \( E \sim E_c \) in the laboratory\(^{(1)}\). For \( E \ll E_c \), the exponential suppression of the rate makes this process practically unobservable at present. However, if millicharged particles with fractional charge \( \epsilon = Q_{\epsilon}/e \ll 1 \) and mass \( m_{\epsilon} \) exist in nature, their corresponding critical field,

\[
E_{\epsilon c} \equiv \frac{m_{\epsilon}^2}{e\epsilon} \simeq 4.98 \times 10^6 \text{ V/m} \left( \frac{m_e}{\epsilon eV} \right)^2,
\]

may be much smaller and they may be copiously produced with currently available electric fields.

In this Letter, we want to investigate whether the electric fields reachable at currently developed accelerator cavities will allow for a competitive search for millicharged particles.

For a first estimate, we approximate the electromagnetic field in such a cylindrical cavity as a spatially uniform electric field, pointing along the cylinder \( z \) axis and oscillating with a frequency \( \omega \),

\[
\mathbf{E}(t) = (0, 0, E(t)) = (0, 0, E_0 \sin(\omega t)), \quad \mathbf{B}(t) = (0, 0, 0).
\]

For a real cavity, this corresponds to the field configuration on the \( z \) axis. Typical parameters are \( E_0 = (35 - 150) \text{ MV/m} \) and \( \nu = \omega/2\pi = 1 \text{ GHz} \), corresponding to \( \omega = 4.13 \times 10^{-6} \text{ eV} \) \[24, 25\]. Furthermore, we assume that the frequency \( \omega \) is much smaller than the rest energy of the millicharged particle, \( \omega \ll m_{\epsilon} \). Under these conditions, the dominant contribution to the pair-production rate, i.e., the probability that a pair is produced per unit time and unit volume, is given by the Schwinger formula \[22\],

\[
w = \frac{d^4 n}{d^3 x dt} = \frac{(2s+1)}{2} \frac{m_{\epsilon}^4}{(2\pi)^3} \left( \frac{E}{E_c} \right)^2 \sum_{n=1}^{\infty} \frac{\beta_n}{n^2} \exp \left( -n\pi \frac{E_{\epsilon c}}{E} \right),
\]

where \( \beta_n = (-1)^{n+1} \) for bosons and \( \beta_n = 1 \) for fermions; \( s \) denotes the spin of the produced particles \[26\]. For our quantitative estimates, we will from now on consider fermions, \( s = 1/2, \beta_n = 1 \). Corrections to this leading-order formula for inhomogeneous fields can be computed in a semiclassical manner, using generalized WKB \[27, 28\], imaginary-time methods \[29\], propagator constructions \[30\], or modern worldline/instiont methods \[31–33\], as well as functional techniques \[34\]. For instance, corrections to the Schwinger formula for time-like inhomogeneities as in Eq. \[3\] are controlled by the ratio \( \eta \) of the energy of the laser

\(^{(1)}\)At the focus of standing laser waves, this may eventually be accomplished in the not so distant future \[23\].
Fig. 1 – Laboratory limits on the fractional electric charge \( \varepsilon \equiv Q_e / e \) of a millicharged fermion of mass \( m_\varepsilon \). The “Orthopositronium” limit stems from a limit on the branching fraction of invisible orthopositronium decay [38]. The green “BFRT” upper limits arise [4] from the upper limit on vacuum magnetic dichroism and birefringence placed by the laser polarization experiment BFRT [1]. The red (thin solid) line corresponds to the (too) naive bound obtained from Eq. (7) \( (E_0 = 25 \text{ MV/m}) \). The solid red “Cavity (TESLA)” upper limit arises from the bound on the energy loss caused by Schwinger pair production of millicharged particles in accelerator cavities developed for TESLA [24] \( (E_0 = 25 \text{ MV/m}, L_{\text{cav}} = 10 \text{ cm}, Q_{\text{min, MCP}}^{\text{min}} = 10^{10}) \). The red dashed upper limit demonstrates the possible bounds obtainable in the near future \( (E_0 = 50 \text{ MV/m}, L_{\text{cav}} = 10 \text{ cm}, Q_{\text{min, MCP}}^{\text{min}} = 10^{12}) \).

Let us start with an order-of-magnitude estimate of the sensitivity of accelerator cavities to millicharged fermions. Assuming that significant pair production leads to measurable deviations from the standard electrodynamical behaviour of the cavity, the non-observation of such deviations implies an upper bound on \( \varepsilon \) as a function of \( m_\varepsilon \). Using the observation that sizeable pair production sets in for \( E_0 / E_\varepsilon \sim 0.1 \sim 0.25 [36, 37] \), the equation \( E_0 / E_\varepsilon = \kappa = \mathcal{O}(0.25) \) translates into

\[
\varepsilon \lesssim 2.5 \times 10^{-2} \left( \frac{m_\varepsilon}{\text{eV}} \right)^2 \left( \frac{\kappa}{0.25} \right) \left( \frac{50 \text{ MV/m}}{E_0} \right). \tag{7}
\]

This rough estimate looks very promising. For \( m_\varepsilon \lesssim 1 \text{ meV} \), the sensitivity is better than the one obtained from the observed laboratory limit on vacuum magnetic dichroism due to pair production of millicharged fermions from laser photons in a static magnetic field [4] (cf.
Fig. 1. Note, that the adiabaticity parameter, for \( \omega = 4 \times 10^{-6} \) eV, \( m_\epsilon \geq 4 \times 10^{-4} \) eV, and \( \mathcal{E}_0/\mathcal{E}_c = 0.25 \), is indeed small, \( \eta \leq 0.04 \ll 1 \).

For a more realistic estimate of the sensitivity, we have to take into account that the effects caused by the millicharged particles typically decrease with shrinking \( \epsilon \). Direct detection, for instance, is therefore not straightforward. However, even without a direct detection mechanism at hand, millicharged particles may leave an observable imprint on the properties of the cavity. In particular, if a large number of them is produced, they contribute to the macroscopic energy loss of the cavity. This will be reflected by a decrease of the cavity’s quality factor \( Q \),

\[
Q = 2\pi E_{\text{cav}}/\Delta E,
\]

where \( E_{\text{cav}} \) is the energy stored in the cavity and \( \Delta E \) is the energy loss per oscillation period. In our case, the latter consists of two parts,

\[
\Delta E = \Delta E_{\text{diss}} + \Delta E_{\text{MCP}},
\]

where \( \Delta E_{\text{diss}} \) is the normal dissipative energy loss in absence of millicharged particles, and \( \Delta E_{\text{MCP}} \) the energy loss into millicharged particles.

In the following, \( \Delta E_{\text{MCP}} \) will be estimated by a series of conservative approximations. For our idealized cavity (cf. Eq. 3), our resulting \( \Delta E_{\text{MCP}} \) can hence be viewed as a lower bound to the true energy loss.

First, we consider only the kinetic energy carried away by those millicharged particles which leave the cavity. At sufficiently high field strength \( \mathcal{E} \gtrsim \mathcal{E}_c \), the particles will predominantly be produced moving highly relativistic in the direction of the electric field [36]. Depending on where and when they are produced, they may eventually reach a wall of the cavity. For our conservative estimate, we require that the particle reaches the wall of the cavity before the direction of the electric field is reversed. For example, a particle being produced at a time \( t \) within the first half of the oscillation period, \( 0 \leq t \leq \pi/\omega \), has to reach the wall before \( t_r = \pi/\omega \). Therefore, only particles with a distance less than

\[
L_{\text{max}}(t) = \frac{\pi}{\omega} - t
\]

from the ends of the cylindrical cavity contribute in our estimate. A particle starting at rest at \( t \) and leaving the cavity at \( t_r = \pi/\omega \) has picked up an average energy

\[
E_{\text{av}}(t) = ee_\epsilon \frac{1}{L_{\text{max}}(t)} \int_0^{L_{\text{max}}(t)} dL \int_t^{t+L} dt' \mathcal{E}(t') = ee_\epsilon \mathcal{E}_0 \left( \frac{\cos(\omega t)}{\omega} + \frac{\sin(\omega t)}{\omega(\pi - t\omega)} \right),
\]

where the \( t' \) integral determines the average field which the particle is exposed to if it starts at a distance \( L \) from the wall. The \( L \) integral is an average over the possible initial positions \( L \leq L_{\text{max}} \). This implies for the energy loss in one period,

\[
\Delta E_{\text{MCP}} = 4A_{\text{cav}} \int_0^{\pi/\omega} dt' E_{\text{av}}(t)L_{\text{max}}(t)w(t),
\]

where \( A_{\text{cav}} \) is the area of the cavity perpendicular to the electric field. Here, we took a factor two for the second half of the oscillation period into account, and another factor of two takes care of the fact that the energy loss happens at both ends of the cavity. For the electromagnetic field [3], the maximal energy stored in the cavity is given by

\[
E_{\text{cav}} = \frac{1}{2} \mathcal{E}_0^2 A_{\text{cav}} L_{\text{cav}},
\]
with the total length of the cavity $L_{\text{cav}}$. The cavity’s $Q$ factor is limited by the energy loss into millicharged particles. The maximal value is reached by an ideal cavity where $\Delta E_{\text{diss}} = 0$,

$$Q_{\text{MCP}} = \frac{\pi}{4} \frac{E_0^2 L_{\text{cav}}}{\int_{t_0}^{t_{\text{max}}} \frac{E_{\text{av}}(t)}{L_{\text{max}}(t)} w(t) dt}.$$  \hfill (14)

Modern superconducting cavities of the type developed for the Tera Electronvolt Superconducting Linear Accelerator (TESLA) reach $Q$ factors in excess of $Q_{\text{measured}} > 10^{10}$ at field strength $E_0 \sim 25 \text{ MV/m}$ [24]. Real cavities must have $Q < Q_{\text{MCP}}$. This enforces $Q_{\text{measured}} < Q_{\text{MCP}}$ and constrains the allowed values of $(\epsilon, m_\epsilon)$. The resulting bound is plotted in Fig. 1 and compared to other laboratory bounds on millicharged particles. Note that, for small masses, the upper limit for $\epsilon$ becomes independent of the mass and scales as

$$\epsilon \lesssim 10^{-6} \left( \frac{10^{10}}{Q_{\text{MCP}}^\text{min}} \right)^{1/3} \left( \frac{50 \text{ MV/m}}{E_0} \right)^{1/3} \left( \frac{L_{\text{cav}}}{30 \text{ cm}} \right)^{1/3},$$  \hfill (15)

where $Q_{\text{MCP}}^\text{min}$ is the minimal value for $Q_{\text{MCP}}$ allowed by experiment. With the above argument, $Q_{\text{MCP}}^\text{min}$ agrees with $Q_{\text{measured}}$. In the larger mass region, the limit weakens considerably. Improvement in this region requires stronger electric fields; roughly, the scaling behaviour is as in Eq. (7).

An even better bound could be obtained by comparing the expected $Q$ factor, $Q_{\text{expected}}$, for a real cavity in absence of millicharged particles with the measured $Q$ factor, $Q_{\text{measured}}$. If millicharged particles exist, the measured $Q$ factor should deviate from the expected one by,

$$\left( Q_{\text{measured}}^{-1} - Q_{\text{expected}}^{-1} \right)^{-1} = Q_{\text{MCP}}.$$  \hfill (16)

A 10% accuracy of the quantities contributing to the left-hand side gives already an improvement by a factor of 10 for the lower limit $Q_{\text{MCP}}^\text{min}$ on $Q_{\text{MCP}}$. Using this and some further improvements of cavities, including an increase in the maximal field strength by a factor of two, we plot the corresponding bound in Fig. 1 (red dashed line) to demonstrate the potential sensitivity reachable in the near future. The bound is well competitive with the one coming from laser experiments which currently provide the best laboratory bounds.

Further improvement may come from a more accurate determination of the energy loss caused by millicharged particles. Our criterion for particles to leave the cavity presumably takes into account only a fraction of particles that ultimately leave the cavity. On the other hand, in a real cavity, an additional complication arises due to the non-vanishing magnetic fields. These will lead to more complicated trajectories than the ones used in our simple estimate. Nevertheless, depending on the precise field distribution in the cavity, we expect that only exceptional particle trajectories inside the cavity are stable. Since the pairs are generally created with a continuous momentum distribution, the probability that a particle moves precisely on a stable trajectory inside the cavity is presumably very small. Therefore, it may well be that a large fraction of all produced particles ultimately leaves the cavity; this fraction would thus contribute to $\Delta E_{\text{MCP}}$ rather independently of the initial position. Moreover, the set of produced particles on stable trajectories will undergo plasma oscillations [36, 37, 39, 40] and potentially contribute indirectly to energy loss. Finally, it has to be checked for a real cavity whether some produced pairs could evade to contribute to the energy loss by subsequent coherent pair annihilation; for the special case of a spatially homogeneous field, this effect can lead to a reduction of pair accumulation [37] in comparison with the Schwinger formula [41]. Future estimates may also include the particles’ rest mass which we have not added to the energy loss so far. Also, thermal fluctuations can lead to an enhancement of the pair-production
Fig. 2 – Schematic set up for a “dark current shining through a wall” experiment. The alternating dark current (frequency $\nu$), comprised of the produced millicharged particles (dashed line), escapes from the accelerator cavity and traverses also a thick shielding (“wall”), in which the conventional dark current of electrons is stopped. The dark current induces a magnetic field in a resonant (frequency $\nu$) detector cavity behind the wall, which is detected by a SQUID [43].

rate [41]. In total, all these considerations may well lead to a factor $0.1 - 100$ in the energy loss compared to our conservative estimate. Since, for small masses, the sensitivity in $\epsilon$ scales with $\Delta E_{\text{MCP}}^{-1/3}$, this leads only to a moderate change of the bound by a factor of $2 - 0.2$.

Ultimately, one would like to probe also the region of larger masses. This requires much stronger electric fields. For example, one may envisage an experiment which exploits the electric field in an antinode of a standing wave produced by a superposition of two petawatt laser beams, focussed to the diffraction limit (cf. [23, 42]). In such an experiment, field strengths in excess of $10^8 \text{MV/m}$ may be reached, and consequently particles with larger masses may be produced. However, measuring the energy loss into millicharged particles is not straightforward in this set-up. Nevertheless, this possibility should be seriously studied, because it would be capable of testing the millicharged-fermion interpretation of the PVLAS dichroism signal, which requires $\epsilon \sim 3 \times 10^{-6}$ and $m_{\epsilon} \sim 0.1 \text{ eV}$ [4].

Above, we have discussed how one can obtain bounds on millicharged particles from the regular operation of accelerator cavities. A more direct approach to infer the existence of such particles may be based on the detection of the electrical current comprised of them. In Fig. 2 we show schematically how one could set up an experiment to detect this current.

In summary: Schwinger pair production in strong electric fields could turn accelerator cavities into factories for light millicharged particles, whose possible existence is, in many extensions of the standard model, directly tied to physics at very large energy scales, even up to the Planck scale, $M_P \sim 10^{19} \text{ GeV}$. Hence, parts of accelerators may probe higher energy scales than the accelerator beams themselves.

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We would like to thank E.-A. Knabbe, L. Lilje, A. Lindner, P. Schmüser, and H. Weise for valuable information on accelerator cavities. HG acknowledges support by DFG Gi 328/1-3.

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