Unconditional Bell-type state generation for spatially separate trapped ions

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We propose a scheme for generation of maximally entangled states involving internal electronic degrees of freedom of two distant trapped ions, each of them located in a cavity. This is achieved by using a single flying atom to distribute entanglement. For certain specific interaction times, the proposed scheme leads to the non-probabilistic generation of a perfect Bell-type state. At the end of the protocol, the flying atom completely disentangles from the rest of the system, leaving both ions in a Bell-type state. Moreover, the scheme is insensitive to the cavity field state and cavity losses. The issue of the practical implementation of our scheme is addressed by considering the realistic situations in which dephasing and dissipation are taken into account for the flying atom in its way from one cavity to the other. We then discuss the applicability of the resulting noisy channel for performing quantum teleportation.

PACS numbers: 03.67.Mn, 42.50.Pq

The idea of combining stationary and flying qubits in a quantum network has recently attracted much interest from the quantum information community. In such a network, the nodes are formed by the stationary qubits (typically matter qubits) and the information is carried between them by the flying qubits, usually photons. This concept forms the base for an alternative route to finding a scalable technology for building quantum computers in a distributed way. Actually, the ability to inter-convert stationary and flying qubits and also faithfully transmit the latter between distant locations is part of what is known as DiVincenzo requirements for the physical implementation of quantum computation and information. These requirements seem to provide the necessary resources for any useful use of quantum computers. In order to practically implement those networks, distributed quantum systems in typical cavity-QED settings have been considered in several papers. Proposals for the generation of entanglement between two spatially separated matter qubits, without direct interaction, have often made use of detection of another system, normally photons. Most of those schemes make use of either Λ-type three-level or four-level atoms/ions trapped in distant cavities where the entanglement between them is established via interference induced effects. Bose et. al and Cabrillo et. al. independently made the seminal proposals. In Bose et. al scheme, atoms inside lossy cavities become entangled by Bell measurements on photons escaping from the cavities (achieved with the use of beamsplitters) and entanglement swapping. In Cabrillo et. al proposal, entanglement is created by driving the atoms with a laser pulse and detecting subsequent spontaneous emission; Feng et.al. proposed a scheme using interference of photons leaking out the cavities, and a generalization for 3-qubits and GHZ/W state generation has been proposed by Ou et. al. Recently Barrett and Kok used Bose et. al scheme to propose scalable distributed QC with nondeterministic entangling operations, and Lim et al proposed a repeat-until-success gate operation allowing to eventually perform it in a deterministic fashion. Despite the evident importance and applicability of such probabilistic and repeat-until-success schemes, it is always desirable to have a non-probabilistic way of generating entanglement and related gate operations. In this context, Clark et al have proposed a scheme to unconditional preparation of entanglement between atoms trapped in separate cavities by employing quantum reservoir engineering in a appropriate cascade cavity-QED setting.

Here, we propose a new unconditional generation method of the maximally entangled state (Bell-type state)

\[ |\Psi_{1,2}\rangle = \frac{1}{\sqrt{2}}(|e_1 \rangle g_2 + |g_1 \rangle e_2\rangle), \tag{1} \]

between two distant two-level trapped ions (1 and 2), where \( |e_i\rangle \) and \( |g_i\rangle \) stand for the electronic excited and ground state of the ion \( i \). This non-probabilistic generation protocol makes use of a flying two-level atom sequentially interacting with both ions and promoting the establishment of entanglement between them. The current cavity-QED experiments already manipulate flying atoms with high degree control and employ it to implement quantum dynamics. On the other hand, the trapping and local laser manipulation of ions has also improved and many important experiments have been performed. Therefore, thinking of an union of both settings seems to be a very promising idea and it is the very core of our proposal. Although there are many proposals for quantum state manipulation or quantum computing using trapped ions in cavities, our scheme seems to be the first one based on the interaction of flying atoms with trapped ions.

In this letter, we show how the state \( |\Psi\rangle \) may be perfectly generated in the ideal case, and then we bring imperfections into the scene by including dissipation and dephasing for the flying qubit (atom) during its course
between the cavities containing stationary qubits (ions). The establishment of entanglement between distant parties forms a quantum channel which in association with classical communication may be used for several applications such as superdense coding [18] and quantum teleportation [19]. Thus, we evaluate the fully entangled fraction [20] which is directly related to the fidelity of those applications [21, 22].

The system under consideration consists of two distant cavities A and B, each of them containing one trapped ion inside, 1 and 2, and a flying two-level atom crossing both cavities, as shown schematically in Figure 1.

\[ H = H_0 + H_{\text{int}}, \]

where

\[ H_0 = \hbar \omega \hat{a}^\dagger \hat{a} + \frac{1}{2} \hbar \omega_{1(2)} \hat{\sigma}_z^{(2)} + \frac{1}{2} \hbar \omega_f \hat{\sigma}_z, \]

and

\[ H_{\text{int}} = \hbar g_1 \hat{\sigma}_+ \hat{b}^\dagger + \hbar g_2 \hat{\sigma}_- \hat{b}, \]

where the indexes 1, 2, and f are used to label the respective ions, and the flying atom. The \( \hat{\sigma}'s \) are the Pauli atomic operators, \( g_{1(2)}[f] \) is the coupling constant between the trapped ion 1(2)[flying atom] and the cavity mode, \( \omega_{1(2)}[f] \) is the atomic transition frequency of the ion 1(2)[flying atom], \( \hat{b}^\dagger (\hat{b}) \) is the creation (annihilation) operator of the cavity mode (frequency \( \omega \)), \( \hat{a}^\dagger (\hat{a}) \) is the bosonic operator of the ion vibrational mode (frequency \( \nu \)), and \( \eta = 2\pi a_0 / \lambda_0 \) is the Lamb-Dicke parameter, being \( a_0 \) the rms fluctuation of the ion’s position in the lowest trap eigenstate and \( \lambda_0 \) is the cavity field wavelength.

We are now going to use an effective Hamiltonian in the case which the trapped ions and the flying atom are kept far off-resonance with the field (\( \Delta = \omega - \omega_{1(2)}[f] \gg g_{1(2)}[f] \)). In this case, there is no energy exchange between the cavity field and the matter qubits. If one also chooses the frequencies of the system such that no sidebands are to be excited \( \Delta \neq k \nu \), with the \( k \) integer, and maintain the resonance between ions and flying atom (\( \omega_1 = \omega_2 = \omega_f \)), it is possible to end up with the following Hamiltonian in the interaction picture

\[ \hat{H}_{\text{int}}^{(2)} = \lambda_1(2) (\hat{\sigma}_+^{(2)} + \hat{\sigma}_-^{(2)}), \]

where \( \lambda_1(2) = g_{1(2)} g_f / \Delta \) is the effective coupling constant. For the sake of simplicity, we are going to set \( g_1 = g_2 = g \) which leads to \( \lambda_1 = \lambda_2 = \lambda \). The final form is essentially the Hamiltonian used by Zeng and Guo in [23]. This particularly simple form of the Hamiltonian has been obtained by also considering that both ions are initially cooled down to their lowest trap state. Otherwise, the motion would couple to the electronic degrees of freedom. Since the Hamiltonian does not depend on the cavity field, the Rabi frequency will be simply \( \lambda \) in A(B). This fact makes our scheme insensitive to the cavity field state and cavity decay [24, 25]. It would work even for a thermal field with a few photons. Now, consider that both ions are initially in their electronic ground state and the flying atom is in its excited state, i.e. \( |\psi(0)\rangle = |e_f, g_1, g_2\rangle \). According to [24], if the flying atom takes \( \lambda_A = \pi / 4 \) to cross cavity A, the state of the system will be

\[ |\psi(t_A)\rangle = \frac{1}{\sqrt{2}} (|e_f, g_1\rangle - i|g_f, e_1\rangle) \otimes |g_2\rangle. \]

Then, the atom is let to fly from one cavity to the other and it takes a time of flight \( t_f \). In the ideal case (without decoherence), the evolution of the three subsystems during the interval \( t_f \) is local and unitary leading to no change in the entanglement shared between them. However, if losses or dephasing are included, the entanglement shared between them will change due to the coupling with the environment. We will deal with these aspects later on in this paper. Now, the flying atom reaches cavity B, and after spending \( \lambda_B = \pi / 2 \) to cross it, the global state of the system will be

\[ |\psi\rangle = |g_f\rangle \otimes |\Psi_{1,2}\rangle, \]

where \( |\Psi_{1,2}\rangle \) is the Bell-type state [14]. We can see from [14] that at the end of the protocol, the state of the flying atom completely factorizes from the rest of the system which is left in a perfect maximally entangled state.
\[\Psi_{1,2}\]. Therefore, the scheme proposed here does not rely on probabilistic outcomes of any measurement process, thus being an unconditional generation protocol.

In real experiments, the flying atom in its way from one cavity to the other may collide with other atoms or molecules resulting in dephasing. Also, depending on how far the cavities are set from each other, the flying atom may spontaneously decay due to the coupling with the electromagnetic modes of free space. Once cavity decay does not destructively affect our scheme, as explained before, phase and amplitude damping of the flying atom seems to be the most important source of loss of quantum coherence here. We are now going to model such noise mechanisms by the standard method of master equations. The system master equation in the interaction picture describing spontaneous emission and phase damping of the flying atom in its way from cavity A to cavity B is given by

\[
\frac{\partial \hat{\rho}(t)}{\partial t} = \frac{\gamma}{2} \left[ 2\hat{\sigma}_z^f \hat{\rho}(t) \hat{\sigma}_z^f - \hat{\sigma}_z^f \hat{\sigma}_z^f \hat{\rho}(t) - \hat{\rho}(t) \hat{\sigma}_z^f \hat{\sigma}_z^f \right] \\
+ \frac{\gamma}{2} \left[ \hat{\sigma}_z^f \hat{\rho}(t) \hat{\sigma}_z^f - \hat{\rho}(t) \right],
\]

where \(\gamma\) is the atomic decay (dephasing) rate. This equation is to be solved with the initial condition \(\hat{\rho}(0) = |\psi(t_A)\rangle\langle\psi(t_A)|\), where \(|\psi(t_A)\rangle\) is the global state after the flying atom has left the cavity A. The solution of the master equation for such an initial condition is

\[
\hat{\rho}(t_f) = \frac{1}{2} \left[ |gf, e1\rangle\langle gf, e1| + ie^{-\gamma t_f} |gf, e1\rangle\langle ef, g1| \\
+ ie^{\gamma t_f} |ef, g1\rangle\langle gf, e1| \\
+ (1 - e^{-\gamma t_f}) |gf, g1\rangle\langle gf, g1| \right] \otimes |g2\rangle\langle g2|,
\]

where \(t_f\) is the time of flight. Now, the flying atom reaches cavity B, and there it follows a unitary evolution according to \(\mathcal{U}\). Considering the initial state of the system to be \(|\Psi\rangle\) and just like in the ideal case the atom still spends \(\Delta t = \pi/2\), one will obtain after tracing out the flying atom

\[
\hat{\rho}_{1,2} = \frac{1}{2} \left[ |e1, g2\rangle\langle e1, g2| + e^{-(\gamma + \gamma_p)t_f} |e1, g2\rangle\langle g1, e2| + \\
+ e^{-(\gamma + \gamma_p)t_f} |g1, e2\rangle\langle e1, g2| + e^{-\gamma t_f} |g1, e2\rangle\langle g1, e2| \\
+ (1 - e^{-\gamma t_f}) |g1, g2\rangle\langle g1, g2| \right].
\]

The state involving the spatially separated trapped ions is the result of our generation protocol when the flying atom employed to distribute entanglement is affected by amplitude and phase damping. The first thing to be noted is that the state has not as much entanglement as the Bell-type state generated in the ideal case. Actually, the entanglement in decreases exponentially with the time of flight as measured by the concurrence \(C\), which for is given by \(C(t_f) = e^{-2\gamma t_f}\).

If Alice and Bob share a two-qubit mixed state such as \(|\Psi\rangle\), they can try to teleport the unknown state of a third qubit using local quantum operations and classical communication (LQCC). It has been demonstrated that the optimal fidelity of teleportation \(F_{\text{max}}\) attainable using LQCC is connected to a quantity called fully entanglement fraction \(F_{\text{max}}\) which is defined as \(F_{\text{max}} = \max_{\Psi} \langle\Psi|\hat{\rho}_{1,2}|\Psi\rangle\). The maximization here is taken over all maximally entangled states \(|\Psi\rangle\), i.e. all states that can be obtained from a singlet using LQCC. The relation between both quantities is \(F_{\text{max}} = (2F_{\text{max}} + 1)/3\) \(F_{\text{max}}\). The fully entangled fraction may be readily evaluated writing \(\hat{\rho}_{1,2}\) in a suitable basis and finding the eigenvalues of its real part as suggested in \(\frac{1}{2} \max_{\rho_{1,2}}\). Following their receipt it is not difficult to see that for the noisy channel \(\hat{\rho}_{1,2}\) given by the fully entangled fraction reads

\[
F_{\text{max}} = \frac{1}{4} \left( 1 + e^{-\gamma t_f} + 2e^{-(\gamma + \gamma_p) t_f} \right).
\]

A classical channel can give at most a fidelity equal to \(2/3\) that is achieved when Alice simply performs a measurement on the unknown qubit and tells Bob the outcome. It follows that in order to gain a real improvement coming from quantum mechanics one must have \(F_{\text{max}} \geq 1/2\). For special cases, we can get simple but very important upper limits for the time of flight \(t_f\). For the pure amplitude damping channel, for instance, one must have \(t_f \leq \ln(3)\gamma^{-1}\), and for equal dephasing and damping \((\gamma_p = \gamma)\) channels, \(t_f \leq \ln(2)\gamma^{-1}\). Different choices of \(\gamma\) and \(\gamma_p\) lead to other maximal values of \(t_f\) obtained from \(F_{\text{max}}\). As an example, should one consider the flying qubit to be a Rydberg atom with principal quantum number \(n \approx 50\) having \(\gamma \approx 2 \times 10^2 s^{-1}\), it will lead to \(t_f^{\max} = 5\) ms for the pure amplitude damping case. Considering the typical velocity of the flying Rydberg atom to be \(v \approx 3 \times 10^2 m/s\), the maximal distance between the cavities and so the distance between the entangled pair of ions should be around 1.5m. This example indicates that our scheme might be useful for generating entangled pairs for performing quantum teleportation involving macroscopically separated parties. Rather than using atomic highly excited states as flying qubits, another possibility would be the use of atomic qubits of optical frequency. This would be specially suitable for the interaction with \(^{40}\text{Ca}^-\)-qubits \(|S_{1/2}/2, e_1, 2 = D_{3/2}/2\rangle\) inserted in optical cavities.

In summary, we have shown a new scheme to generate entangled states involving distant parties. To our knowledge, it is the first proposal based upon the interaction of flying atoms with trapped ions. Though the cavity is used to induce an indirect interaction between them, our scheme is robust against cavity losses and insensitive to the cavity field state. In this case, our scheme results in the unconditional generation of a perfect Bell-type state. We have also considered a situation where spontaneous atomic decay and dephasing are included. For this realistic setting, we have discussed limits of the applicability of the generated state for performing quantum teleportation. Our findings seem to indicate that our proposal...
might be useful for the generation of distributed entanglement over macroscopic distances.

Acknowledgments

FLS would like to thank R. Blatt for valuable discussions; RJM acknowledges financial support from CAPES, FLS from FAPESP under project 05/04533–7 and KF to CNPq (partial) under project #300651/85–6.