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DEEP INELASTIC PHOTON - PHOTON SCATTERING
IN AN ASYMPTOTICALLY FREE GAUGE THEORY

M.A. Ahmed *)
I.C.T.P., Trieste
and
G. G. Ross
CERN -- Geneva

A B S T R A C T

Two-photon scattering is analyzed assuming the strong interactions are described by an asymptotically free gauge theory. The box diagram is found to be enhanced relative to the "strong" contribution in the deep inelastic region but only as a power of $\ln \nu$. An exception is the structure function W_3 in which the box diagram dominates like a power of ν . The validity of a fixed pole sum rule is also considered.

*) Permanent address : University of Khartoum, Khartoum.

It has been observed ¹⁾ that the cross-section for the two-photon process

$$e^+ + e^- \rightarrow e^+ + e^- + \text{hadrons} \quad (1a)$$

can be large and will dominate the annihilation process $e^+e^- \rightarrow \gamma \rightarrow \text{hadrons}$ at high energies. This process allows the study of $\gamma\gamma$ interactions. The amplitude depends on three independent variables q^2 , p^2 and $\nu = p \cdot q$ where q and p are the momenta of the virtual photons, see Fig. 1a. $\gamma\gamma$ scattering may also be studied via the reactions

$$e^- + e^- \rightarrow e^- + e^- + \text{hadrons} \quad (1b)$$

and

$$e^+ + e^- \rightarrow \gamma + \text{hadrons} \quad (1c)$$

(see Figs. 1a and 1b)

In the reactions (1a) and (1b), the cross-section drops fast at high momenta due to the photon propagators and is difficult to measure when both q^2 and p^2 are large. It is more practical to look at the situation where one electron (say the q electron) is detected at large four-momentum and the other (nearly real) remains inside the small momentum transfer forward peak. We will be principally interested in this region.

In the reaction (1c), the produced photon is, of course, real. Other two photon contributions to this reaction may be removed using knowledge of the process $e^+e^- \rightarrow \text{hadrons}$.

We are interested in the deep inelastic region ($p^2 \simeq 0$, $-q^2 \rightarrow \infty$), $\nu = p \cdot q \rightarrow \infty$, $\omega = -2\nu/q^2 = \text{const.}$, $1 < \omega < \infty$) where counting rates should be high. Several authors ²⁾ have discussed what is expected for these processes in this limit using the parton model or light-cone expansions. They find that two types of contribution are to be expected. The first, the "strong" contribution, corresponds to vector meson dominance of the (nearly real) photon and the scattering then measures the deep inelastic structure

functions of the vector mesons. These structure functions scale in the Bjorken limit. The second type of contribution corresponds to the case that the (nearly real) photon couples in a pointlike manner to partons. This term, the box term, violates Bjorken scaling by a $\log[v^2/q^2 p^2]$ factor and also gives non-vanishing contributions to the equivalent of longitudinal structure functions in electroproduction, even for spin $\frac{1}{2}$ constituents.

Thus in the infinite q^2 limit, the box term should dominate. At finite (but large) q^2 the relative magnitude of the two terms is at present unknown. It has been suggested that look for the box terms one should study the longitudinal type structure functions. In the parton model, the strong contribution should vanish like $1/v$. The equivalent term in electroproduction is already only $\sim 10\%$ of the transverse structure function at $v = 2 \text{ GeV}^2$. One might hope that the box term may dominate here.

In this note we consider what happens if the strong interactions are described by an asymptotically free gauge theory (AFGT) coupled to the quark colour³⁾. In electroproduction it is known that such models give Bjorken scaling up to powers of $\log q^2$ ⁴⁾. Here again we find Bjorken scaling terms modified by powers of $\log q^2$. We compute these powers. We also find the box term which, in leading order, is unmodified by $\log q^2$ terms.

From an analysis of electroproduction, as noted above, the longitudinal terms are expected to be small at $v \sim 2 \text{ GeV}^2$. Thus at this value of v the box term may dominate the structure function W_L . If it does not, however, it is unlikely that going to high v will help as we find only $\log v$ dominance of the box term. This is in contrast to the parton model result which gives a dominance like v to the box term.

However, we do find that for the structure function W_3 , defined below, the box term should dominate like a power of v . Study of W_3 should be the best way of isolating the box contribution.

The cross-sections for the processes (1a), (1b) and (1c), where the hadronic states are summed, may be expressed in terms of $W_{\mu\nu\text{pt}}$, the absorptive part of the forward $\gamma\gamma$ scattering amplitude (see Fig. 2)

$$W_{\mu\nu\rho\tau} = \frac{1}{2} \int dx dy dz \exp \{ i q \cdot x + i p \cdot (y-z) \} \\ \langle 0 | T^* (J_\mu(x) J_\rho(y)) T(J_\nu(0) J_\tau(z)) | 0 \rangle_{(2)}$$

where $J_\mu(x)$ is the electromagnetic current.

The matrix element in Eq. (2) is a function of six mutual distances. The phase oscillations in the exponential leads, using the usual light cone arguments, to the dominance of the integral by certain regions in this space. The regions dominating depend on the values of the kinematic variables p^2 , q^2 and s , and they have been given by Chernyak and Serbo²⁾.

We first consider the choice

$$\mu^2 \ll s \ll p^2 q^2 / \mu^2 \quad ; \quad |q^2| \gg \mu^2 \quad (3)$$

where μ is a mass setting the scale for the onset of scaling ($\mu^2 \approx 1 \text{ GeV}^2$). For p^2 small s will not be very large so only near $\omega = 1$ will this region overlap the deep inelastic region [since $s = p^2 + (1-\omega)q^2$]. However, we consider it first since it illustrates the origin of the box term.

For this choice of kinematics all six distances are small for the significant part of the integral in Eq. (2). We want therefore the matrix element in Eq. (2) for short distances only. To obtain this we apply the renormalization group equations⁵⁾ directly to the Green's function $\Gamma(x,y,z,g)$ for the scattering of four photons.

Here g is the coupling constant of the strong interaction gauge group G . All mass terms have been dropped. G is taken to be $SU(n)$ coupling to colour indices on quark fields. The colour group is assumed to commute with internal symmetry indices such as isospin and hypercharge. Then the solution of the renormalization group equations is

$$\Gamma(x/\lambda, y/\lambda, z/\lambda, g) = [1 + 2b\bar{g}^2 \ln \lambda]^\gamma \Gamma(x, y, z, \bar{g}) \quad (4)$$

where b is a number determined by the group G , γ is an anomalous dimension determined from the renormalization group equations and $\bar{g}$ is the effective strong coupling constant. $\bar{g}$ is given, in an AFGT, by ³⁾

$$\bar{g}^2(t) = \frac{g^2}{1 + 2bg^2 t} \quad ; \quad t = \ln \lambda \quad (5)$$

Note that $\bar{g}^2 \rightarrow 0$ as $t \rightarrow \infty$ so that for large t we may use a perturbation expansion to determine $\Gamma(x, y, z, \bar{g})$.

For the product of four currents the leading term is the box graph of Fig.

In this case the anomalous dimension γ is zero, the product of the four currents being proportional to the unit matrix. Corrections to this term arise from the exchange of colour vector gluons and are of order $\bar{g}^2$ and above. Consequently they vanish like $\log q^2$ [cf. Eq. (5)]. For a computation of the graph in Fig. 3, see, for instance, Ref. 6). The contributions of relevance in the deep inelastic region are given below.

We now turn to the Bjorken scaling region

$$\begin{aligned} |p^2| &\leq \mu^2 \\ |q^2| &\gg \mu^2 \\ s &\gg p^2 q^2 / \mu^2 \end{aligned} \quad (6)$$

In this case only $(x-0)^2$ is necessarily small due to phase cancellation arguments. The other five distances may be small or not. In the case that they are small the analysis given above again applies and there is a term corresponding to the box diagram of Fig. 3.

If some distances are not small we cannot apply the renormalization group techniques directly. It is necessary first to make an operator product expansion of the currents that are at short distances and apply the renormalization group to the coefficients of each of the operators entering in the expansion.

We first expand $W_{\mu\nu\rho\tau}$ on a tensor basis ⁷⁾

$$\begin{aligned}
 W_{\mu\nu\rho\tau} = & (-W_1 g_{\mu\nu} + W_2 p_\mu p_\nu) g_{\rho\tau} \\
 & + W_3 \left[\left(-g_{\mu\rho} + \frac{p_\mu q_\rho}{p \cdot q} \right) \left(-g_{\nu\tau} + \frac{p_\nu q_\tau}{p \cdot q} \right) + \left(-g_{\nu\rho} + \frac{p_\nu q_\rho}{p \cdot q} \right) \right. \\
 & \left. \left(-g_{\mu\tau} + \frac{p_\mu q_\tau}{p \cdot q} \right) \right] + W_4 \left[\left(-g_{\mu\rho} + \frac{p_\mu q_\rho}{p \cdot q} \right) \left(-g_{\nu\tau} + \frac{p_\nu q_\tau}{p \cdot q} \right) \right. \\
 & \left. - \left(-g_{\nu\rho} + \frac{p_\nu q_\rho}{p \cdot q} \right) \left(-g_{\mu\tau} + \frac{p_\mu q_\tau}{p \cdot q} \right) \right] \quad (7)
 \end{aligned}$$

Here we have dropped terms proportional to q_μ , q_ν , p_μ and p_ν . Since we are working with $p^2 \simeq 0$ only four structure functions contribute.

Measurements of the Bjorken scaling region with unpolarized electrons determine W_1 , W_2 and W_3 . W_1 and W_2 are related to the usual structure functions as defined for any unpolarized target. W_3 is proportional to the difference between the cross-sections when the two photons are polarized in the same plane and the cross-section when they are polarized in perpendicular planes in their centre-of-mass system. In order to measure W_4 both the electron beams must be transversely polarized ¹⁾ in reaction (1a) or (1b). For reaction (1c) W_4 may be measured by observing circularly polarized final photons from annihilations of transversely polarized electrons on unpolarized positrons ⁷⁾.

The light cone expansion for the currents at short distances is

$$J_\mu(x) J_\nu(0) = (-g_{\mu\nu} \square + \partial_\mu \partial_\nu) \frac{1}{x^2 - i\epsilon x_0} \sum_{n=0}^{\infty} \sum_i \frac{1}{n!} \partial_\mu^n \partial_\nu^n$$

$$\begin{aligned}
 & C_{i,1}^{(n)}(x^2 - i\epsilon x_0, g) O^{i\mu_1 \dots \mu_n}(0) x_{\mu_1} \dots x_{\mu_n} \\
 & - (g_{\mu\lambda} g_{\nu\sigma} \square - g_{\mu\nu} \partial_\lambda \partial_\sigma - g_{\nu\sigma} \partial_\mu \partial_\lambda + g_{\mu\sigma} \partial_\lambda \partial_\nu) \\
 & \sum_{n=0}^{\infty} \sum_i C_{i,2}^{(n)}(x^2 - i\epsilon x_0, g) O^{i\lambda\sigma\mu_1 \dots \mu_n}(0) x_{\mu_1} \dots x_{\mu_n} \\
 & \text{(new)} \\
 & - i\epsilon_{\mu\nu\lambda\sigma} \partial^\lambda \frac{1}{x^2 - i\epsilon x_0} \sum_{n=0}^{\infty} \sum_i E_{i,1}^{(n)}(x^2 - i\epsilon x_0, g) \\
 & R_i^{i\sigma\mu_1 \dots \mu_n}(0) x_{\mu_1} \dots x_{\mu_n} + \text{nonleading terms} \quad (8)
 \end{aligned}$$

The sum i is over all operators of equal twist contributing to the product of two currents.

It is evident that W_1 and W_2 , which are given by the operators $O^{i\mu_1 \dots \mu_n}$ and $O^{i\lambda\sigma\mu_1 \dots \mu_n}$ have the same q^2 behaviour as their counterparts in unpolarized electroproduction. The prediction of this behaviour in an APCT is well known ⁴⁾.

We will be concerned mainly with the "new" structure functions W_3 and W_4 which we expect to provide the new physics interest as far as the "strong" contribution is concerned. For completeness, however, we include all structure functions in the following analysis.

The matrix elements of the operators are taken between the (nearly real) spin-one photon states. We write

$$\langle p, \phi^a | O_{\mu_1 \dots \mu_n}^i | p, \phi^c \rangle = \phi_\mu^a(p) M_{\mu_1 \dots \mu_n}^{i\mu p} \phi_\rho^c(p) \quad (9)$$

where ϕ_τ^c is a polarization vector. With a similar notation for the other operators we have the following matrix elements

$$M_{\mu_1 \dots \mu_n}^{i\mu p} = (i)^n g^{\mu p} p_{\mu_1} \dots p_{\mu_n} a_n^i(p^2) + \text{nonleading terms} \quad (10)$$

$$\begin{aligned}
 M_{\lambda\sigma\mu_1\ldots\mu_n}^{i\ell\rho} &= (i)^n g^{\ell\rho} p_\lambda p_\sigma p_{\mu_1}\ldots p_{\mu_n} a_n^i(p^2) \\
 &+ (i)^n S\{g_\lambda^\ell g_\sigma^\rho p_{\mu_1}\ldots p_{\mu_n}\} b_n^i(p^2) \\
 &+ \text{nonleading terms}
 \end{aligned} \tag{11}$$

where S denotes symmetrization with respect to the indices $\lambda, \sigma, \mu_1 \dots \mu_n$

$$\begin{aligned}
 M_{\sigma\mu_1\ldots\mu_n}^{i\ell\rho} &= (i)^n \epsilon_{\sigma\gamma}^{\ell\rho} p^\gamma p_{\mu_1}\ldots p_{\mu_n} C_n^i(p^2) \\
 &+ \text{nonleading terms}
 \end{aligned} \tag{12}$$

The structure functions defined in Eq. (7) are related to the Fourier transforms of the coefficients appearing in Eq. (8), times the matrix elements of the operators as defined above. This gives the sum rules

$$\begin{aligned}
 \int_1^\infty d\omega \omega^{-n-1} W_1^{\text{str}}(\omega, q^2, p^2) &= \sum_i a_n^i(p^2) \left\{ \tilde{C}_{i,1}^n(q^2, g) \right. \\
 &\quad \left. + \frac{1}{2} \tilde{C}_{i,2}^{n-2}(q^2, g) \right\} \\
 \int_1^\infty d\omega \omega^{-n-2} \nu W_2^{\text{str}}(\omega, q^2, p^2) &= \sum_i a_n^i(p^2) \tilde{C}_{i,2}^n(q^2, g) \\
 \int_1^\infty d\omega \omega^{-n-2} \nu W_3^{\text{str}}(\omega, q^2, p^2) &= \sum_i b_n^i(p^2) \tilde{C}_{i,2}^n(q^2, g) \\
 \int_1^\infty d\omega \omega^{-n-2} W_4^{\text{str}}(\omega, q^2, p^2) &= \frac{1}{2} \sum_i C_n^i(p^2) \tilde{E}_i^{(n)}(q^2, g)
 \end{aligned} \tag{13}$$

The superscript "str" is to remind us that this is the part with x small, where the soft photon behaves as a vector meson target.

The Fourier transforms are defined by

$$\begin{aligned}\tilde{C}_{i,1}^n(q^2, g) &= \frac{i}{4} (-q^2)^{n+1} \left(\frac{\partial}{\partial q^2} \right)^n \int d^4x e^{iq \cdot x} \frac{1}{x^2 - i\epsilon} C_{i,1}^{(n)}(x^2 - i\epsilon, g) \\ \tilde{C}_{i,2}^n(q^2, g) &= \frac{i}{8} (-q^2)^{n+2} \left(\frac{\partial}{\partial q^2} \right)^n \int d^4x e^{iq \cdot x} C_{i,2}^{(n)}(x^2 - i\epsilon, g) \\ \tilde{E}_i^n(q^2, g) &= \frac{i}{4} (-q^2)^{n+1} \left(\frac{\partial}{\partial q^2} \right)^n \int d^4x e^{iq \cdot x} \frac{1}{x^2 - i\epsilon} E_i^{(n)}(x^2 - i\epsilon, g)\end{aligned}\tag{14}$$

In order to study the small x behaviour, we apply the renormalization group techniques to the operator product expansion of Eq. (8) ⁴⁾. The Wilson coefficients obey a renormalization group equation whose solution may be written in the form

$$\tilde{C}_{i,1}^n(q^2, g) = \tilde{C}_{i,1}^n(1, \bar{g}(t, g)) \exp \left[- \int_0^t dx \gamma_{i,1}^n(\bar{g}(x, g)) \right]\tag{15}$$

where

$$t = \frac{1}{2} \ln \left[\frac{-q^2}{\mu^2} \right]$$

and similarly for $\tilde{C}_{i,2}^n$ and $\tilde{E}_i^n$. The determination of the anomalous dimensions for the operators of Eq. (8) given by a coloured quark model with colour vector gluons has been done in Ref. 4).

For the $SU(3)$ non-singlet piece of the structure functions only one operator contributes ($i=0$) and we find on integrating Eq. (15)

$$\begin{aligned}\tilde{C}_{0,j}^n(q^2/\mu^2, g) &= \tilde{C}_{0,j}^n(1, \bar{g}) \left[1 + b g^2 \ln \frac{q^2}{\mu^2} \right]^{-A_{n,j}} \\ \tilde{E}_0^n(q^2/\mu^2, g) &= \tilde{E}_0^n(1, \bar{g}) \left[1 + b g^2 \ln \frac{q^2}{\mu^2} \right]^{-B_n}\end{aligned}\quad (16)$$

where

$$\begin{aligned}B_{n+1} &= A_{n,1} = A_{(n-2),2} \\ &= \frac{3C_2(R)}{11C_2(G) - 4T(R)} \left[1 - \frac{2}{n(n+1)} + 4 \sum_{k=2}^n \frac{1}{k} \right]\end{aligned}$$

For the SU(3) singlet piece $A_{2,1} = A_{0,2} = 0$ corresponding to $\gamma = 0$ for the energy momentum tensor. For other values of n , and for B for all values of n , Eq. (16) is a good approximation. The exact expressions are given in Ref. 4).

Since $\tilde{C}_{0,j}^n(1, \bar{g})$ and $\tilde{E}_0^n(1, \bar{g})$ in Eq. (16), in lowest order in $\bar{g}$, are given by the free quark model, we recover the Bjorken scaling results up to the $\log q^2$ factors.

To this we have to add the contribution due to the box graphs. These are

$$\begin{aligned}W_1^{\text{box}} &= - \frac{\langle Q^4 \rangle}{4\pi} \left[\ln \frac{4\nu^2}{q^2 p^2} - 2 \right] \left(1 - \frac{2(\omega-1)}{\omega^2} \right) + \frac{1}{\omega^2} \\ W_L^{\text{box}} &= W_1^{\text{box}} - \frac{\omega}{2} \nu W_2^{\text{box}} = \frac{\langle Q^4 \rangle}{\pi} \frac{(\omega-1)}{\omega^2} \\ W_3^{\text{box}} &= \frac{\langle Q^4 \rangle}{2\pi} \left(-\frac{1}{2\omega^2} \right)\end{aligned}\quad (17)$$

$$W_4^{\text{box}} = \frac{\langle Q^4 \rangle}{4\pi} \left(\ln \frac{4y^2}{q^2 p^2} - 2 \right) \left(\frac{2}{\omega} - 1 \right)$$

where $\langle Q^4 \rangle$ is the sum of the quark charges to the fourth power.

As $p^2 \rightarrow 0$, the box contribution for W_1 and W_4 has an infra-red singularity. This occurs since we have been ignoring all masses; the renormalization group equations we have used being applicable only in the deep Euclidean region. To apply the results at $p^2 = 0$ we must continue outside the region of validity of our equations. It is clear that in the continuation a cut-off will occur due to some effective mass. If one introduces a cut-off via a quark mass m_q then when $p^2 \rightarrow 0$, Eqs. (17) are modified for W_1^{box} and W_4^{box} to read⁽²⁾

$$W_1^{\text{box}} = - \frac{\langle Q^4 \rangle}{4\pi} \left[\left(\ln \frac{s}{m_q^2} - 2 \right) \left(1 - 2 \frac{\omega-1}{\omega^2} \right) + \frac{(2\omega-1)}{\omega^2} \right]$$

$$W_4^{\text{box}} = \frac{\langle Q^4 \rangle}{4\pi} \left[\left(\ln \frac{s}{m_q^2} - 2 \right) \left(\frac{2}{\omega} - 1 \right) + 1 \right] \quad (18)$$

(We ignore here the possibility that m_q may itself have a logarithmic momentum dependence due to renormalization effects.)

The box terms, which violate Bjorken scaling in W_1 (and W_4) by a $\log s$ ($\equiv \log[(1-\omega)q^2]$) term will ultimately dominate the strong terms which are suppressed relative to Bjorken scaling by powers of $\log q^2$ [see Eq. (16)]. This may not happen until $\log q^2$ is very large, possibly beyond the range of present experiments. To have the best chance of seeing the box contribution we should look at combinations of the structure functions that have small contributions from the strong part⁽⁴⁾. Since in Eq. (16) $\tilde{G}_{0,j}^n(1, \bar{g})$ and $\tilde{E}_0^n(1, \bar{g})$ are given to zeroth order in $\bar{g}$ by the free quark model, they will satisfy parton model relations in this order. In particular if charged spin $\frac{1}{2}$ constituents only are present we have the generalization of the Callan-Gross relation to $\gamma\gamma$ scattering

$$\int_1^\infty d\omega \omega^{-n-2} W_L^{\text{str}}(\omega, q^2) \underset{-q^2 \rightarrow \infty}{=} 0 + O(\bar{g}^2) \quad (19)$$

The correction to the Callan-Gross relation is of order $\bar{g}^2$ and by Eq. (5) we see that the moments in Eq. (19) tend to zero like inverse powers of $\log q^2$. What this implies for the structure functions is known at least for the SU(3) non-singlet part. In this case the operators contributing to W_L^{str} are the same as those contributing to W_2^{str} . The operator matrix elements may therefore be obtained from W_2^{str} and, evaluating $\tilde{C}_{0,j}^n(1, \bar{g})$ to order $\bar{g}^2$, we may solve for W_L^{str} in terms of W_2^{str} . This gives for the SU(3) non-singlet piece (8), (9)

$$W_L^{\text{str}}(\omega, q^2) = 4C_2(R) \frac{\bar{g}^2}{16\pi^2} \frac{1}{\omega} \int_1^\omega d\omega' \omega' \nu W_2^{\text{str}}(\omega', q^2) \quad (20)$$

From an analysis of electroproduction ⁹⁾ we expect $4C_2(R)(\bar{g}^2/16\pi^2)$ to be less than 0.2 at $-q^2 = 2 \text{ GeV}^2$. We may hope that this suppression will mean the box contribution dominates. For q^2 increasing $\bar{g}^2 \rightarrow 0$ as $(\log q^2)^{-1}$ but the integral in Eq. (20) may increase or decrease. For small ω , νW_2 decreases (like a power of $1/\ln q^2$) and so the integral will decrease. For large ω , νW_2 increases. The box contribution of Eq. (18) remains constant.

Thus we may expect at small ω an improvement in the ratio of the box to strong contributions, but this will not be nearly as dramatic as the parton model prediction which gives a q^2 rise for the ratio. For large ω we cannot even expect a $\log q^2$ suppression of the strong term relative to the box term.

Of course, Eq. (20) and the consequent discussion only applied to the SU(3) non-singlet piece of the structure function. However, as discussed in Ref. 8), the analysis may be extended to the SU(3) singlet piece to obtain a lower bound for the strong contribution with similar conclusions.

In comparison to the rather slow dominance of the box term in W_L , we may see from Eqs. (17) and (19) that in W_3 the box term gives a scaling contribution to W_3 but that the strong term gives a scaling contribution (up to $\log q^2$ terms) to νW_3 . Thus for high ν the contribution will be suppressed relative to the box contribution by a $1/\nu$ factor. This result is unaffected by higher order (in $\bar{g}^2$) contributions to W_3 . A study of W_3 for large values of ν should then give the best hope for seeing the box contribution on its own.

Finally we turn to a sum rule proposed by Worden ¹⁰⁾ for W_3 . In the absence of fixed poles and high-lying Regge cuts this structure function would satisfy a superconvergence relation

$$\frac{1}{\pi} \int_{\nu_0}^{\infty} \frac{d\nu}{\nu} W_3 = 0 \quad (21)$$

However, he finds that in the parton model the box diagram of Fig. 3 gives a $\sigma=0$ fixed pole which contributes to the sum rule so that it becomes

$$\frac{1}{\pi} \int_{\nu_0}^{\infty} \frac{d\nu}{\nu} W_3 = -\frac{1}{2\pi^2} \langle Q^4 \rangle \quad (22)$$

The fixed $J=0$ poles arising from the strong contribution he finds do not contribute to W_3 . As Eq. (22) is proposed, it would be correct for all values of q^2 since the residues of fixed poles are expected to be polynomials in q^2 and only a constant everywhere can be a constant at $q^2 = \infty$.

What can be said about the sum rule Eq. (22) in an AFGT? The large ν behaviour of the strong contribution to the structure functions is governed by the nearest n singularity in the moments given by Eq. (13). The anomalous dimensions for the $SU(3)$ singlet piece has a pole at $n = -1$ ⁴⁾. This leads ¹¹⁾ to a growth in W_3 with ν , faster than any power of $\log \nu$ but slower than a power which, if actually present, would violate the sum rule. However, higher order (in $\bar{g}^2$) terms are of the same order of magnitude in the Regge region and we may hope that they sum to give the conventional Regge singularity ¹²⁾. Then with these singularities subtracted the strong contribution would satisfy Eq. (21).

The remaining term, the box contribution, is to zeroth order in $\bar{g}^2$ the same as the parton model result. It will then lead to Eq. (22) for the full structure function W_3 . However, non-leading terms from the box contribution may also be expected to contribute to Eq. (22). Thus we expect the sum rule to be approached only logarithmically in $\log q^2$ rather than to be universally true. The reason the sum rule fails for low q^2 is that these higher order terms introduce fixed poles with non-polynomial residues, in contradiction to the usual assumptions.

In conclusion we have found that the deep inelastic structure functions of the photon should, in an asymptotically free gauge theory, exhibit two parts. The first is a hadronic-like part where the photon interacts as though it were a vector meson with a transverse momentum cut-off. The second is characteristic of the photon, the pointlike coupling part of the soft photon leading to the box term. Isolation of the second part should be easiest by looking at W_3 for large ν . A measurement of this part would give a measurement of the sum of the charges to the fourth power of the fundamental constituents entering in deep inelastic scattering.

The strong contribution is expected to be significant in W_1 and W_4 and also W_L although suppressed in the latter by a factor $\lesssim 0.2$.

Finally, the fixed pole sum rule is difficult to justify in an AFGT, higher order terms introducing fixed poles with non-polynomial residues. Whether it is possible to avoid these, or if not, how they can be reconciled with conventional ideas about fixed poles, is unclear at present.

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FIGURE CAPTIONS

Figure 1a Two-photon contribution to $e^+e^- \rightarrow e^+e^- + \text{hadrons}$.

Figure 1b The two-photon contribution to $e^+e^- \rightarrow \text{hadrons}$ related to $\gamma\gamma$ scattering.

Figure 2 The forward $\gamma\gamma$ scattering amplitude.

Figure 3 Zeroth order (in $\bar{g}$) contribution to $\Gamma(x,y,z,\bar{g})$.

FIG.1a

FIG.1b

FIG.2

FIG.3