THE M.I.T. BAG 1975

Victor F. Weisskopf
M.I.T. and CERN, Geneva

ABSTRACT

This is a review of the M.I.T. bag model and its recent advances, in particular of the successful calculations of the masses of the lowest baryon and meson octets and of the baryon decuplet. Effects of the quark kinetic energy, bag energy, strange quark mass and coloured gluon exchange in lowest order are included. The latter effects give rise to a spin-spin interaction. Magnetic moments, charge radii and weak decay constants are also calculated. A reasonable agreement is found with only four adjustable constants. The reported work was done by T. DeGrand, R. Jaffe, K. Johnson and J. Kiskis.
I. INTRODUCTION

The M.I.T. bag description of hadrons was greatly developed since I reported it here at Erice in 1974. Today I will give a personal interpretation of the fundamental ideas and of the recent developments. My interpretation may or may not be accepted by the authors and developers of this theory: A. Chodos, T. DeGrand, R.L. Jaffe, K. Johnson, J. Kiskis and C. Thorn, naturally all of M.I.T.\(^1\). I have not contributed much except what could be referred to as the necessary don't-know-how.

The M.I.T. bag is a way to describe the strong interactions between quarks. So far the model was successful only in explaining some static properties of the low-lying hadron states. There exists an interesting attempt by F. Low to extend it to the scattering of hadrons at high energy (see Erice Lectures, 1974). We will not discuss it here.

It is assumed that the hadrons consist of quarks in the usual way (three for baryons, a quark-antiquark pair for mesons), and that there are interactions between the quarks. These interactions are considered to be strong and "soft", i.e., very strong for small momentum transfers \(\Delta q\) and considerably weaker for large ones. The transition lies around \(\Delta q \sim 0.2\text{ GeV}/c\) or \(1\text{ (fermi)}^{-1}\). In particular the strong effects at low \(\Delta q\) are responsible for the confinement of the quarks; they produce an attractive force which increases strongly with distance, such that quarks cannot be separated.

In my interpretation of the model, the bag is a particularly simple phenomenological way of describing the low \(\Delta q\) effects of the strong interaction. One assumes that the quarks are "enclosed" in a finite volume \(V\) which, in the simple cases treated here, will be a sphere of radius \(R\). The confinement is expressed by a boundary condition that the quark current across the boundary of the volume \(V\) be zero. The size of the volume is variable and there is an energy term \(B\cdot V\) proportional to the volume, expressing in the simplest possible way the action of the confining forces. The short-range (high \(\Delta q\)) effects of the strong interaction will be considered as a weak "residual" interaction which can be treated by perturbative methods. It will be expressed in the form of a gluon vector field coupled to the colour of the quarks.

In some respects, this interpretation establishes a parallel to the treatment of the nuclear forces in the shell model. There also the strong over-all long-range effects of the nuclear attraction are expressed by a common potential well, albeit one with fixed radius and finite depth. This well is a close analogy to the bag. The residual short-range effects are then added by a perturbation treatment to the freely moving nucleons in the well. Still, the nuclear physicists are convinced that both, well and residual forces, are a description of two aspects of the same force.

I am going to describe what I believe is the present state of this theory, by proceeding in two consecutive steps. In the first step, we consider the quarks as massless and free within the bag. In this step the SU₆ symmetry is not yet broken. One gets a reasonable description of the nucleon, its size, magnetic moment and $g_A/g_V$ ratios. In the second step we introduce a gluon field with massless gluons coupled to colour, in analogy to the electromagnetic field. We treat this interaction to first approximation in the coupling constant $g^2/4\pi$ which we suppose is smaller than unity. We assume that the gluon field also is confined in the bag volume. This is achieved by a boundary condition expressing the condition that the "glu-electric" field lines (the analogue of the electric field lines) must never cross the confining surface.

We note here three consequences following from the introduction of the gluon field and will give the explanations later on:

1) only "colour-less" quark combinations can exist in a confining bag;

2) the zero-point oscillations of the gluon and quark fields contribute an infinite positive energy proportional to the volume whose renormalized value may be the source of the B+V bag energy. Furthermore, they give rise to a characteristic negative energy term $-Z/\mathcal{R}$, where $\mathcal{R}$ is the radius of the bag;

3) there is a spin-spin interaction between quarks, splitting the SU₆ degeneracies.

It will turn out that the low-lying mass spectrum of the baryons and mesons, the magnetic moments, and the $g_A/g_V$ ratios are reasonably well reproduced if, in addition to the afore-mentioned points, a finite mass $m_s$ is ascribed to the strange quark. There are four adjustable constants in these calculations: $B$, $Z$, $g^2/4\pi$, $m_s$. The constant $Z$ is calculable in principle, as we will see below.
II. THE PHEE-QUARK MODEL

In the most primitive version of our model, we assume free massless quarks, moving freely within the bag without interactions. This simple model exhibits a similar intriguing simplicity, and leads to a similar surprising number of qualitatively significant results, as the approximation of the shell model with free nucleons in a potential well did for nuclear structure.

Most of these results can be found in my 1974 Erice report. We summarize them here.

In this simple form, the bag model assumes that hadrons are bubbles in an ideal liquid (the ether) under a pressure \( B \). The bubbles are kept from collapsing by the counter-pressure of the free quarks moving inside them. At this stage of the theory no reason is apparent for the fact that the number of quarks in the bubbles must be multiples of three or of quark-antiquark pairs.

Assume a massless quark in a sphere of radius \( R \) with the boundary condition of zero-current across the surface. Its wave functions can be calculated rather simply from the Dirac equation. We face here an extreme relativistic analogue to a particle in a potential well with infinite wells. The lowest solution \( \psi_0 \) is a well-defined four-component Dirac wave function, with an energy eigenvalue \( \epsilon_0 \), which cannot depend upon anything but the radius \( R \) and therefore must have the form

\[
\epsilon_0 = \frac{x}{R}, \quad x = 2.043
\]

where \( x \) is a numerical constant. The total energy of the lowest three-quark state, therefore, is

\[
E = \frac{3x}{R} + \frac{4\pi}{3} R^3 B = m_p
\]

where \( m_p \) is the mass of the proton. The second term is the volume energy of the bag. A minimization of (2) with respect to \( R \) gives us a virial theorem, stating that the volume energy must be a quarter of the total one. It also gives us a relation between \( B \) and \( m_p \) by eliminating \( R \):

\[
B^{1/4} = \left( \frac{3}{4\pi} \right)^{1/4} \frac{1}{4x^{3/4}} m_p = 0.102 m_p = 96 \text{ MeV}
\]
B is supposed to be a universal constant. The radius \( R \) of the three-quark system, if all three quarks are in the lowest quantum state \( \psi_0 \), is also determined by these equations:

\[
R = \frac{\gamma x}{m_p} = 1.71 \, \ell
\]  

(4)

The root mean square radius can be calculated from the knowledge of the wave function \( \psi_0 \) and we get

\[
\langle r^2 \rangle^{\frac{1}{2}} = 0.74 \, R = 1.28 \, \ell
\]  

(4.a)

This is somewhat high in this primitive theory; it will be reduced by further refinements. It will be interesting to note that, for a non-relativistic particle enclosed in an infinite potential well, the following relation holds:

\[
\langle r^2 \rangle^{\frac{1}{2}} = 0.53 \, R
\]  

(4.b)

The massless particle is less concentrated toward the origin than the massive one.

The knowledge of \( \psi_0 \) also allows the calculation of the magnetic moment. The moment of one quark with charge \( e_q \) is given by

\[
\mu_q = e_q \int \overline{\psi}_0 (\vec{r} \times \hat{r}) \psi_0 \, d^3 x
\]

where \( \psi \) is the current operator. Because of dimensional reasons it must be proportional to \( R \). The result is:

\[
\mu_q = e_q \cdot R \ell, \quad \ell = 0.202
\]  

(5)

If we express it in units of the proton Bohr magneton \( 1/2m_p \), we get from (4):

\[
\mu_q = 3.3 \, e_q \frac{1}{2m_p}
\]  

(5.a)

One can calculate from this the magnetic moment of the proton and the neutron since we know how the spins of the quarks combine (see my 1974 Erice Lectures), and one gets
\[ \mu_p = 3.3 \frac{1}{2m_p}, \quad \mu_N = -2.2 \frac{1}{2m_p} \]  

(6)

values that are a little high but satisfactory.

It should be mentioned that no other quark model except this one has yet yielded any absolute values for the magnetic moments but only ratios.

Another magnitude which can be calculated in this simple quark model is the ratio \( g_A / g_V \) of the axial and vector coupling constants in the \( B \) decay of baryons. The experimental value of that ratio for the nucleon is 1.24, very different from the value 5/3 obtained by the old quark model. This ratio can be expressed for all baryon-decays of the basic octet by two constants \( F \) and \( D \) on the basis of \( SU_3 \) symmetry. We give a few of these ratios for the decays from baryon \( A \) to baryon \( B \):

<table>
<thead>
<tr>
<th>AB</th>
<th>Vector</th>
<th>Axial</th>
</tr>
</thead>
<tbody>
<tr>
<td>np</td>
<td>1</td>
<td>( F+D )</td>
</tr>
<tr>
<td>( \Sigma )</td>
<td>0</td>
<td>( \sqrt{2/3} D )</td>
</tr>
<tr>
<td>( \Lambda )</td>
<td>( \sqrt{3/2} )</td>
<td>( \sqrt{3/2}(F+1/3D) )</td>
</tr>
<tr>
<td>( \Sigma^0 )</td>
<td>( 1/\sqrt{2} )</td>
<td>( 1/\sqrt{2}(F-D) )</td>
</tr>
</tbody>
</table>

**TABLE 1**

The experimental values are

\[ F = 0.41 \pm 0.02, \quad D = 0.83 \pm 0.02 \]  

(7)

and the old non-relativistic quark model gives \( F = 2/3, \ D = 1 \). The new massless confined quark model indeed gives better values. The reason is as follows. The determination of that ratio hinges on the matrix element of the quark spin:

\[ (q|\sigma_2|q) \]  

(8)

It is unity for a non-relativistic quark whose spin is up. The situation is different for a massless quark since the spin is opposite in the so-called "small components" which are not small in this case. The matrix element (8) can be computed for the state \( \psi_0 \) and the result is
\( (q \sigma_i q) = \frac{X}{3(x-1)} = 0.65 \).

The final result is simply that the non-relativistic values for \( F \) and \( D \) must be multiplied with that number. One then gets

\[
F = 0.44, \quad D = 0.65 \quad (\text{relativized})
\]

which are much nearer to the experimental values (7).

We now apply the primitive free massless quark model to the mesons. Here are the results: because of the validity of \( SU_3 \) all meson masses will be equal. The meson mass is given by an expression analogous to (2):

\[
m_m = \frac{2x}{R} + \frac{4\pi}{3} R^3 B
\]

with \( x \) given by (1) and \( B \) is already fixed by (3). A minimization of (9) with respect to \( R \) gives

\[
R = \left( \frac{2x}{4\pi B} \right)^{1/4} = 1.55
\]

and

\[
m_m = \frac{8}{3} x^{3/4} (2\pi)^{1/4} B^{1/4} = 692 \text{ NeV}.
\]

This is not too bad for an average value of meson masses in the two octets. The magnetic moment of the charged vector mesons would be given by (5). The moment would be \( \mu = 3.0 \) (1/2\( m_p \)).

It is necessary to point out a serious deficiency of this first approximation. Let us consider more than three non-strange quarks in a bag. The Pauli principle allows up to 12 non-strange quarks to be in the lowest quantum state because of the colour quantum number. The energy of an \( n \) quark system with \( n \) quarks in \( \Psi_0 \) is

\[
E_n = \frac{nx}{R} + \frac{4\pi}{3} R^3 B
\]
and, after minimization and elimination of $R$:

$$E_n = \frac{2}{3} \pi^{3/4} n^{3/4} (4\pi)^{1/4} B^{1/4}.$$  \hfill (13)

This expression shows that the energy is proportional to $n^{3/4}$. This leads to contradictions, even if we assume that only multiples of three quarks can be contained in a bag. It would follow that two nucleons have less than twice the mass of one nucleon if they decide to unite their quarks within one bag. A deuteron, for example, would have a state with a binding energy of $(2-2^{3/4})m_p = 300$ MeV when all six quarks are in one bag. A similar situation occurs for the $\alpha$ particle which would have a state with a binding energy of 1100 MeV! Obviously this is not so. This difficulty will disappear in the next step of development of our model.

III. THE GLUON FIELD

We now improve (and complicate) our model by introducing a special quark-quark interaction in order to describe those parts of the strong interaction which are not included in the bag. It is assumed to be transmitted by a field of massless vector gluons coupled to the colour of the quarks. We consider it as a non-Abelian coupling in which the gluons carry colour. Apart from this, the coupling is similar to electromagnetism with charge being replaced by colour. We will rely on this analogy in this report. There is another difference to ordinary electromagnetism. We assume the gluon field also to be confined in the bag by a boundary condition which forces the gluonic field lines never to cross the surface.

We now discuss some of the effects of this gluon field.

1) The fact that the field is confined to the bag, has a very interesting consequence: the quarks inside must be in a colour singlet ("colourless"). If they were not, Gauss' law would be violated, since the boundary condition says that the total gluonic flux out of the bag must be zero. Hence only a quark triplet, or multiples of triplets, or quark-antiquark pairs, can be in the bag. Single quarks or quark pairs are impossible because they necessarily carry colour. Thus, the confinement of a massless gluon field, together with its coupling to colour, automatically assures the correct number of quarks in the bag.
2) As the next effect, we consider the energy of the zero-point oscillations of both the quark field and the gluon field in a bag of radius \( R \). Usually, for non-confined fields, one disregards that energy since it is constant though infinite. Here, however, it will be a function of the radius \( R \) of the confinement. The problem is similar but not identical to the so-called Casimir-Forder effect. There one calculates the difference in the zero-point energy of the electromagnetic field in empty space and in a space where there is a metallic container (spherical or plane slab). Indeed, that difference is finite and depends on the linear dimensions of the container.

However, in the Casimir-Forder problem, the field is not suppressed outside of the container as it is in our case. The M.I.T. crew was not able yet to calculate the effect for a sphere, but was able to calculate it for a confinement in form of a plane slab. There, they discovered the interesting fact that the zero-point energy of a confined field in a plane slab of thickness \( L \) per area \( L^2 \) is given by

\[
E_o = a \Omega^4 L^3 + \frac{b}{L^2} + \text{neg. powers of } \Omega.
\]

Here \( \Omega \) is the cut-off momentum and \( a, b \) are numerical constants. Two facts are remarkable: no terms of the form \( \Omega^2 L^2, \Omega^2 L \) appear, and \( b \) is negative for vector and spinor fields. We therefore conclude that for the sphere also

\[
E_o = a' \Omega^4 R^3 - \frac{Z}{R}
\]

where \( a' \) and \( Z \) are finite positive numerical constants. The interesting features here are the appearance of an energy proportional to the volume and a negative finite energy proportional to \( R^{-1} \). The first energy is infinite but its renormalized value might well be the source of the term \( B+V \), that is the source of the pressure on the bubble \( l \). In principle, the constant \( Z \) in the second term of (14) is finite and calculable. Since this calculation has not yet been done, we will add this term to our energy expressions (2), (9) and (12), and determine \( Z \) by fitting to the facts.

3) We now discuss the glu-electromagnetic interactions between the quarks. I am going to use the electromagnetic analogy in discussing it; expressions between quote-marks refer to gluon fields. There are two kinds of quark-quark interactions: "electrostatic" and "magnetic". We are going to leave out the "electric" interaction because it has the character of a long-range low
\( \Delta q \) interaction, and we therefore assume that it is somehow included in the phenomenological bag \(^*)\).

We therefore consider only "magnetic", that is spin-spin interactions. This is done by perturbation theory, to the first approximation in the coupling constant. Note that the interaction comes from the "magnetic moments" of the quarks. They are not the real magnetic moments but the gluo-magnetic analogues. They are proportional to the "charges", that is, to the colour of the quarks. The coupling with the field is a minimal interaction, just as in the case of the electron spin. The calculation is almost the same as the electromagnetic one. The only difference is the boundary condition for the field, which does not constitute any fundamental difficulty here since the "magnetic" field of each quark spin is divergence free.

The result is as follows. The interaction energy between two quarks \((i,j)\) in the lowest state within a bag of radius \(R\) is given by

\[
\Delta E^{(ij)}_{m_q} = \frac{e^2}{4\pi} \sum_a \left( \lambda^a_i \sigma^i \cdot \lambda^a_j \sigma^j \right) \frac{I(k_i, k_j)}{R} \tag{15}
\]

Here it is taken in consideration that one or both of the quarks may be ascribed a mass \(m_s\) different from zero. Here \(\lambda^a_i\) and \(\sigma_i\) are the eight colour indices and the spins of the quarks,

\[
\kappa_i = m_i R,
\]

\(^*)\) This interpretation resulted from a discussion with Ken Wilson. The M.I.T. authors give a different reason for leaving out the "electrostatic" energy. They argue that it consists of the mutual interactions of different quarks and of the self-interaction of each quark. Usually the latter is not counted because it is included in the mass. Here the situation is more difficult. The split into those two parts is impossible because each part separately does not fulfil the boundary conditions. The mutual interaction in first approximation is the interaction of the average "charge" distributions. The self-interaction can be divided into two parts: the self-energy of the average "charge" distribution of the particle and the rest of the self-energy which comes from the field carried along by the particle. If the particle is massless and moves with light velocity, not much of the field is carried along. Indeed, one can show that the (non-confined) self-energy of a massless particle is finite and most of the self-energy is in the first part.

Now, if we neglect the second part, it is easily seen that the total "electrostatic" energy of a system of quarks vanishes, since the total average "charge" density is zero if all quarks are in the same quantum state, and their total colour is zero. If some quarks are massive, there remains a small energy coming from the different charge distributions; we will neglect this difference.
and \( I(K_1, K_2) \) is a slowly varying function of the two variables, such that

\[
I(0, 0) = 0.147, \quad I(2, 0) \sim \frac{1}{2} I(0, 0),
I(1, 1) \sim I(2, 0),
\]

\( g^2 \) is the gluon coupling constant. It will turn out that the best fit with
facts gives \( g^2/4\pi \sim 0.55 \).

The spin-spin energy (15) is proportional to \( R^{-1} \) for massless quarks
\( (\kappa_1 = 0) \) and the dependence is only weakly different from \( R^{-1} \) if one or two
quarks have a mass. Please note the sign of the interaction. As in electrodynamics, parallel magnetic moments attract, antiparallel repel. But in a
hadron, two quarks in general have opposite colour (the total colour is zero)
so that parallel spins will give rise to repulsion, antiparallel spins to
attraction. This will be the reason why the \( \Delta \) is heavier than the nucleon,
and the \( p \) is heavier than the \( n \).

It is easy to sum up the spin-spin interactions within a given hadron.
We now already count with a finite mass for the strange quark of roughly
\( m_\Xi \sim 300 \), whereas the non-strange quarks are still assumed to be massless.
Then we can write the total spin-spin energy :

\[
\Delta E_{\text{spin}} = \sum_{i>j} \Delta E_{i,j}^{(i,j)} =
\frac{g^2}{3} \frac{g^2}{2\pi} \left[ a_{nn} \frac{I(0,0)}{R} + a_{ns} \frac{I(0,0)}{R} + a_{ss} \frac{I(0,0)}{R} \right]
\]

(17)

Here the first term contains the interactions of non-strange quarks, the
second the ones where one quark is strange, the third the ones where both are
strange. The coefficients \( a_{ik} \) are easily determined from Clebsch-Gordan
coefficients and colour counting. We have for the different hadrons :

<table>
<thead>
<tr>
<th>( N )</th>
<th>( \Delta )</th>
<th>( \pi )</th>
<th>( p )</th>
<th>( \omega )</th>
<th>( \varphi )</th>
<th>( \Lambda )</th>
<th>( \Sigma )</th>
<th>( \Sigma^* )</th>
<th>( \Xi )</th>
<th>( \Xi^* )</th>
<th>( \Omega )</th>
<th>( K )</th>
<th>( K^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_{nn} )</td>
<td>-3</td>
<td>+3</td>
<td>-6</td>
<td>+2</td>
<td>+2</td>
<td>0</td>
<td>-3</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( a_{ns} )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-4</td>
<td>2</td>
<td>-4</td>
<td>2</td>
<td>0</td>
<td>-6</td>
</tr>
<tr>
<td>( a_{ss} )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

**TABLE II**
We now collect all the contributions (2), (14), (17) to the hadron energy. We also must take into account that the lowest quark eigenvalue $\varepsilon_0$ is no longer given by (1) in the case of the strange quark, because of its mass. We therefore write

$$\varepsilon_0^{(i)} = \frac{x(K_i)}{R}$$  

(18)

where $x$ is a rather smooth function of $x_1$:

$$x(0) = 2.04, \quad x(1) = 2.4, \quad x(\infty) = \pi.$$

The energy of a hadron is

$$E = \frac{1}{R} \left[ \sum_i \frac{x(K_i)}{R} - Z + \frac{g^2}{4\pi} \frac{g^2}{4\pi} \sum_{l=3} a_{el} T(K_l, K_m) \right] + \frac{9}{8} \frac{R^3}{R}$$  

(19)

Here, the first summation is taken over all quarks in the hadron; in the second summation the indices $l, m$ stand for $n$ and $s$ as the case may be.

IV. COMPARISON WITH REALITY

a) Hadron masses

Expression (19) contains four adjustable constants: $R, m_s, g^2/4\pi, Z$. An attempt was made to determine these constants by equating the following four masses with the corresponding expression (19): proton, $\Omega$, $\Lambda$, $\phi$. Once this is done, all other masses can be determined. The result is given in Fig. 1. The hadron masses are reproduced surprisingly well with the exception of the $\Lambda-\Sigma$ difference and the pion mass. The latter turns out to be somewhat too large. We should not expect a first approximation to work well when the mass of the pion is so close to zero. After all, the approximations must break down completely when the energy comes near to this singular point. We have no obvious excuse why the $\Lambda-\Sigma$ difference is only half of what it should be.

Let us look at some qualitative features: clearly expression (19) lifts the $SU_3$ degeneracy of the expression (2) of the primitive model. The split of the baryon decuplet is caused by the introduction of $m_s$, as in the usual quark models. The split between the nucleon and the $\Delta$ is due to the
spin-spin term. We know that parallel spins have higher energy than anti-parallel. The same effect splits the \( p \) and the \( \rho \). For the mesons, this split is even larger because of the fact that the colour of a quark and its antiquark are exactly and fully opposite, whereas in the baryon, we have to deal with a colour triality, which reduces the effect.

The split between the \( \Sigma \) and the \( \Sigma^* \) is less than the \( N-A \) split because of an interesting reason: the gluomagnetic moment of the strange quark is somewhat smaller than the one of the non-strange quark. This is due to the fact that the massive wave function is somewhat more concentrated than the massless one, as we saw comparing (4.a) and (4.b). Hence, the "magnetic" moment is reduced (smaller circular current), and thus the spin-spin interaction. This is also the reason why the \( K^*-K \) difference is less than the \( \rho-\pi \) difference.

b) Magnetic moments

The introduction of a massive strange quark destroys the \( SU_6 \) symmetry of the primitive model. Therefore, the ratios of hadron magnetic moments will differ from the \( SU_6 \) symmetric ratios. It is easy to calculate the mass dependence of the magnetic moment; as mentioned before, it becomes somewhat smaller with increasing mass, and therefore the moments for strange hadrons are smaller than the \( SU_6 \) predictions, as indeed they are found to be.

Table II shows the \( SU_6 \) predictions, the predictions of this model on the basis of \( m_s = 279 \text{ MeV} \) (the value which fits best the hadron masses), and the experimental values. The Table contains the relative ratios to \( \mu_p \), the proton magnetic moment, since this is all that the classical \( SU_6 \) based non-relativistic quark model can do.

<table>
<thead>
<tr>
<th>Hadron</th>
<th>Magnetic moments ((\mu/\mu_p))</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>Experiment: (-0.685)</td>
</tr>
<tr>
<td></td>
<td>Bag model: (-2/3)</td>
</tr>
<tr>
<td></td>
<td>Non-relativistic quark model: (-2/3)</td>
</tr>
<tr>
<td>( \Lambda )</td>
<td>Experiment: (-0.240 \pm 0.02)</td>
</tr>
<tr>
<td></td>
<td>Bag model: (-0.255)</td>
</tr>
<tr>
<td></td>
<td>Non-relativistic quark model: (-1/3)</td>
</tr>
<tr>
<td>( \Sigma^+ )</td>
<td>Experiment: (0.93 \pm 0.16)</td>
</tr>
<tr>
<td></td>
<td>Bag model: (0.97)</td>
</tr>
<tr>
<td></td>
<td>Non-relativistic quark model: (1)</td>
</tr>
<tr>
<td>( \Sigma^0 )</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>Bag model: (0.31)</td>
</tr>
<tr>
<td></td>
<td>Non-relativistic quark model: (1/3)</td>
</tr>
<tr>
<td>( \Sigma^- )</td>
<td>Experiment: (&lt;-1.6 \text{ to } 0.6)</td>
</tr>
<tr>
<td></td>
<td>Bag model: (-0.36)</td>
</tr>
<tr>
<td></td>
<td>Non-relativistic quark model: (-1/3)</td>
</tr>
<tr>
<td>( \Xi^0 )</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>Bag model: (-0.56)</td>
</tr>
<tr>
<td></td>
<td>Non-relativistic quark model: (-2/3)</td>
</tr>
<tr>
<td>( \Xi^- )</td>
<td>Experiment: (-0.69 \pm 0.27)</td>
</tr>
<tr>
<td></td>
<td>Bag model: (-0.23)</td>
</tr>
<tr>
<td></td>
<td>Non-relativistic quark model: (-1/3)</td>
</tr>
</tbody>
</table>

**TABLE III**
The bag-model, as we know, also gives the absolute value of the proton moment \( \mu_p \). Here the "improvements" coming from quark-quark interactions reduce the predicted value to the not very satisfactory value of \( \mu_p = 2.0 \, (1/\mu_p) \) instead of \( 2.7 \, (1/\mu_p) \). This is explained as follows: the primitive version gave too big a value \( \mu_p = 3.3 \, (1/\mu_p) \). The magnetic moment is proportional to the radius \( R \) of the bag as shown in (5). The radius, in turn, is determined by the equilibrium between the tendency of compressing the bubble due to the pressure \( B \) and the counter-pressure of the quarks expressed by their kinetic energy (1). The gluon field reduces this pressure, first by the term \(-Z/R\), and then by the spin-spin interaction which is negative in the nucleon. Hence the radius of the nucleon becomes \( R = 1.0 \, f \) instead of (4). This reduction is responsible for the small value of the magnetic moments. It is the most serious drawback of the present bag model. It may be that higher approximations will increase the value. A list of hadron bag radii is found in Table IV.

<table>
<thead>
<tr>
<th>Hadron</th>
<th>N</th>
<th>( \Delta )</th>
<th>( \Lambda )</th>
<th>( \Sigma )</th>
<th>( \Xi^* )</th>
<th>( \Xi )</th>
<th>( \Omega )</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>5.00</td>
<td>5.48</td>
<td>4.95</td>
<td>4.95</td>
<td>5.43</td>
<td>4.91</td>
<td>5.35</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Hadron</th>
<th>( \pi )</th>
<th>( \rho )</th>
<th>( \varphi )</th>
<th>( K )</th>
<th>( K^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>3.34</td>
<td>4.71</td>
<td>4.61</td>
<td>3.26</td>
<td>4.65</td>
</tr>
</tbody>
</table>

**TABLE IV**  
Bag radii in units of \( \mu_p^{-1} \)

This list shows the effect of the attraction between quarks with opposite spin and the repulsion between quarks with parallel spin. The former reduces, the latter increases the radius. Strange quarks exert a little less counter-pressure in the "bubble" because of their mass, thus reducing the radius.

The predicted root mean square radii, of course, are also smaller than in the primitive model. For the proton, we get \( <r^2>^{1/2} = 0.73 \, f \) which is nearer to the experimental value \( 0.88 \pm 0.03 \, f \) than (4.1a), but too small, roughly in the same ratio as the magnetic moments. The following results are obtained for mesons: the magnetic moment of the charged \( \rho \) meson is \( 0.86 \, \mu_p \) or \( 1.9(1/2\mu_p) \). The root mean square radius of the pion is \( 0.51 \, f \).
c) **Axial currents**

The ratio of the axial to vector current coupling in baryon $\beta$ decay is the same as in the primitive theory, as long as we only consider non-strange quarks. After all, the wave functions do not change to the first approximation in the coupling constant $g$. There is a change, however, for strange quarks, since the wave function, and therefore the value of the matrix element $\langle q | \sigma_\mu | q \rangle$, is indeed different for massive quarks. Assuming the mass $m_\beta = 279$ MeV, the reduction factor which must be applied to the non-relativistic values of $P$ and $D$ is increased from 0.65 (massless) to 0.71. Thus the constants $P$ and $D$ which determine the axial to vector coupling, as indicated in Table I, are different according to the bag model for $\Delta S = 0$ and $\Delta S = 1$ transitions, since in the former only non-strange quarks are involved, whereas in the latter a strange quark must be involved. Table V gives the values of $P$ and $D$ calculated by the bag model.

Experimentally, no distinction between $\Delta S = 0$ and $\Delta S = 1$ is yet observable. Future more accurate experiments would have to decide whether there is a dependence on $\Delta S$.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Bag model</th>
<th>Classical quark model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta S = 0$</td>
<td>$\Delta S = 1$</td>
</tr>
<tr>
<td>$P$</td>
<td>0.41 ± 0.02</td>
<td>0.44</td>
</tr>
<tr>
<td>$D$</td>
<td>0.83 ± 0.02</td>
<td>0.65</td>
</tr>
</tbody>
</table>

**TABLE V**

d) **Bags with more than three quarks**

The improved model completely solves the deficiency pointed out in Section II of the primitive model in regard to the lowest state of systems with $3n$ non-strange quarks where $n = 2, 3, 4$. According to formula (13) the lowest energy of these systems was found to be proportional to $n^{3/4}$ which would make it energetically favourable to put the quarks of 2, 3 or 4 nucleons into one bag, which is not the case in nature.

The more complicated expression (19), in contrast to (12), does no longer show this effect. The reason lies partially in the appearance of the term $-Z/R$, but mainly in the spin-spin interaction. Indeed, for $n$ non-strange quarks, the gluomagnetic energy (17) can be written in the form
\[ \Delta E_{mag} = \frac{8}{3} \frac{e^2}{4\pi \hbar c} \left[ J(J+1) + I(I+1) \right] \frac{I(g_0)}{R} \]

where \( J \) and \( I \) are the total spin and isospin of the \( n \) quark system. The first term \( n(n-6) \) is negative only for three quarks. It comes from the fact that there is an increasing number of parallel spin pairs when \( n \) goes from 3 to 6 quarks, or higher. This raises the spin-spin energy. As an example, the lowest values allowed by the Pauli principle for \( n=6 \) are \( J=1, I=0 \) (the deuteron quantum numbers). Putting this into (20) and using (19) and minimizing in respect to \( R \), we get \( (2=1.85) \) an energy of 2.16 GeV for the six-quark body. This is higher by 280 MeV than the sum of two nucleons. The paradox has disappeared.

One can go a step further and maintain that the positive energy surplus of six quarks in a bag compared to two nucleons is connected with the repulsive core in the nuclear potential between two nucleons at small distance. It would be that this energy surplus is an indication of the energy needed to press two nucleons into the same spatial area. It is interesting that the \( J=0, I=1 \) state of six quarks in one bag has an energy which is 60 MeV higher. It may be connected with the fact that this state in the actual deuteron indeed is not bound.

At the end of these deliberations, I would like to repeat the statement that none of the results in this paper are found by myself. They are published or unpublished results reached by A. Chodos, T. DeGrand, R. Jaffe, K. Johnson, J. Kiskis and C. Thorn. I am grateful to them for communicating their results to me prior to publication and to let me participate in the excitement of discovery.
** experimentation

---

** bag model predictions

---

masses used to determine model parameter

\(B^{1/4} = 0.146 \text{ GeV}\)

\(Z_0 = 1.84\)

\(\rho_c = 0.55\)

\(m_s = 0.279 \text{ GeV}\)

---

** m GeV

---

\(M(\text{GeV})\)

---

\(J = \frac{3}{2}\)  \(J = \frac{1}{2}\)  \(J = 1\)  \(J = 0\)

---

** baryons

---

** mesons

---