HADRONS (USUAL AND NEWLY DISCOVERED) IN GAUGE THEORIES

A DREAM?

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1. INTRODUCTION

In this talk I will concentrate on the following two questions:

i) Given the relative success of the description within the framework of a non-Abelian gauge field theory of the newly discovered phenomena in $e^+e^-$ physics *, can we also learn something about usual hadrons in the same theoretical framework?

ii) In particular, is it possible to incorporate in the phenomenological scheme proposed for the new particles, interpreted as "charmonia", also the usual hadrons? This achievement might produce restrictions that would be relevant to understanding the same starting point of the analysis ($\psi$ particles and $e^+e^-$ physics above 3 GeV).

It is worthwhile to start by giving a qualification of the ideas and the calculations that I will present in the following when trying to answer these questions. They have a high speculative content, and the resulting picture of hadron dynamics is so simple that many people will find it unbelievable; furthermore, the bridge between this phenomenological picture and the supposed underlying field theory is far from being built. Nevertheless, the importance of the questions that these ideas try to answer, as well as some of the results that will come out, make me confident about the usefulness of their presentation.

2. UNDERLYING FIELD THEORY FOR STRONG INTERACTIONS

The reference field theory for strong interactions will be the SU(4) $\otimes$ SU(3) non-Abelian gauge theory -- about which we have heard a lot in Appelquist's lectures -- involving four types of fractionally charged quarks, each in three colours and coupled to an SU(3) octet of massless gauge gluons. Here SU(4) is the minimal extension of the usual SU(3) group to incorporate the new quark, whose bound states with a corresponding antiquark we are eventually seeing in the $e^+e^-$ channel. By this I mean that the theory is likely to be, but it is not necessarily, the original SU(4) model of Glashow, Iliopoulos and Maiani; what I am going to say can easily be modified to apply to more elaborate "charmed" quark models.

I would like to remind you that, in usual notation, the gluon-quark coupling is given in the theory as

\[
\mu \quad = \quad g \gamma_\mu \frac{(\lambda^i)}{2}
\]

*) See the lectures by T. Appelquist in these Proceedings.
where $\mu$ stands for the Lorentz index of the vector gluon and $i,j,k$ are colour indices.

Standard concepts associated with this kind of theory are: i) the asymptotic freedom; ii) the "infrared slavery", the latter being a conjecture. They have already been discussed here by several people. As to (i), I will assume that the effective "strong fine-structure constant" $g^2(M)/4\pi \equiv \alpha_S(M)$, renormalized at a running energy scale $M$, is small compared to 1 already at $M \sim 1$ GeV. From this point on, asymptotic freedom sets in and allows us to compute a smaller and smaller coupling $\alpha_S(M)$ in terms of $\alpha_S(1)$. The smallness of $\alpha_S(1)$, which seems to be required by deep inelastic experiments, is within the gauge theory an open question which has eventually become more mysterious with the need to introduce a mass parameter for the "charmed" quark of about 1.6 GeV (see below).

As to the speculation of "infrared slavery" and the consequent permanent confinement of quarks and gluons in colourless hadrons, let me only point out the possible relevance of the recent investigation by Cornwall and Tiktopoulos\(^1\), who give arguments for the vanishing in the zero-mass limit of Yang-Mills theories both of the exclusive and inclusive cross-sections of colour non-neutral objects.

3. PHENOMENOLOGICAL SCHEME

The phenomenological scheme proposed from the theory by De Rujula, Georgi and Glashow\(^2\) can be summarized in the following four points:

i) The hadrons (usual, "charmed", and "hidden charmed") are mainly described by non-relativistic dynamics. This hypothesis has clearly nothing to do with the non-Abelian gauge theory, but only with the actual values of the parameters appearing in the theory. Its self-consistency has to be checked.

ii) The "infrared slavery" produces an effective long-range binding force which is independent of quark spins and quark masses. The breaking of SU(4) enters only in the kinetic term of the effective Hamiltonian since different masses for quarks of different "flavours" are allowed ($m_p = m_n = m_d, m_s, m_c$).

iii) Since short distances are a regime where perturbation theory is meaningful (asymptotic freedom), the short-distance interaction is mediated by the one-gluon exchange with a small coupling $\alpha_S$. The non-relativistic reduction of the one-gluon potential, $g^2 \gamma_\mu \gamma^2/k^2$, gives then in coordinate space the usual Coulomb interaction $\alpha_S/r$ plus the Breit-Fermi potential $S_{BF}$, which contains spin-spin and spin-orbit interactions.

iv) The role played by the colour group in getting out of the theory the phenomenological scheme that we are discussing is presumably fundamental, but escapes our present knowledge of non-Abelian gauge field theories. Part of
this ignorance is expressed in making the assumption that hadrons are bound states of colour singlet wave functions. The graphical notation that I will use in the following for the meson and baryon wave functions is, respectively,

\[ M_{ij} = (1/\sqrt{3})\delta_{ij}, \]

whereas for baryons \( B_{ijk} = (1/\sqrt{6})\varepsilon_{ijk} \). The non-Abelian structure of the theory is reflected here only in the different coefficients that appear in front of the one-gluon exchange potential due to the colour coupling. For mesons

\[ 1 \Rightarrow \text{Tr} \left[ M (\frac{\not{s} + 1}{2}) M (\frac{\not{s} + 1}{2}) \right] \frac{\gamma_\mu \gamma_\nu}{k^2} = \frac{g^2}{3} \frac{\gamma_\mu \gamma_\nu}{k^2} \]

and for baryons

\[ 1 \Rightarrow \left[ B_{ij} \gamma_\mu (\frac{\not{s} + 1}{2}) B_{jm} \gamma_\mu (\frac{\not{s} + 1}{2}) \right] \frac{\gamma_\mu \gamma_\nu}{k^2} = \frac{2}{3} \frac{g^2}{k^2} \frac{\gamma_\mu \gamma_\nu}{k^2} \]

4. APPLICATIONS TO USUAL S-WAVE BARYONS

Let us assume that \((m_\lambda - m_d)/(m_\lambda + m_d)\) is a small parameter. The SU(3) breaking term in the kinetic energy of the effective Hamiltonian is then treatable as a perturbation of an SU(6) symmetric theory. One has therefore a degenerate 56-plet of S-wave baryons, with a common O(3) wave function \( \psi_0(\vec{r}_1, \vec{r}_2, \vec{r}_3) \).

We also assume that this wave function \( \psi_0 \) contains predominantly S-waves in any of the relative quark coordinates \((\vec{r}_1 - \vec{r}_3)\).

It follows for states of equal quark content that the breaking in mass is given only by the Fermi spin-spin interaction which is contained in \( S_{BF} \) and is a contact interaction.
\[ \langle \delta M \rangle = -\frac{2}{3} \alpha_s \langle \psi \mid \delta^3 (\vec{r}_1 - \vec{r}_2) \mid \psi \rangle \sum_{i,j} \frac{1}{m_i m_j} \langle \vec{s}_i \cdot \vec{s}_j \rangle \]

The following mass relations are then easily derived\(^2\) (particle names stand for particle masses):

\[
\frac{2}{\ell} \left( \frac{\Sigma^*-\Sigma}{\Sigma^*+\Sigma-3\Lambda} \right) = \frac{m_d}{m_\lambda} = 0.622 \\
\frac{\Sigma}{-\Lambda} = \frac{2}{3} \left( 1 - \frac{m_d}{m_\lambda} \right) (\Lambda - N)
\]

If the first equation is taken as fixing the ratio \(m_d/m_\lambda\), the second relation is a successful prediction. It goes without saying that more common SU(3) or SU(6) predictions, such as the Gell-Mann/Okubo formula or the equal spacing rules for the decuplet, are also recovered here.

Coming to the electromagnetic properties of baryons, from our non-relativistic picture and the knowledge of the SU(6) wave functions we get for the magnetic moments\(^3\)

\[
\mu (N) = -\frac{2}{3} \mu (P) ; \mu (\Lambda) = -\frac{m_d}{3m_\lambda} \mu (P) = -0.6 ; \mu (\Sigma) = \frac{P}{m_\lambda}
\]

The first relation is the usual successful SU(6) relation, which gets corrections in our scheme only by electromagnetism. The second relation is closer to the experimental values than the "naïve" SU(6) relation \(\mu (\Lambda) = -\mu (P)/3\), because of the previously determined \(m_d/m_\lambda\) mass ratio; it gets corrections in the theory of order \(\alpha_s (m_\lambda - m_d)/(m_\lambda + m_d)\). Finally, the prediction for the proton magnetic moment \(\mu (P) = P/m_d\) fixes \(m_d = 336\) MeV (and consequently \(m_\lambda = 540\) MeV); this is the least firm of the three relations because corrections to it of order \(\alpha_s\) are expected.

5. SPECIFICATION OF THE FULL HAMILTONIAN

Given the relative success of this starting point, and leaving aside for the moment the mesons, we would like to extend the analysis to excited multiplets of different O(3) content and also to incorporate the "charmed" states. However, because of our ignorance of the binding potential, both these extensions require the introduction of new free parameters. In fact: i) the spatial wave functions
for different multiplets are not correlated; and ii) the expansion in the quark mass breaking is untenable if, as expected, \( m_c \gg m_d \). On the other hand, the actual dependence of the wave functions on the constituent quark masses is totally related to the potential itself.

Since we want to avoid the introduction of too many free parameters, we choose to work with a definite binding potential\(^3\). For a two-quark mesonic bound state -- a technically simpler problem to handle than the three-body problem for a baryonic system -- the fully specified total Hamiltonian that we are going to diagonalize is

\[
H = m_1 + m_2 + \frac{\lambda}{\mu} \vec{p}^2 + V_1 + V_2, \quad \mu = \frac{m_1 m_2}{m_1 + m_2}
\]

\[
V_1 = \lambda \tau + V_2 - \frac{4}{3} \frac{\alpha_s}{\pi}
\]

\[
V_2 = -\left( \frac{\lambda}{m_1} + \frac{\lambda}{m_2} \right) \frac{\vec{p}^4}{p^2} - \frac{4}{3} \frac{\alpha_s}{\pi} \vec{p} \cdot \vec{r}
\]

A linear binding potential, together with a Coulomb component, has also been used successfully by Eichten et al.\(^*\) to analyse the \( \bar{c}c \) bound states. The constant \( V_2 \) represents the effect of intermediate range forces. The potential \( V_2 \), which will be treated in first order of perturbation, contains, together with the Breit-Fermi interaction, the \( \vec{p}^2 \) term coming from a consistent expansion of the relativistic kinetic energy. The coupling strengths \( \lambda \) and \( V_2 \) are taken as universal constants, i.e. independent of the quark masses.

As to the value of \( \alpha_s \), we should not forget that the presence in \( H \) of the term proportional to \( \alpha_s \) relies on the use of perturbation theory at short distances. Since the mean distances that are relevant for bound states of different quark content are different, to optimize the convergence of the perturbation expansion I will take \( \alpha_s(M) \) renormalized at the mass scale \( M \), when describing a bound state of the same mass \( M \). The reference value of \( \alpha_s \) is given at the \( \psi(3100) \) mass by the ratio \( \Gamma(\psi + e^+e^-)/\Gamma(\psi \to \text{hadrons}) \) analysed as in Appelquist's lectures. Typical values are then: \( 4/3 \alpha_s(\bar{c}c) = 0.27, 4/3 \alpha_s(\bar{\lambda}\lambda) = 0.36, 4/3 \alpha_s(\bar{d}d) = 0.42. \)

6. **MESON SPECTROSCOPY**

The diagonalization of the Hamiltonian \( H \) is straightforwardly done numerically. The results are shown in Table 1 for the S-wave spin-triplet (vector) and spin-singlet (pseudoscalar) mesons. The column denoted \( E_{RC} \) gives the relativistic corrections to the spin-triplet states coming from the perturbative treatment of the potential \( V_2 \). The triplet-singlet splitting is produced by the spin-spin Fermi

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interaction contained in \( S_{\text{BF}} \), which in the baryon case was responsible for the \( \Sigma-\Lambda \) mass splitting. In this table the \( \psi(3095) \) and \( \psi'(3684) \) masses are inputs. In fact, after the discussion of baryons and having taken the value of \( \alpha_S \) from the analysis of the \( \psi \)-decay branching ratio, we are left in the Hamiltonian \( H \) with three free parameters: \( \lambda, V_\theta, m_c \), which are fixed by the \( \psi, \psi' \) masses and by the leptonic decay width of the \( \psi \) (see below). The full set of parameters used in the calculation with their values is then: \( m_d = 336 \text{ MeV}, m_{\lambda} = 540 \text{ MeV}, m_c = 1.640 \text{ GeV}, \lambda = 0.25 \text{ GeV}^2, V_\theta = -0.76 \text{ GeV}, \alpha_S(3100) = 0.20 \).

Let me make the following comments to Table 1. The agreement with the experimental data for the vectors is quite good. The same cannot be said for the low-lying pseudoscalars, but this is not un-understandable; relativistic corrections and mixings of quark content should be very important in this case (see below).

The recently reported\(^{1}) \) \( \mu e \) events with \( \geq 2 \) neutrals in \( e^+e^- \) annihilation can perhaps be interpreted in this scheme as the weak decay products of two charmed vector mesons (\( D^*, F^* \) in the usual notation)*). However, in order to have vector mesons stable against all but the weak interactions, we should not have their pseudoscalar partners (\( D,F \)) lying below them, as is the case in Table 1. This may be a difficulty of our specific phenomenological analysis.

Without introducing any new input, the calculations can also be performed for the \( P^- \) and \( D \)-wave bound states. The results are shown in Table 2, where, for the \( D \)-waves, only the \( cc \) bound states are reported. Again, even if the experimental knowledge is quite uncertain in several cases, the agreement is acceptable. Note in particular the tentative insertion in this table of the very recently found\(^{2}) \) \( C^- \text{-even objects, seen from the radiative decays of the } \psi' \text{ particle, and interpreted here as } P^- \text{-wave charmonia.} \)

Restricting to the states that are likely to be produced in an \( e^+e^- \)-initiated reaction, the spectroscopic table of the \( cc \) bound states is almost filled up. We are waiting for the finding of a third \( P^- \) state (\( 2^{++} \)) and for the pseudoscalar partners (\( n_c, n'_c \)) of the \( \psi, \psi' \) particles.

As a last remark, note that the tensor term present in the Breit-Fermi potential introduces a mixing between the almost degenerate \( S^- \) and \( D \)-wave \( 1^{--} \) charmonia of masses 3680 and 3778 MeV, respectively. From the computation\(^{3}) \), it turns out, however, that the mixing angle is very small \( \theta \approx 2 \times 10^{-2} \), implying an essential decoupling of the \( D \)-wave in \( e^+e^- \) annihilation because of the vanishing at the origin of the \( L = 2 \) wave function.

\* The heavy lepton pair production is also an appealing interpretation for these events.
7. POSSIBLE SOURCES OF CORRECTIONS TO THE SPECTROSCOPY

I would like to point out and briefly comment on three possible sources of corrections to the results that I have presented.

i) Importance of the relativistic corrections: If, in Tables 1 and 2, one naively compares the size of $E_{RC}$ with the over-all mass values, one concludes that the relativistic corrections, although non-negligible, are indeed relatively small except for the low-lying pseudoscalars. Let us look, however, in the last column of both tables, where the values are reported of the ratio $\xi$ between the mean value of the $7^0$ term present in $V_2$ and the mean non-relativistic kinetic energy ($\frac{\vec{p}^2}{2\mu}$). Since $\xi$ is a sort of relativistic expansion parameter, a better criterion for deciding whether the non-relativistic expansion is tenable or not is to see how small $\xi$ is with respect to 1. And now the conclusion is that whenever a bound state contains a d-type quark, its description as a mainly non-relativistic system seems quite inadequate. If we want to do better, given the underlying field theoretic framework, a Bethe-Salpeter equation treatment should be the only adequate one. And we would be immediately faced with at least serious technical, if not fundamental, difficulties: in brief "big relativistic corrections" is equivalent to "lack of dynamical information". The observation that gluonic states could be a non-negligible component for some of the known particles is perhaps relevant here.

ii) The force due to strong three-meson couplings is completely neglected in our potential-model calculation. Considering, for example, the $\psi$ particles, we expect in the real case for their wave functions an admixture of $c\bar{c}$ and $D\bar{D},F\bar{F}$ components\(^5\), with the pure quark-antiquark component presumably decreasing for the higher radial excitations. We could also say that the effective quark-antiquark potential should not go to infinity at large distances, but from some distance on it should show a Yukawa-like behaviour appropriate to meson-meson interaction\(^6\). To make these observations quantitative is difficult but not impossible\(^*)\).

iii) Finally, let us note that the admixture of quark content for the physical particles is neglected in the results of Tables 1 and 2. And this admixture is known to be important at least for the low-lying pseudoscalars.

8. MESON DECAYS

The explicit knowledge of the meson wave functions enables us to compute some of the meson decays.

\(^*)\) See E. Eichten's talk in these Proceedings.
8.1 Leptonic decays of neutral vector mesons

From the diagram

\[ \Gamma (\nu \rightarrow e^+e^-) = \varepsilon_\nu^2 \chi^2 \frac{\left| \psi_\nu^1 (\nu) \right|^2}{m_\nu^2} \]

where \( \varepsilon_\nu = \langle \nu | Q | \nu \rangle \) is the mean value of the SU(4) charge operator over the state \( | \nu \rangle \). Including first-order corrections in the one-gluon exchange, one gets\(^7\)

\[ \left| \psi_\nu^{NR} (\nu) \right|^2 = \left| \psi_\nu^{NR} (\nu) \right|^2 \left( 1 - \frac{4 \varepsilon}{3 \pi} \chi_s (M_\nu) \right) \]

where \( \psi_\nu^{NR} \) are the non-relativistic wave functions obtained from the diagonalization of the potential \( V_1 \). Here again \( \chi_s (M_\nu) \) is changing with the mass of the vector meson. The results compared with the experimental values are

<table>
<thead>
<tr>
<th>( \psi )</th>
<th>( \psi' )</th>
<th>( \psi''(4100) )</th>
<th>( \phi )</th>
<th>( \rho )</th>
<th>( \omega )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Gamma^{th} (\text{keV}) )</td>
<td>5.2 *</td>
<td>3.1</td>
<td>2.7</td>
<td>2.1</td>
<td>10.6</td>
</tr>
<tr>
<td>( \Gamma^{exp} )</td>
<td>5.2</td>
<td>2.2</td>
<td>-</td>
<td>1.44</td>
<td>6.0</td>
</tr>
</tbody>
</table>

\( \Gamma (\psi \rightarrow e^+e^-) \) is the input, as I have already stated. The relative value of the widths for the usual vector mesons (\( \rho, \omega, \phi \)) is a known successful prediction of SU(3). The introduction of the one-gluon correction in Eq. (1) is then quite important for having a satisfactory description for the new mesons too. Note that despite the reduction of the widths owing to the \( \chi_s \) correction, the predicted values for the usual mesons are still higher than the experimental ones.

8.2 Weak decays of charmed mesons

In the specific GIM model for the weak interactions, we can go through the calculation of the leptonic weak decays quite similarly to the calculation of the electromagnetic widths. The relevant diagram here is the following one
For a meson bound state of mass M, wave function \( \psi(\mathbf{r}) \), and total spin \( S = 0,1 \) we get (to leading order in \( m_\lambda/M \))

\[
\Gamma \left( M \rightarrow e^+e^- \right) = \frac{G_F}{\sqrt{2}} C_\lambda^2 \left| \int \psi^*(\mathbf{r}) \right|^2 \left[ \frac{2M^2}{3} \left\langle \frac{S^2}{2} \right\rangle + m_\lambda^2 \left\langle \frac{2 - S^2}{2} \right\rangle \right]
\]

where \( C_\lambda \) should be read as \( G_F \cos \theta_C \) for pn and c\( \lambda \) type states and \( G_F \sin \theta_C \) for p\( \lambda \) and c\( \lambda \) type bound states. The first term on the right-hand side, proportional to \( \langle S^2 \rangle \) is contributing to the vector decays, whereas the second term, proportional to \( \langle 2 - S^2 \rangle \), gives the pseudoscalar decays. Incidentally this formula makes clear why the already mentioned \( \mu \nu \) events in \( e^+e^- \) annihilation could come, in the chain

\[
e^+ + e^- \rightarrow M^+ + M^- \rightarrow \mu^+ + \bar{\nu}_\mu \left( e^+ + \bar{\nu}_e \right) \rightarrow \mu^+ + \nu_\mu \left( \mu^+ + \nu_\mu \right)
\]

from the decays of two charmed vector mesons, and not from pseudoscalar meson pair production: the electronic decays of the pseudoscalars are suppressed with respect to the muonic decays by the factor \( (m_\nu/m_\mu)^2 \). The numerical results for the charmed mesons are

\[
\Gamma \left( \Delta^+ \rightarrow \mu^+ + \nu_\mu \right) \approx \Gamma \left( \Delta^+ \rightarrow e^+ + \nu_e \right) \approx 2 \times 10^{11} \text{ sec}^{-1}
\]

\[
\Gamma \left( \Xi^* \rightarrow \mu^+ + \nu_\mu \right) \approx \Gamma \left( \Xi^* \rightarrow e^+ + \nu_e \right) \approx 7 \times 10^{12} \text{ sec}^{-1}
\]

\[
\Gamma \left( \Sigma^+ \rightarrow \mu^+ + \nu_\mu \right) \approx 3 \times 10^{10} \text{ sec}^{-1}
\]

\[
\Gamma \left( \Xi^0 \rightarrow \mu^+ + \nu_\mu \right) \approx 3 \times 10^{10} \text{ sec}^{-1}
\]

The predictions for the pseudoscalars are roughly an order of magnitude higher than the analogous ones by Gaillard, Lee and Rosner\(^4\), essentially based on extrapolation from K-leptonic decay. Indeed, in our calculation the K-decay width
comes out an order of magnitude higher than the experimental value, as in the pion case. If, however, this well-known failure of the non-relativistic quark model for the low-lying pseudoscalars is simply interpreted as a reflection of the already observed inadequate description of these light particles in the model, the predictions for the heavier charmed particles should be considered on a sounder basis.

Coming to the semi-leptonic and non-leptonic decays -- by now a crucial problem for the charmed vector mesons -- our computational ability becomes lower and lower, since here the strong interactions effects can play a major role.

For the semi-leptonic decays of any charmed meson, Gaillard, Lee and Rosner give the following estimate:

$$
\Gamma (M \rightarrow e\nu + k\bar{u}d) \cong \Gamma (M \rightarrow \mu\nu + k\bar{u}d) \cong \left( \frac{m_c}{m_{\mu}} \right)^5 \Gamma (\mu \rightarrow e\nu\bar{\nu}) \times 3 \times 10^{11} \text{ sec}^{-1}
$$

This is what we would expect if the charmed particle decays are viewed as occurring due to elementary quark processes, such as $c \rightarrow \lambda + k^+ + \nu$ or $c \rightarrow \eta + k^+ + \nu$, followed by quark decays with unit probability into stable hadrons.

A similar attitude can perhaps be taken when discussing non-leptonic decays. For processes due to charm quark decays, such as $c \rightarrow \lambda + p + \bar{n}$, one would have

$$
\Gamma_1 (M \rightarrow k\bar{u}d) \cong 9 \times 10^{11} \text{ sec}^{-1}
$$

In particular I neglect here any asymptotic freedom enhancement factor, which for the heavy charmed mesons is suggested not to be as important as for lighter strange particle decays. In addition to $c \rightarrow \lambda + p + \bar{n}$, a second kind of elementary process, such as $c + \bar{\lambda} \rightarrow p + \bar{n}$, can contribute to the non-leptonic decays of charmed mesons. This second kind of mechanism, which adds incoherently with the previous one to the total rate, is similar to the one leading to leptonic decays. Its contribution to the total rate is then particularly important for the vector decays (to the extent that the quark masses $m_\lambda, m_d$ are small compared with the charm meson masses) and can be estimated as

$$
\begin{align*}
\Gamma_2 (F^{*+} \rightarrow k\bar{u}d) & \cong 3 \omega \bar{\tau} \bar{\theta}_c \Gamma (F^{*+} \rightarrow \mu\nu) \cong 21 \times 10^{12} \\
\Gamma_2 (D^{*+} \rightarrow k\bar{u}d) & \cong 3 \omega \bar{\tau} \bar{\theta}_c \Gamma (D^{*+} \rightarrow \mu\nu) \cong 9 \times (10^{12}) \\
\Gamma_2 (D^{*+} \rightarrow k\bar{u}d) & \cong 3 \omega \bar{\tau} \bar{\theta}_c \Gamma (D^{*+} \rightarrow \mu\nu) \cong 6 \times 10^{11}
\end{align*}
$$

Summing up

$$
\begin{align*}
\Gamma_{\text{tot}} (F^{*+}) & \cong \Gamma_{\text{tot}} (D^{*+}) \cong \Gamma_{\text{tot}} (D^c) \cong 10^{12} \text{ sec}^{-1} \\
\Gamma_{\text{tot}} (F^{*+}) & \cong 3 \times 10^{13} \text{ sec}^{-1} \quad \Gamma_{\text{tot}} (D^{*+}) \cong 2 \times 10^{12} \text{ sec}^{-1} \quad \Gamma_{\text{tot}} (D^c) \cong 3 \times 10^{12} \text{ sec}^{-1}
\end{align*}
$$
Which of these two lines is going to be relevant for the experimental comparison depends obviously, as we discussed already, on the actual spectrum. As to the branching ratios, an obvious general conclusion is that, not including any enhancement factor of the charmed non-leptonic decays, makes the semi-leptonic decays (for pseudoscalars) and the leptonic decays (for vectors) an important factor of the total weak decays. And this may be quite consistent with recent experimental evidence.\(^6,10\).

8.3 Radiative decays of $\bar{c}c$ states

The importance of these decays for the newly found particles has been emphasized since the first suggestions of their interpretation as charmonia. And in fact the discovery of some of these decays has recently been reported.\(^5\) As to their quantitative predictions, the use of our wave functions would not lead to any substantial difference with respect to the Cornell group results.\(^11\) The problem then remains of understanding why the experimental values are definitely lower than these predictions.

The criticism to the calculation of the spectrum, especially in connection with the neglected meson-pair contamination of the $\bar{c}c$ wave function, applies here as well.\(^*)\)

8.4 Hadronic decays and Zweig's rule

As we have heard in Appelquist's lectures, the asymptotic freedom mechanism is also likely to give an explanation of Zweig's rule for the hadronic decays. Indeed the extrapolation of the lepton/hadron decay branching ratio from the $\psi$ to the $\phi$ case according to the asymptotic freedom formulae is quite in agreement with the experimental values.\(^7\) However, I think that I would feel better with the asymptotic freedom explanation of Zweig's rule if I would not be disturbed by the following problems, in order of importance: i) The smallness of the $\psi' \to \psi + 2\pi$ decay. (Although the quark diagram for this process is a "disconnected" one, it may be connected by relatively soft gluon lines.) ii) The apparent difficulty in seeing $\psi'$ decays into normal hadrons. (The three-gluon decay process does not substantially distinguish the $\psi$ from the $\psi'$.) iii) The experimental limit on the $\eta'$ width comparable to the $\phi \to 3\pi$ decay width. (One would argue that the two-gluon decay of the $\eta'$, leading to pionic final states, should be bigger than the three-gluon $\phi$-decay.) As to this last point, it is clear that a clean test of these ideas would be the finding of a definitely broader ($\geq 1$ MeV width) pseudoscalar partner of the $\psi$.

\(^*)\) See E. Eichten's talk in these Proceedings.
9. $e^+e^-$ ANNIHILATION AT THE CHARM THRESHOLD

Let me very briefly address my attention finally to the $e^+e^-$ annihilation total cross-section data in the region $\sqrt{s} \gtrsim 3.5$ GeV. A fit to the experimental curve can be made$^3$ for the total charmed hadron production cross-section by assuming that: i) this cross-section is mainly given by quasi-two-body charmed meson final states; ii) their form factors are dominated by the vector meson poles ($\psi$'s excitations), and iii) the three meson vertices ($\psi \rightarrow DD$, $\psi' \rightarrow DD$, $\psi \rightarrow FF$, etc.) are given in the quark pair creation model by diagrams of the type

![Diagram]

The results are shown in Fig. 1: the full line for no interference between the various $\psi$-poles, and the dotted line with the interference taken into account. In this last curve the bump at $\sqrt{s} = 4.1$ GeV is accounted for despite the smallness of the predicted leptonic width (2.7 keV) of the $\psi'(4100)$ resonance. Note also that the opening of the numerous thresholds keeps the value of $R(s)$ substantially high even at $\sqrt{s} = 6$ GeV. Despite this fact, the four-quark model is very probably getting into trouble with preliminary data from SLAC at higher values of $\sqrt{s}$ (up to 7.4 GeV), which show $R$ essentially constant between 5 and 6$^1$. As a matter of fact, a claim that even the data shown in Fig. 1 call for new quarks (and/or for new heavy leptons) comes both from asymptotic freedom arguments$^2$ and from the application to $e^+e^-$ annihilation of finite energy sum rules$^1$. This is essentially because of the observation that, averaging $R(s)$ over energy intervals of order of 1 GeV, we find remarkably constant values $\overline{R}$ around 5, whereas we would expect $3.3 \lesssim \overline{R} \lesssim 4$. This may support the existence of other quarks or of new heavy leptons.

*) See T. Appelquist's lectures in these Proceedings.
REFERENCES


5) W. Braunschweig et al., DESY preprint 75/20 (1975).


Table 1

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Fig. 1