RADIATIVE DECAYS AS A TEST FOR RIGHT-HANDED CURRENTS

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A B S T R A C T

It is shown that the parity violating amplitude for the decays $\Sigma^+ \rightarrow p\gamma$ and $\Xi^- \rightarrow \Sigma^-\gamma$ vanish in the $SU(3)$ symmetry limit if the charged weak currents possess a U spin symmetry for strangeness changing processes. This applies to Cabibbo currents and their $SU(4)$ generalization. Assuming an asymptotically free model for the strong interactions this result applies to all models with only left-handed charged currents. Observation of a large parity violating amplitude would indicate right-handed charged weak currents. Tests involving other radiative decays are also considered.

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The study of semileptonic decay processes at low energies suggests that hadronic weak interactions involving conventional hadrons may be well described by the Cabibbo scheme involving left-handed charged currents alone. However, the structure of the hadronic weak interactions at higher energies, possibly involving new hadronic degrees of freedom, is not clear. One way to get information on the hadronic weak interactions at all energies is through non-leptonic weak processes, because these processes involve a sum over all possible intermediate states.

In this note we examine what can be learnt from a study of the weak radiative decays of hadrons. We find that for models which have a U spin symmetry for strangeness changing processes for the charged weak currents, the parity violating (p.v.) amplitude for $\Sigma^+ \rightarrow p\gamma$ and $\Xi^- \rightarrow \Sigma^-\gamma$ should vanish when U spin breaking is zero. Both conventional Cabibbo currents and the SU(4) generalization of these currents due to Glashow, Iliopoulos and Maiani¹⁾ have this U spin symmetry and so observation of a large parity violating amplitude in these decays would be indicative of components in the hadronic current which violate the U spin symmetry or of large symmetry breaking effects arising from the short distance piece. If we assume that the strong interactions are described by an asymptotically free gauge theory (AFGT) coupled to colour, then we find that for all models with only left-handed currents the contribution of the short-distance piece to the parity violating amplitude should vanish in leading order.

In this case, observation of a large parity violating amplitude in $\Sigma^+ \rightarrow p\gamma$ or $\Xi^- \rightarrow \Sigma^-\gamma$ would be indicative of right-handed hadronic weak currents which violate the U spin symmetry. Implications for other radiative decays are discussed.

Non-leptonic strangeness changing radiative decays involve the time-ordered product of three currents (two charged weak and one electromagnetic) coming from Fig. 1a and the time-ordered product of two (weak) currents coming from the graphs of Fig. 1b (neutral strangeness changing currents are not, of course, expected in first order).

In the Cabibbo theory the strangeness conserving current $J_\mu^{\Delta S=0}$ and the strangeness changing current $J_\mu^{\Delta S=1}$ are given by

$$\begin{aligned} J_\mu^{\Delta S=0} &= \frac{1}{2} (\bar{\psi} \gamma_\mu (1-\gamma_5) \psi) \\ J_\mu^{\Delta S=1} &= \frac{1}{2} (\bar{\psi} \gamma_\mu (1-\gamma_5) \lambda) \end{aligned} \quad (1)$$

Then under a U spin rotation $R = \exp(i\pi U_2)$ they transform as U spin spinors

$$J_{\mu}^{\Delta S=0 (\Delta S=1)} = \mp R J_{\mu}^{\Delta S=1 (\Delta S=0)} \quad (2)$$

Since the photon is a U spin scalar the time-ordered products $T(J_{\mu}^{\Delta S=0 \dagger}(x) J_{\nu}^{\Delta S=1}(y) J_{\lambda}^{\text{em}}(0))$ and $T(J_{\mu}^{\Delta S=0 \dagger}(x) J_{\nu}^{\Delta S=1}(y))$ will both transform as $U_3 = -1$ components of a U spin vector. For example,

$$R T(J_{\mu}^{\Delta S=0 \dagger}(x) J_{\nu}^{\Delta S=1}(y)) R^{-1} = -T(J_{\mu}^{\Delta S=1 \dagger}(x) J_{\nu}^{\Delta S=0}(y)) \quad (3)$$

Then assuming U spin and CP invariance we may immediately deduce ²⁾ for the p.v. amplitude A_{pv} between U spin spinor states p and Σ^+ (or Σ^- and Ξ^-)

$$\begin{aligned} A_{pv}(\Sigma^+ \bar{p} \rightarrow \gamma) &= A_{pv}(\bar{p} \Sigma^+ \rightarrow \gamma) \\ A_{pv}(\Xi^- \bar{\Xi}^- \rightarrow \gamma) &= A_{pv}(\bar{\Xi}^- \Xi^- \rightarrow \gamma) \end{aligned} \quad (4)$$

This result combined with the CP relation ²⁾

$$A_{pv}(B_1 \bar{B}_2 \rightarrow \gamma) + A_{pv}(\bar{B}_2 B_1 \rightarrow \gamma) = 0 \quad (5)$$

implies the vanishing of the p.v. amplitude in the absence of U spin violation.

Evidently this argument proceeds for any model in which the strangeness changing and strangeness conserving currents are related by a U spin rotation as in Eq. (2). This is the case in the SU(4) generalization of the Cabibbo currents where the charmed currents $J_{\mu}^{\Delta S=0, c}$ and $J_{\mu}^{\Delta S=1, c}$ are given by

$$\begin{aligned} J_{\mu}^{\Delta S=0, c} &= \frac{1}{2} (\bar{p}' \gamma_{\mu} (1 - \gamma_5) n) \\ J_{\mu}^{\Delta S=1, c} &= \frac{1}{2} (\bar{p}' \gamma_{\mu} (1 - \gamma_5) \lambda) \end{aligned} \quad (6)$$

Here p' is the fourth (charmed) quark.

How big could the U spin breaking effects be ? Phenomenological analysis ³⁾ of the non-short distance piece suggests only p.v. terms of order 10% of the parity conserving (p.c.) amplitude arise. The p.c. amplitude is expected to be of order $G_F \sin \theta_c$. Experiment ⁴⁾ at present is consistent with an amplitude of order $G_F \sin \theta_c$ but the p.v. amplitude is large, the asymmetry parameter α being $(-1.03^{+0.52}_{-0.42})$.

It has been suggested ⁵⁾ that symmetry breaking effects in the short distance piece could be enhanced and this could explain a large p.v. amplitude. Such an enhancement could occur if the amplitude were proportional to external masses for then, in the free quark model for example, the p.c. piece would be proportional to the quark masses $m_n + m_\lambda$ while the p.v. piece would be proportional to $m_n - m_\lambda$. If $m_n \ll m_\lambda$ there would be no significant suppression of the p.v. piece. For a model with only charged left-handed currents this mass structure is indeed expected. Due to gauge invariance, the amplitude for the radiative baryon decay $I \rightarrow F + \gamma$ has the form

$$T(I \rightarrow F + \gamma) = i e^\lambda(q) \bar{u}(p_F) [C + D \gamma_5] \sigma_{\lambda\mu} q^\mu u(p_I) \quad (7)$$

where q and ϵ^λ are the photon momentum and polarization.

It therefore flips the nucleon helicity. However, both the electromagnetic currents and the vector weak currents conserve helicity so in order for them to generate a helicity flip amplitude a mass insertion is necessary. In a theory with only left-handed charged weak currents one mass insertion cannot be within the weak boson loop and is therefore an external mass, m_n or m_λ in the free quark model. In a theory with both left-handed and right-handed currents one mass insertion can be within the boson loop, m_p or $m_{p'}$ in the SU(4) quark model.

Large p.v. effects due to U spin breaking could therefore arise from the short distance piece in a model with only left-handed currents. However, if we now restrict our attention to renormalizable models of the weak and electromagnetic interactions in which the strong interactions are described by an AFGT coupled to colour, these short-distance effects are calculable and will turn out to be negligible.

As described in Ref. 6) the time-ordered product of currents is given by

$$T[J_\alpha(\mu^-x)J_\beta(\mu^-y)\dots J_\gamma(0)] \underset{\mu \rightarrow \infty}{\sim} \sum_n \mu^{D_{O_n}} \exp \left\{ \int_1^\mu \gamma_{O_n}(g(x')) \frac{dx'}{x'} \right\} G^n(x,y,\dots,g(\mu),m(\mu)) O^n(0) \quad (8)$$

where the sum n runs over all the operators O_n contributing to the Wilson expansion. D_{O_n} is the dimension of the operator O_n , γ_{O_n} is the anomalous dimension of O_n and $g(\mu)$ and $m(\mu)$ are the effective strong coupling constant and effective mass as defined in Ref. 6). Since $g(\mu) \rightarrow 0$ for μ large the SU(3) non-singlet operators contributing to this expansion will, to order $g(\mu)^2$, be those of the free quark model.

The leading operators that may contribute to the radiative baryon decay amplitude defined by Eq. (7) are $O_{\alpha\beta} \equiv \bar{n}_\sigma \alpha_\beta \lambda$ and $O_\alpha \equiv \partial^\beta \bar{n}_\sigma \alpha_\beta \lambda$ for the p.c. amplitude and $O_{\alpha\beta}^5 \equiv \bar{n}_\sigma \alpha_\beta \gamma_5 \lambda$ and $O_\alpha^5 \equiv \partial^\beta \bar{n}_\sigma \alpha_\beta \gamma_5 \lambda$, for p.v. amplitude. Can these operators arise?

We consider models in which strangeness changing processes are mediated by charged current only; we assume no neutral strangeness changing currents in first order. The Cabibbo current will generate such operators in the free quark model and, as discussed above, they contribute to Eq. (7) terms proportional to the masses of the external quark lines m_n and m_λ . However, the free quark model operators with only Cabibbo currents will also generate divergences which, in a renormalizable model, must be cancelled by counter terms or contributions from other currents. In particular there is, in the 't Hooft-Feynmann gauge, a logarithmic divergence in the irreducible $\bar{n}\lambda\gamma$ vertex coming from Fig. 1a. Since there is no $\bar{n}\lambda\gamma$ counter term in our class of model this divergence must be cancelled by other charged current contributions and since both this divergence and the contribution, in leading order, to Eq. (7) are independent of internal masses both these contributions will be cancelled when all charged currents are considered.

Thus models with only left-handed charged currents coupling to the n and λ will not contribute to the short distance piece of radiative decays in leading order. The possible contributions will be of order $G_F \sin \theta_c (\Delta m^2 / M_W^2)$ where Δm^2 is the mass difference between the p quark

and the other quarks coupling to the n and λ via a charged current. This result applies not only to the $SU(4)$ model, where the amplitude has been computed in Ref. 7), but also to other models, such as the Georgi-Glashow eight quark model ⁸⁾, in which only the charged currents coupling to the n and λ are purely left-handed.

Since the non-short piece is expected to have only 10% U spin symmetry breaking effects we conclude that observation of a large p.v. amplitude in $\Sigma^+ \rightarrow p\gamma$ or $\Xi^- \rightarrow \Sigma^- \gamma$ would be evidence for a right-handed component of the charged weak current which does not possess the U spin symmetry. (For right-handed currents the leading contribution is not proportional to external masses and so symmetry breaking effects are not expected to be enhanced in the short distance piece.)

Finally we compute the expected size of the radiative decay amplitudes in a model with such currents. We take the weak and electromagnetic interactions to be described by the $SU(2) \times U(1)$ Salam-Weinberg gauge model with the GIM mechanism. In addition we introduce a right-handed doublet coupling the p' to the n , or the λ . The former occurs in models suggested by De Rujula, Georgi and Glashow ⁹⁾, the latter in models suggested by Fritzsch, Gell-Mann and Minkowski ¹⁰⁾.

The graph of Fig. 1a may be evaluated in the 't Hooft-Feynmann gauge with a contribution $M(q,a)$ for the process $I \rightarrow F + \gamma$

$$M(q,a) = \frac{e^{\lambda}(q) e g^2}{2(2\pi)^4} \int d^4k \frac{1}{k^2 - M_w^2} \int d^4z \int d^4y e^{ik \cdot (z-y)} \langle F | T(J_{\alpha}^w(z) J_{\alpha}^w(y) J_{\lambda}^{em}(0) | I \rangle \quad (9)$$

where

$$J_{\alpha}^w = \frac{1}{2} \bar{p} \gamma_{\alpha} (1 - \gamma_5) (n \cos \theta_c + \lambda \sin \theta_c) + \frac{1}{2} \bar{p}' \gamma_{\alpha} (1 - \gamma_5) (-n \sin \theta_c + \lambda \cos \theta_c) + \frac{1}{2} \bar{p}' \gamma_{\alpha} (1 + \gamma_5) n \quad \{ \text{or } \lambda \}$$

The contribution of (unphysical) scalars to this graph are suppressed by a factor $1/M_W^2$.

Figure 1b gives a contribution $M(q,b)$ from the vector boson piece

$$M(q,b) = - \frac{ie g^2 E^\lambda(q)}{2(2\pi)^4} \int d^4 k \left(\frac{1}{k^2 - M_W^2} \right) \left(\frac{1}{(q-k)^2 - M_W^2} \right) [g_{\beta\lambda} (k-2q)_\alpha - 2g_{\alpha\beta} k_\lambda + g_{\alpha\lambda} (k+q)_\beta] \int d^4 x e^{ik \cdot x} \langle F | T(J_\alpha^{W^\dagger}(x) J_\beta^W(0)) | I \rangle \quad (10)$$

The (unphysical) Higgs scalars give a contribution $M(q,c)$ in leading order given by the graphs of Fig. 2.

$$M(q,c) = - \frac{ie g^2 E^\lambda(q)}{2(2\pi)^4} \int d^4 k \left(\frac{1}{k^2 - M_W^2} \right) \left(\frac{1}{(q-k)^2 - M_W^2} \right) \int d^4 x e^{ik \cdot x} \langle F | T(J_\alpha^{W^\dagger}(x) S(q)) | I \rangle + \text{crossed} \quad (11)$$

where

$$S(x) = \frac{1}{2} \cos \Theta_c \bar{\psi}(x) [m_n(1+\gamma_5) - m_p(1-\gamma_5)] \psi(x) + \frac{1}{2} \sin \Theta_c \bar{\psi}(x) [m_n(1+\gamma_5) - m_p(1-\gamma_5)] \lambda(x) + \frac{1}{2} \sin \Theta_c \bar{\psi}'(x) [-m_n(1+\gamma_5) + m_{p'}(1-\gamma_5)] \lambda(x) + \frac{1}{2} \cos \Theta_c \bar{\psi}'(x) [m_n(1+\gamma_5) - m_{p'}(1-\gamma_5)] \lambda(x) + \frac{1}{2} \bar{\psi}(x) [m_n(1-\gamma_5) - m_p(1+\gamma_5)] \lambda(x) \} \propto \lambda(x)$$

The operator product expansion is given by Eq. (8). We find that $T(J_\alpha^{W^\dagger}(x) J_\alpha^W(y) J_\lambda^{\text{em}}(0))$ does not contribute in leading order to Eq. (7). For $M(q,b)$ we find

$$T(J_\alpha^{W^\dagger}(\mu^{-1}x) J_\beta^W(0)) \underset{\mu \rightarrow \infty}{\sim} \frac{\mu^{+2}}{8\pi^2} [1 + b \ln(\frac{x^2}{\mu^2})]^{(-\gamma_0 + \gamma_m)/2b}$$

$$\frac{m_{p'}}{x^2} (O_{\alpha\beta}(0) - O_{\alpha\beta}^S(0)) \cos \Theta_c \left\{ \alpha (O_{\alpha\beta}(0) + O_{\alpha\beta}^S(0)) \sin \Theta_c \right\} \quad (12)$$

where for n colours

$$\gamma_m = \gamma_0 = \frac{3g^2}{16\pi^2} \left(\frac{n^2-1}{n} \right), \quad b = -\frac{g^2}{16\pi^2} \left(\frac{11n-8}{3} \right)$$

$m_{p'}$ is the p' quark mass evaluated at the Euclidean co-ordinate x .

Inserting this in Eq. (10) we find for the short distance contribution to the radiative decay amplitude ¹¹⁾

$$M(q, b) = \left(\frac{3}{4} \right) \frac{i e \gamma_5 G_F m_{p'}}{\sqrt{2} 2\pi^2} \bar{u}(p_F) i \sigma_{\lambda\alpha} q^\alpha c \\ (1-\gamma_5) \cos \theta_c \left\{ \text{or } (1+\gamma_5) \sin \theta_c \right\} u(p_I) \quad (13)$$

where c is given by

$$\langle F | O_{\alpha\beta} | I \rangle = c \bar{u}(p_F) \sigma_{\alpha\beta} u(p_I) \\ \langle F | O_{\alpha\beta}^5 | I \rangle = c \bar{u}(p_F) \gamma_5 \sigma_{\alpha\beta} u(p_I) \quad (14)$$

The contribution from the graphs of Fig. 2 gives $1/3 M(q, b)$ altogether $M(q)$ is given by $(4/3)M(q, b)$.

Thus with these right-handed currents, we expect short distance contribution to the decays $\Sigma^+ \rightarrow p \gamma$ and $\Xi^- \rightarrow \Sigma^- \gamma$ of order $e G_F \cos \theta_c C$ (or $e G_F \sin \theta_c C$) with the p.v. amplitude equal to (or opposite to) the p.c. amplitude. The corresponding asymmetry parameter α is $+1$ (or -1).

What about other radiative decays? If we have only charged left-handed currents then the short distance contribution vanishes in the symmetry limit for the neutral decays $\Xi^0 \rightarrow \Sigma^0 \gamma$, $\Xi^0 \rightarrow \Lambda \gamma$, $\Lambda \rightarrow n \gamma$ as well as for the charged decays $\Sigma^+ \rightarrow p \gamma$ and $\Xi^- \rightarrow \Sigma^- \gamma$. The non-short piece for the neutral decays has been analyzed elsewhere ¹²⁾. The relative size of the p.v. and p.c. amplitudes depends on the f/d ratio so there is no direct test for the absence of the short distance piece. However, we note that using the U spin part of octet dominance of $\mathcal{L}_W$ in the non-short piece the following relations were obtained for the parity violating amplitudes $b_{I,F}$

$$b_{\Lambda, n} = -b_{\Xi^0, \Lambda} = -\frac{1}{3} \sqrt{3} b_{\Xi^0, \Sigma^0} \equiv \beta$$

If right-handed currents are present giving rise to a parity violating short distance piece, these relations become

$$b_{\Xi^0, \Sigma^0} = -\frac{1}{2} \sqrt{2} b_{\Xi^0, \Sigma^+} - \sqrt{3} \beta$$

$$b_{\Xi^0, \Lambda} = \frac{1}{6} \sqrt{6} (b_{\Xi^0, \Sigma^+} - 2 b_{\Sigma^+, \Lambda}) - \beta$$

$$b_{\Lambda, n} = \frac{1}{6} \sqrt{6} (b_{\Sigma^+, \Lambda} - 2 b_{\Xi^0, \Sigma^+}) + \beta$$

Radiative non-leptonic K decays $K_a \rightarrow \pi_b \pi_c \gamma$, where a can be $\pm$, L or S and b, c can be $\pm$ and 0, have also been analyzed by Gaillard and Roy⁵⁾. Using octet dominance of $\mathcal{A}_W$ they derived a relation for the non-short part of parity violating electric term E

$$E_{\Sigma^+ -} = -2 E_{\pm 0 \pm}$$

In the absence of right-handed currents this relation would be true for the full electric term. If it does not hold [up to SU(3) breaking] this would be evidence for right-handed currents.

In conclusion, we have found that the observation of a large p.v. amplitude in $\Sigma^- \rightarrow p \gamma$ and $\Xi^- \rightarrow \Sigma^- \gamma$ decays would be evidence for the presence of right-handed currents. Moreover, the sign of the asymmetry parameter should distinguish between various possible right-handed currents^{10), 11)}. Present experimental data suggests that right-handed currents coupling to the λ quark are present, but a more accurate measurement is necessary to confirm the effect.

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FIGURE CAPTIONS

Fig. 1 : Diagrams giving rise to radiative hadron decay.

Fig. 2 : Higgs scalar contribution to radiative decays.

FIG. 1

FIG. 2