Effect of exchange interaction on fidelity of quantum state transfer from a photon qubit to an electron-spin qubit

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We analyzed the fidelity of the quantum state transfer (QST) from a photon-polarization qubit to an electron-spin-polarization qubit in a semiconductor quantum dot, with special attention to the exchange interaction between the electron and the simultaneously created hole. In order to realize a high-fidelity QST we had to separate the electron and hole as soon as possible, since the electron-hole exchange interaction modifies the orientation of the electron spin. Thus, we propose a double-dot structure to separate the electron and hole quickly, and show that the fidelity of the QST can reach as high as 0.996 if the resonant tunneling condition is satisfied.

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Quantum state transfer (QST) has attracted enormous attention as one of the key concepts in quantum information science. Quantum information can take several different forms such as photons, nuclear spin of atoms, and electron spin of quantum dots. All of these physical realizations of quantum information are called “qubits”. Since each qubit has its own merits and demerits, we have to choose the right qubit for each process. The photon-polarization qubit is the most convenient medium for sharing quantum information between distant locations. Presently, we can distribute quantum keys over 122km of standard telecom fiber. On the other hand, the electron-spin qubit is the most convenient medium for quantum gate and quantum memory in a semiconductor quantum dot, since coupling among the qubits can easily be controlled by gate voltage. Electron-spin qubits are a promising candidate for the realization of a scalable quantum computer. It is then a logical next step to study the QST from a photon qubit to an electron-spin qubit in order to construct efficient quantum information processing devices.

In 2001, Vrijen and Yablonovitch proposed a spin-coherent semiconductor photo-detector which transfers the quantum information from a photon-polarization qubit to an electron-spin qubit. Such a quantum-state-coherent photo-detector is a basic element of a quantum repeater, which enables us to drastically expand the distance of quantum key distribution. They showed that the well-known optical orientation in semiconductor heterostructure can be used for the QST.

The spin-coherent semiconductor photo-detector has an optically active quantum well where the quantum information is transferred from photon polarization to electron spin. The k-vector of the incident photon is parallel to the growth direction of the well. The energy levels of the well are shown in Fig. 1(a). In order to carry out the photon-spin QST, the spin states $|\uparrow\rangle$ and $|\downarrow\rangle$ of an electron should be degenerate in the presence of a magnetic field. Therefore, we have to tune the electron spin $g$-factor to be zero, $g_e = 0$, with the help of $g$-factor engineering. The $g$-factor engineering can be realized by using the proximity of the electron wave function into the barrier layer. In the quantum well system, the $g_e$ can be estimated as $g_e = w g_W + (1 - w) g_B$, where $g_W$ and $g_B$ are the $g$-factor of the well and that of the barrier, respectively, and $w$ is the occupation probability of the electron in the well. Appropriate choice of the structure and the material enables us to obtain $g_e = 0$. The degeneracy between the heavy-hole states and light-hole states is lifted if the material is placed under tensile strain. The uniform magnetic field $B$ is applied along the $z$-direction to lift the degeneracy of the light-hole states $|\psi^+\rangle_{lh} = \sqrt{1/2} (|J = 3/2, m_J = 1/2\rangle + |J = 3/2, m_J = -1/2\rangle)/\sqrt{2}$ and $|\psi^-\rangle_{lh} = \sqrt{1/2} (|J = 3/2, m_J = 1/2\rangle - |J = 3/2, m_J = -1/2\rangle)/\sqrt{2}$. The Zeeman splitting between these two states is given by $g_h \mu_B B$, where $\mu_B$ is the Bohr magneton. The Zeeman splitting of the electron spin states is assumed to be zero. According to the selection rule, the electron with the spin up state along the $z$-direction $|\uparrow\rangle$ is excited in the quantum dot by a right-handed circularly polarized photon $|\sigma^+\rangle$. Similarly, a left-handed circularly polarized photon $|\sigma^-\rangle$ excites the electron in the spin-down state $|\downarrow\rangle$. In these two cases, a hole in the $|\psi^+\rangle_{lh}$ state is created in the dot simultaneously. After elimination of the hole, the superposition of the polarized photon $\alpha_+ |\sigma^+\rangle + \alpha_- |\sigma^-\rangle$ is transferred to the superposition of the electron spin $\alpha_+ |\uparrow\rangle + \alpha_- |\downarrow\rangle$.

One of the main obstacles to high-fidelity QST in a spin-coherent semiconductor photo detector is the exchange interaction between the electron and the simultaneously created hole. In this paper, we analyze the effect of the exchange interaction on the fidelity of the QST from a photon-polarization qubit to an electron-spin qubit. For high-fidelity QST we have to extract the hole as soon as possible. We propose a double-well structure to separate the electron and hole quickly via resonant tunneling. Quick extraction of the carrier using resonant tunneling in the double-well structure was extensively studied by Gurvitz, and experimentally demonstrated by Cohen. Using the double-well structure, quick extraction of the hole can be realized without...
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state, the electron with spin state $\sigma$ is polarized photon
optically excited by the right-handed (left-handed) circularly
polarized photon $|\sigma^+\rangle (|\sigma^-\rangle)$. (b) Energy band of the system.
The electron-hole pair is excited in dot1. The created hole is
extracted from dot1 to the continuum of the hole via dot2.

From the $|\psi^+\rangle_{th}$ state, the electron with $|\uparrow\rangle (|\downarrow\rangle)$ spin state is
optically excited by the right-handed (left-handed) circularly
polarized photon $|\sigma^+\rangle (|\sigma^-\rangle)$. (b) Energy band of the system.
The electron-hole pair is excited in dot1. The created hole is
extracted from dot1 to the continuum of the hole via dot2.

The wave function of the system can be written as

$$|\Psi(t)\rangle = \sum_{s=\uparrow,\downarrow} \phi_{1s}(t) |sh_1\rangle + \sum_{s=\uparrow,\downarrow} \phi_{2s}(t) |sh_2\rangle + \sum_{s=\uparrow,\downarrow} \psi_{ls}(t) |sl\rangle,$$

where $s=\uparrow, \downarrow$ denotes the electron state with spin $s$, $h_{1(2)}$ the hole state in dot1(2), and $l$ the hole state in the
continuum. Note that the state of the hole in the dot1 $h_1$ is
assumed to be restricted into the top-most light-hole
state $|\psi^+\rangle$. Here, $\phi_{1s}(t)$, $\phi_{2s}(t)$, and $\psi_{ls}(t)$ are coefficients
to be determined by solving the Schrödinger equation.

The Hamiltonian of the system is expressed as

$$H = \sum_{s=\uparrow,\downarrow} (\omega_e + \omega_1) |sh_1\rangle \langle sh_1| + \sum_{s=\uparrow,\downarrow} \omega_f |sh_1\rangle \langle sh_1|$$
$$+ \sum_{s=\uparrow,\downarrow} \{\delta |sh_1\rangle \langle sh_2| + h.c.\} + \sum_{s=\uparrow,\downarrow} (\omega_e + \omega_2) |sh_2\rangle \langle sh_2|$$
$$+ \sum_{s=\uparrow,\downarrow} \sum_{l} \{W_l |sh_2\rangle \langle sl| + h.c.\}$$
$$+ \sum_{s=\uparrow,\downarrow} \sum_{l} (\omega_e + \omega_l) |sl\rangle \langle sl|,$$

where $\omega_e$ is the energy level of the electron in dot1, $\omega_{1(2)}$ the hole energy level in dot1(2), $\omega_f$ the electron-hole
exchange interaction, $\omega_l$ the hole energy level in the
continuum, $\delta$ the coupling between dot1 and dot2, $W_l$ the coupling between dot2 and continuum state $l$, and $s$ the electron spin opposite to $\sigma$. Here, we set $\hbar = 1$. In
a zincblende crystal, the electron-hole exchange interaction
is given by $a s \cdot J + b \sum_{\lambda=x,y,z} s_\lambda J_\lambda$, where $a, b$
are coefficients, $s$ and $J$ represent the electron and hole
spin, respectively. In Eq. (2), we consider the coupling
term between two degenerated states $|\uparrow h_1\rangle$ and $|\downarrow h_1\rangle$.

The dynamics of the system are obtained by solving
the following Schrödinger equation:

$$\dot{\phi}_{1s}(t) = -i(\omega_e + \omega_1)\phi_{1s}(t) - i\omega_f\phi_{2s}(t) - i\delta\phi_{2\sigma}(t),$$
$$\dot{\phi}_{2s}(t) = -i(\omega_e + \omega_2)\phi_{2s}(t) - i\delta\phi_{1s}(t)$$
$$- i \sum_{l} W_l \psi_{ls}(t),$$
$$\dot{\psi}_{ls}(t) = -i(\omega_e + \omega_l)\psi_{ls}(t) - iW^*_l \phi_{2s}.$$  

These equations can be simplified by changing the electron
spin basis from the eigenstates $(|\uparrow\rangle, |\downarrow\rangle)$ of $\sigma_z$ to the
eigenstates $(|+\rangle, |--\rangle)$ of $\sigma_z$, introducing

$$\phi_{1\pm}(t) = (\phi_{1\uparrow}(t) \pm \phi_{1\downarrow}(t))/\sqrt{2},$$
$$\phi_{2\pm}(t) = (\phi_{2\uparrow}(t) \pm \phi_{2\downarrow}(t))/\sqrt{2},$$
$$\psi_{\pm}(t) = (\psi_{l\uparrow}(t) \pm \psi_{l\downarrow}(t))/\sqrt{2}.$$

Eqs (8)–(11) are rewritten as

$$\dot{\phi}_{1\sigma}(t) = -i(\omega_e + \omega_1 + \omega_{J\sigma})\phi_{1\sigma}(t) - i\delta\phi_{2\sigma}(t),$$
$$\dot{\phi}_{2\sigma}(t) = -i(\omega_e + \omega_2)\phi_{2\sigma}(t) - i\delta\phi_{1\sigma}(t)$$
$$- i \sum_{l} W_l \psi_{ls}(t),$$
$$\dot{\psi}_{\sigma}(t) = -i(\omega_e + \omega_l)\psi_{\sigma}(t) - iW^*_l \phi_{2\sigma}(t),$$

where $\sigma = \pm$, $\omega_{J\pm} = \pm \omega_J$. One can easily see that Eqs
(8)–(11) are separable with respect to the index $\sigma = \pm$. 

![FIG. 1: (a) Selection rule for the quantum-state transfer from photon polarization to electron spin. The light-hole levels are split into $|\psi^+\rangle_{th}$ and $|\psi^-\rangle_{th}$ by the applied magnetic field $B$.](image)
The solution of Eq. (11) is obtained as
\[
\psi_{\sigma}(t) = -iW^*_t \int_0^t dt' e^{-i(\omega_c + \omega_l)(t-t')} \phi_{2\sigma}(t').
\] (12)

The tunneling process of the hole from the dot2 to the continuum is characterized by the spectral density function \( \gamma_h(\omega) \equiv \pi \sum_i |W^*_i|^2 \delta(\omega - \omega_i) \). Given the density of the state of the hole in the continuum is dense around \( \omega \sim \omega_2 \), we can treat \( \gamma_h(\omega) \) as a constant, which corresponds to the Markov approximation. Substituting Eq. (12) into Eq. (10), and applying the Markov approximation, we have
\[
\phi_{2\sigma}(t) = -i(\omega_c + \omega_2 - i\gamma_h) \phi_{2\sigma}(t) - i\delta^* \psi_{1\sigma}(t).
\] (13)

Here, \( \gamma_h \) represents the tunneling rate of the hole from the dot2 to the continuum.

Applying the Laplace transformation \( \hat{\phi}_{\sigma}(p) = \int_0^\infty dt e^{-pt} \phi_{\sigma}(t) \), Eqs (12) and (13) can be expressed as
\[
p\hat{\phi}_{1\sigma}(p) - \beta_\sigma = -i(\omega_c + \omega_1 + \omega_J) \hat{\phi}_{1\sigma}(p) - i\delta \hat{\phi}_{2\sigma}(p),
\] (14)
\[
p\hat{\phi}_{2\sigma}(p) = -i(\omega_c + \omega_2 - i\gamma_h) \hat{\phi}_{2\sigma}(p) - i\delta^* \hat{\phi}_{1\sigma}(p),
\] (15)

where \( \beta_\sigma \) are the coefficients for the linear combination of electron spin states \( |\pm\rangle \) at \( t = 0 \). Then we obtain
\[
\hat{\phi}_{1\sigma}(p) = \beta_\sigma \left[ p + i(\omega_c + \omega_1 + \omega_J) - i\delta \right],
\] (16)
\[
\hat{\phi}_{2\sigma}(p) = -i\delta^* \hat{\phi}_{1\sigma}(p).
\] (17)

The fidelity of the QST is defined as \( F = \langle \Psi(0)| \hat{\rho}(\infty) |\Psi(0) \rangle \), where \( \hat{\rho}(t) \) is the reduced density matrix, and \( |\Psi(0)\rangle = \beta_+ |+\rangle + \beta_- |-\rangle \) is the initial state of the spin. Each component of \( \hat{\rho}(t) \) is defined as
\[
\rho_{\sigma\sigma'}(t) = \psi_{\sigma}(t) \psi_{\sigma'}^*(t) + \sum_i \psi_i(t) \psi_i^*(t).
\] (18)

In the limit of \( t \rightarrow \infty \), the hole is in the continuum and \( \phi_{1\sigma}(\infty) = \phi_{2\sigma}(\infty) = 0 \). Hence, we have \( \rho_{\sigma\sigma'}(\infty) = \sum_i \psi_i(\infty) \psi_i^*(\infty) \). The reduced density matrix \( \rho_{\sigma\sigma'}(\infty) \) can be easily evaluated by moving to the interaction picture. In the interaction picture, the reduced density matrix is expressed as \( \rho_{\sigma\sigma'}(\infty) = \sum_i \psi_i(\infty) \psi_i^*(\infty) \), where \( \tilde{\psi}_i(\infty) = e^{i(\omega_i + \omega_l)t} \psi_i(t) \). From Eqs (12) and (17), \( \tilde{\psi}_i(\infty) \) is given by
\[
\tilde{\psi}_i(\infty) = -iW^*_i \delta_{2\sigma}(p) + i(\omega_c + \omega_l)) \beta_\sigma \frac{\delta W_i}{f_\sigma(\omega_i)}.
\] (19)

where \( f_\sigma(\omega) = (\omega_1 - \omega + \omega_J(\omega_2 - \omega - i\gamma_h) - |\delta|^2 \). Thus we have \( \rho_{\sigma\sigma'}(\infty) = \beta_\sigma \beta_\sigma^* I_{\sigma\sigma'} \), where
\[
I_{\sigma\sigma'} = \frac{|\delta|^2 \gamma_h}{\pi} \int_0^\infty d\omega \frac{1}{f_\sigma(\omega) f_\sigma^*(\omega)}. \] (20)

Finally, the fidelity of the QST is obtained as
\[
F = 1 - 2|\beta_+|^2|\beta_-|^2(1 - \Re I_{+\bar{+}}).
\] (21)

Equation (21) shows that the fidelity depends strongly on the initial state of the electron spin, \( \beta_\sigma \). If the initial state of the electron spin is \( (|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2} \) \( (\beta_+ = 1, \beta_- = 0) \) or \( (|\uparrow\rangle - |\downarrow\rangle)/\sqrt{2} \) \( (\beta_+ = 0, \beta_- = 1) \), the electron-hole exchange interaction does not affect the fidelity since the initial state is the eigenstate of the electron-hole exchange interaction. For the general initial states with \( |\beta_+|^2|\beta_-|^2 \neq 0 \), the fidelity is reduced from unity by the electron-hole exchange interaction. In Fig. 2 we plot the fidelity \( F \) as a function of the electron-hole exchange interaction, \( \omega_J \), and the tunneling rate of the hole from dot2 to the continuum, \( \gamma_h \). We assume that the hole energy levels in dot1 and dot2 are the same, i.e., \( \omega_1 = \omega_2 \). The initial state of the electron spin is taken to be \( |\uparrow\rangle \) \( (|\beta_+|^2 = |\beta_-|^2 = 1/2) \). These two axis values are normalized by the inter-dot coupling \( |\delta| \).

As shown in Fig. 2 the fidelity \( F \) is a monotonic decreasing function of \( \omega_J \) for \( \omega_J < |\delta| \). The fidelity becomes lower than 1/2 for \( \omega_J > |\delta| \) because the electron-hole exchange interaction flips the electron-spin state before the hole is extracted from dot1. Therefore, the first condition for high-fidelity QST is \( \omega_J \ll |\delta| \). The second condition for high-fidelity QST is for \( \gamma_h \). The fidelity is not a monotonic function of \( \gamma_h \) but has a maximum around \( \gamma_h \sim |\delta| \) as shown in Fig. 2. For \( \gamma_h \ll |\delta| \), the escape time of the hole is dominated by the tunneling rate from dot2 to the continuum, \( \gamma_h \). As we increase \( \gamma_h \), the escape time of the hole decreases. Therefore, the fidelity increases with increasing \( \gamma_h \) as long as \( \gamma_h \ll |\delta| \). On the contrary, for \( \gamma_h \gg |\delta| \), coherent oscillation between hole states in dot1 and dot2 is suppressed by the strong coupling between dot2 and the continuum. Therefore, the hole tends to localize in dot1 and the fidelity decreases with increasing \( \gamma_h \). The quickest extraction of the hole is performed at \( \gamma_h \sim |\delta| \), which is called the resonant tunneling regime. Hence, the conditions for high-fidelity QST are given by \( \omega_J \ll |\delta| \sim \gamma_h \).

We now proceed to the estimation of the fidelity

FIG. 2: Contour plot of the fidelity \( F \) as a function of \( \omega_J/|\delta| \) and \( \gamma_h/|\delta| \). The initial spin state is assumed to be \( |\uparrow\rangle \).
of the electron spin using realistic parameters of GaAs/Al_{0.8}In_{0.2}As heterostructures. We can set the electron $g$-factor in the quantum dot to be zero by adjusting the thickness of the GaAs layer. The energy levels shown in Fig. 1(a) can be realized in this heterostructure since tensile strain is applied to the GaAs layer. The inter-dot coupling, $\delta$, can be calculated by considering the boundary conditions for the wave function for the hole $\psi_x$.

In conclusion, we analyzed the effect of the electron-hole exchange interaction on the QST in a spin-coherent semiconductor photo-detector. We have shown that the fidelity decreases as the strength of the exchange interaction increases, and that it depends on the initial state of the electron spin. We have also shown that a high-fidelity ($F \sim 0.996$) QST is possible using the double-dot structure under the resonant tunneling condition of a hole.

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[35] M. Bayer, G. Ortmeyer, O. Stern, A. Kather, A. Gorbunov,