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THE EFFECTS OF SU(3) BREAKING IN THE CERN-BAG MODELK. Szegö^{*)}

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ABSTRACT

Introducing the SU(3) breaking via the mass of the strange quark, the ratios of one-particle distributions are calculated in the central region. The agreement with the experimental data is good.

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1. INTRODUCTION

Many efforts have been made in the present literature to cope with the description of the hadronic structure. A wide branch of those are the different big-models which assume the hadrons are made of point-like constituents confined to a finite region. Probably the best known model is the MIT-bag ¹⁾. It starts from a Lagrangian of quarks and gluons confined by volume tension, and by applying the standard field theoretic formalism, the particle spectrum can be accounted for. The scattering processes of hadrons are less easy to handle, though visually one can imagine how the processes go on.

At first sight the model of Kuti et al. ²⁾ seems to be just a modification where the confining force for gluons comes from a surface tension too, and the quarks are confined by their interaction with gluons. However, the proper quantization of the bag excitations can be carried out only in this model, and it has some other pleasant features as well, e.g. it maintains chiral-invariance in the appropriate regime.

Another kind of model, not based on field theory, was invented by G. Preparata ³⁾. It is less sophisticated in formulation without losing power to describe the hadron collisions as well as the particle spectrum. This model is the topic of the present paper. Since its construction is reviewed in the next chapter, only some general remarks are made here. At present the model can deal with mesons built from spinless quarks without any internal degree of freedom. It gives rise to a mass-spectrum lying on linear Regge trajectories and reproduces the main qualitative features of high-energy scatterings including high $p_{\perp}$ behaviour. Scaling is predicted in high-energy electron-proton scattering.

Of course these results should be reproduced with realistic quarks. The question of quark spin will be treated elsewhere; this paper is devoted to the study of the breaking of the internal symmetry group SU(3). The wave functions and related topics are treated in Section 3, whereas in Section 4 the one- and two-particle distributions are calculated. The results and the experimental data are compared and discussed in Section 5.

2. THE MODEL

In this Section the CERN bag-model is briefly summarized as it has been presented in Refs. 3), 4) and 5).

The basic building-block of the model is the meson-to-quarks wave function $\psi(p, j, m, x)$, where p, j, m is the four momentum spin and spin projection of the

meson; x_1 and x_2 are to describe the quark degrees of freedom. The quarks are supposed to be scalars and isoscalars. By definition ψ satisfies the equations

$$\begin{aligned} (K_1 + K_2) \psi &= 0 \\ (K_1 - K_2)^2 \psi &= -\omega_0^2 \psi \end{aligned} \quad (2.1)$$

where $K_i = \square_{x_i} - m_i$ is the Klein-Gordon operator for the quarks, m_i is the quark mass, ω_0 is a free parameter which in the CM is connected to the time-extension of the bag. It is required that ψ should vanish on the boundary and outside of a given space-time region. The solution of (2.1) in the CM is

$$\begin{aligned} \psi_n(M, j, m; x_1, x_2) &= e^{-iM \frac{x_{10} - x_{20}}{2}} j_{n_j} \cos \omega_0 t \cdot \\ & j_j \left(\sqrt{\frac{M^2}{4} + \omega_0^2 - \frac{u_1^2}{2} - \frac{u_2^2}{2}} r \right) Y_{jm}(\Omega_x) \end{aligned} \quad (2.2)$$

where $(r, t) = x_1 - x_2$, j_j is the spherical Bessel function, and n labels the radical excitations. This solution gives rise to a mass spectrum

$$M^2 + 4\omega_0^2 - 2u_1^2 - 2u_2^2 = \frac{4\beta_{nj}^2}{R_s^2(M)} \quad (2.3)$$

where β_{nj} is the n^{th} zero of j_j , and R_s measures the space-extension of the bag, $R_s(M) = R^2 M$ has been supposed. R^2 is related to the slope of the Regge trajectories, from $\alpha' = 1/R^2 = \pi$ is fixed. In the momentum space the wave function has the form (after having separated the CM coordinates):

$$\psi_n(p, j, m, k) = (2\pi)^2 \frac{2\pi}{R^2} \delta_{R^2} \left(\frac{p^2}{4} + k^2 - \frac{u_1^2}{2} - \frac{u_2^2}{2} \right) \delta_{R^2}(pk) Y_{jm}(\Omega_k) \quad (2.4)$$

where the functions δ_R are elements of δ convergent function series, i.e., $\delta_{R^2}(x) \rightarrow \delta(x)$ if $R \rightarrow \infty$. ψ_n will be represented by the graph

The next step of the model is to introduce an interaction which is defined as the overlap of three or four wave functions:

$$V_{ijk}(p_1 p_2 p_3) \sim \int dx_1 dx_2 dx_3 \psi(p_1 x_1 x_2) \psi(p_2 x_2 x_3) \psi(p_3 x_1 x_2) \quad (2.5)$$

and similarly for V_{ijkl} , corresponding to the diagrams

As discussed in Ref. 3) there is no need to allow for more than four-particle vertices.

In this framework a set of equations can be derived for the different quark-particle interactions. Let $\sigma_{q\bar{q}}$ be the $q\bar{q}$ total cross-section and $\otimes$ be the $q\bar{q}$ Green function, defined as

one can derive for

$$F(x) = \int \frac{d^2 k_\perp}{(2\pi)^3} \frac{1}{s} A(s, k, q) \Big|_{q=0} \quad (2.10)$$

the equation

$$F(x) = \lambda (1-x) K(x) \phi_{q\bar{q}} + \lambda \int_0^{1-x} \frac{dx'}{x'} (1-x') K(x') F\left(\frac{x}{\sqrt{1-x'}}\right). \quad (2.11)$$

Similarly, for the two-particle distribution the following equation has been obtained:

$$F_2(x_1, x_2) = \lambda (1-x_1) K(x_1) F\left(\frac{x_2}{\sqrt{1-x_1}}\right) + \lambda (1-x_2) K(x_2) F\left(\frac{x_1}{\sqrt{1-x_2}}\right) + \lambda \int_0^{1-x_1-x_2} \frac{dx'}{x'} (1-x') F(x') F_2\left(\frac{x_1}{\sqrt{1-x'}}, \frac{x_2}{\sqrt{1-x'}}\right) \quad (2.12)$$

corresponding to the graphical equation

The diagram shows a graphical equation. On the left, a circle with two horizontal lines passing through it, labeled 1 and 2, is set equal to a sum of three terms. The first term is a diagram with two large circles connected by a line, with a third circle attached to the middle of the connecting line. The second term is the same diagram with the labels 1 and 2 swapped, preceded by a plus sign and a double-headed arrow (1 ↔ 2). The third term is a diagram with two large circles connected by a line, with a third circle attached to the middle of the connecting line, but with a different internal structure. The entire equation is labeled (2.13) on the right.

The two-particle distribution can be used to find out correlations, as Eqs. (3.10) and (3.11) of Ref. 5) do. That yields

$$c(y_1, y_2) \sim \exp(-\text{const.} |y_1 - y_2|).$$

In these equations, only the three-particle vertex was considered. Modifications due to the four-particle one is discussed in Appendix B. In the next Section we discuss how to include the internal symmetry group (SU(3)).

3. WAVE FUNCTIONS

The inclusion of SU(3) is trivial for the wave function of particles containing no mixture of quark states, e.g. $\pi^\pm$, $K^\pm$. For the others the SU(3) pattern is followed: if in SU(3)

$$|\alpha\rangle = \sum_{ij} a_{ij}^\alpha |ij\rangle$$

where $|\alpha\rangle$ is a particle, i, j are the SU(3) quark quantum numbers, $\sum |a_{ij}|^2 = 1$; the wave function will be

$$\alpha = \text{diagram} = \sum_{ij} a_{ij} \text{diagram} \quad (3.1)$$

However, when SU(3) is broken, what we should write into the argument of the Bessel function of the wave function (2.2) is not trivial.

In this paper we take the attitude that the only root of the symmetry breaking is that the strange quark has acquired bigger mass than the others by some unknown mechanism. Since it causes distortion in the wave function, it generates SU(3) breaking in the overlap, but no breaking is allowed in the $\alpha = \text{diagram}$ coupling, i.e., the a_{ij} coefficients of Eq. (3.1) remain unchanged.

Now let us turn our attention to the mass formula (2.3):

$$M^2 + 4\omega_0^2 - 2\omega_1^2 - 2\omega_2^2 = \frac{4\beta_{00}^2}{R_s^2}$$

Of course, inside a multiplet ω_0 is the same. But even then we have two choices:

- consider M^2 as the average mass of the multiplet and allow different size for its members;
- consider M^2 as the real mass and keep R^2 as the average size.

If we would like to interpret (2.3) as an equation for Regge trajectories as well, only the choice (b) is acceptable since R_s counts for the slope. And more, the difference in the quark masses gives an explanation of the different intercepts at the origin.

In the following calculation we choose the masses of the light quarks to be equal to zero. Denoting the mass of the strange quark as Δ , Eq. (2.3) yields

$$\Delta^2 = \frac{1}{2} (M_K^2 - M_\pi^2) \sim (350 \text{ MeV})^2 \quad (3.2)$$

The η mass now could be predicted if we suppose that the weight in the wave function (3.1) counts also for the contribution of the quark pairs to the η mass. Since

$$|\eta\rangle = \frac{1}{\sqrt{6}} |1\bar{1}\rangle + \frac{1}{\sqrt{6}} |2\bar{2}\rangle - \frac{2}{\sqrt{6}} |3\bar{3}\rangle$$

we predict

$$\begin{aligned} m_\eta^2 &= \frac{m_\eta^2}{6} + \frac{m_\pi^2}{6} + \frac{2}{3} (m_\pi^2 + 4\Delta^2) \\ &= \frac{4}{3} m_K^2 - \frac{1}{3} m_\pi^2 \sim (560 \text{ MeV})^2 \end{aligned} \quad (3.3)$$

$$m_\eta^{\text{exp}} = 549 \text{ MeV}$$

Relation (3.3) is just the same as the Gell-Mann-Okubo formula.

Calculating $m_{\eta'}$ in the same way we get

$$\begin{aligned} m_{\eta'}^2 &= \frac{1}{3} m_\pi^2 + \frac{2}{3} m_K^2 \sim (410 \text{ MeV})^2 \\ m_{\eta'}^{\text{exp}} &= 958 \text{ MeV} \end{aligned} \quad (3.4)$$

However, there is no reason to suppose the same ω_0 for different multiplets. Equation (3.4) ought to have been used to find out ω_0 for the singlet representation instead of to predict the mass.

Now we turn back to the wave functions of η and η' . At first sight there is a contradiction between our choice (b) and the way in which the masses have been calculated. The latter would require different M^2 in $\eta = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$, whereas (b) would require different ω_0 for different quarks, provided Eq. (2.3) is fulfilled term by term. In our opinion the presence of this contradiction is a question of interpretation. Though the different quark-antiquark pairs contribute like independent objects to the mass, inside a particle they are somehow not alien to each other which is expressed by (b). We agree that this is a controversial point in the model, but further clarification will be possible only after having introduced quark spin and understand baryon structure, since only then we will have more information on how consistent our assumptions are. In this paper we cannot do better than work with them.

4. ONE- AND TWO-PARTICLE DISTRIBUTIONS FOR THE MESON OCTET

In this Section we generalize and solve Eqs. (2.8) and (2.11) for the stable pseudoscalar mesons. The generalization is trivial; we label each line with a quark-index $i, j, \dots$:

$$\begin{array}{c} i \\ | \\ \text{---} \bigcirc \text{---} \\ | \\ j \end{array} = \sum_k \begin{array}{c} i \\ | \\ \text{---} \bigcirc \text{---} \bigcirc \text{---} \bigcirc \text{---} \\ | \quad | \quad | \quad | \\ j \quad k \quad k \quad k \quad j \end{array} + \sum_k \begin{array}{c} i \\ | \\ \text{---} \bigcirc \text{---} \bigcirc \text{---} \bigcirc \text{---} \\ | \quad | \quad | \quad | \\ j \quad k \quad k \quad k \quad j \end{array} \quad (4.1)$$

As meson intermediate states we allow the pseudoscalar octet. (It has turned out, allowing η' to be exchanged too, that the particle production ratio has not changed.)

The couplings are given by

$$(ij) = \text{---} \bigcirc \text{---} \begin{array}{c} i \\ | \\ \text{---} \\ | \\ j \end{array} \leftrightarrow \tilde{V}_{ij} = \begin{pmatrix} \frac{\gamma}{\sqrt{6}} - \sqrt{\frac{3}{2}} \pi^0 & -\pi^- & -\kappa^- \\ \pi^+ & \frac{\gamma}{\sqrt{6}} + \sqrt{\frac{3}{2}} \pi^0 & \bar{\kappa}^0 \\ \kappa^+ & \kappa^0 & -\frac{2}{\sqrt{6}} \gamma \end{pmatrix} \quad (4.2)$$

To each intermediate state (ij) there is a weight function K which comes from the wave functions. Let alone its momentum dependence, it can depend, in principle, on the quark masses m_i, m_j, m_k and on the mass of the exchanged meson $M_{(ik)}$. But the computation shows that only m_k , the mass of the inner loop quark and $M_{(ik)}$ come into the formula. Since we are interested in (4.1) only for forward scattering, K is evaluated at $t = 0$. We always consider high s limits, hence no explicit s-dependence appears in K. We suppress (i.e., we do the integration) all internal momentum variables except one which is

$$x = 2k_{\parallel} / \sqrt{s} \quad (4.3)$$

where $k_{\parallel}$ is the momentum of the exchanged meson, parallel to the incoming quarks. As we shall calculate in the Appendix:

$$K(x, M, \delta) = \text{const.} \cdot x \exp \left\{ -\frac{4R^2}{\pi} \left(\frac{1-x}{x} M^2 + \delta^2 \right) \right\} \quad (4.4)$$

where M is the exchanged mass, δ is the quark mass in the inner loop, R is the bag radius, and x is given by (4.3).

Later we shall need the quantity

$$\begin{aligned} \tilde{K}(\omega, M, \delta) &= \int_0^1 \frac{(1-x)^{1+\frac{\omega}{2}}}{x} K(x, M, \delta) = \\ &= \text{const.} \cdot e^{-\frac{4R^2}{\pi} \delta^2} e^{\beta/2} \Gamma(2+\frac{\omega}{2}) \beta^{-1-\frac{\omega}{2}} W_{-1, \frac{1}{2}+\frac{\omega}{2}}(\beta) \end{aligned} \quad (4.5)$$

where $\beta = (4R^2/\pi)M^2$, and $W_{m,n}$ is the Whittaker function. In this notation Eq. (2.8) has the form

$$6 \{ 1 - \lambda \tilde{K}(\omega=0, M, \delta=0) \} = 0 \quad (4.6)$$

To make it include SU(3), the coupling (4.2) should be used weighting them with the appropriate kernels. Let us introduce:

$$M_{\pi} = \begin{pmatrix} \frac{1}{2} & 1 & 0 \\ 1 & \frac{1}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix} \tilde{K}(0, m_{\pi}, 0) \quad (4.7)$$

$$M_K = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \tilde{K}(0, m_K, \Delta) + \\ + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix} \tilde{K}(0, m_K, 0) \quad (4.8)$$

$$M_{\eta} = \begin{pmatrix} 1/6 & 0 & 0 \\ 0 & 1/6 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tilde{K}(0, m_{\eta}, 0) + \\ + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2/3 \end{pmatrix} \tilde{K}(0, m_{\eta}, \Delta), \quad (4.9)$$

$$M = M_{\pi} + M_K + M_{\eta} \quad (4.10)$$

Now it is not difficult to see that the generalization of Eq. (4.6), corresponding to graph (4.1), is:

$$\frac{1}{\lambda_{ij}} \delta_{ij} = \sum_n (M_{in} \delta_{nj} + \delta_{in} M_{nj}^T) \quad (4.11)$$

These are eigenvalue equations for λ_{ij} . Although M_K is asymmetrical, Eq. (4.11) treats all kaons symmetrically. When solving Eq. (4.11) we suppose that all σ_{ij} are equal, which is plausible at high energy where mass effects are negligible. The generalization of Eq. (2.11) is

$$\frac{1}{\lambda_{ij}} F_{ij}^{(kl)}(x) = (1-x) \sum_n \left\{ M_{in}^{(kl)}(x) \sigma_{nj} + \sigma_{in} M_{nj}^{T(kl)}(x) \right\} +$$

$$+ \sum_n \int_0^{1-x} \frac{dx'}{x'} (1-x') \left\{ M_{in}(x') F_{nj}^{(kl)}\left(\frac{x}{\sqrt{1-x'}}\right) + F_{in}^{(kl)}\left(\frac{x}{\sqrt{1-x'}}\right) M_{nj}^{T(kl)}(x) \right\} \quad (4.12)$$

where $M^{(kl)}$ is a matrix, the $(k,l)^{th}$ element of which is equal to that of M , and all others are zero. To solve this equation, we introduce -- as in Ref. (5) -- the quantities

$$\tilde{M}(\omega) = \int_0^1 \frac{dx}{x} (1-x)^{1+\frac{\omega}{2}} M(x) \quad ; \quad \tilde{M}^{(kl)}(\omega) = \int_0^1 \frac{dx}{x} x^\omega (1-x) M^{(kl)}(x) \quad (4.13)$$

and

$$\tilde{F}(\omega) = \int_0^1 \frac{dx}{x} x^\omega F(x) \quad (4.14)$$

with its inverse

$$F(x) = \frac{1}{2\pi i} \int_{i\omega}^{-i\omega} d\omega x^{-\omega} \tilde{F}(\omega) \quad (4.14')$$

Of course, $\tilde{M}(\omega) = M$ of Eq. (4.10). As earlier,

$$F_{ij}^{(kl)}(x) = \sum_X \int \frac{d^2 k_\perp}{(2\pi)^3} \frac{1}{S} \quad \begin{array}{c} \text{(kl)} \quad \text{(kl)} \\ \diagup \quad \diagdown \quad \diagup \quad \diagdown \\ i \quad \text{---} \quad j \quad \text{---} \quad i \\ \diagdown \quad \diagup \quad \diagdown \quad \diagup \\ j \quad \text{---} \quad X \quad \text{---} \quad j \end{array} \quad (4.15)$$

Equation (4.12) can now be cast to

$$\begin{aligned} \frac{1}{\lambda_{ij}} \tilde{F}_{ij}^{(kl)}(\omega) = \sum_n \tilde{M}_{in}^{(kl)}(\omega) \delta_{nj} + \delta_{in} \tilde{M}_{nj}^{T(kl)}(\omega) + \\ + \tilde{M}_{in}(\omega) \tilde{F}_{nj}^{(kl)}(\omega) + \tilde{F}_{in}^{(kl)}(\omega) \tilde{M}_{nj}(\omega). \end{aligned} \quad (4.16)$$

Using the eigenvalues of Eq. (4.11), Eq. (4.16) can be solved and -- exactly the same way as it is discussed in Ref. 5) without SU(3) -- we get back $F(x)$ from Eq. (4.14'). This last integral can be evaluated around $x \sim 0$, using the $\omega = 0$ pole of the integrand. Although we cannot calculate absolute magnitudes, we can calculate ratios. Our result is:

$$F_{ij}^{\pi} : F_{ij}^K : F_{ij}^{\eta} = 30 : 1 : \frac{2}{3} \quad \text{for any } ij \quad (4.17)$$

and

$$F_{\text{two light quark}}^{\alpha} : F_{\text{one heavy quark}}^{\alpha} : F_{\text{two heavy quark}}^{\alpha} = 2.6 : 1.6 : 1 \quad (4.18)$$

for any particle α .

Its meaning and comparison with experiment will be discussed in the next Section.

We proceed with the two-particle distributions. The generalization of Eq. (2.12) is now straightforward. Omitting the details here, we get a result very similar to that obtained without SU(3):

$$F_{ij}^{(kl), (mn)}(x_1, x_2)_{x_1 \sim x_2 \sim 0} \sim F_{ij}^{(kl)}(x_1) F_{ij}^{(mn)}(x_2) + (x_1 \leftrightarrow x_2) \quad (4.19)$$

In principle, it is easy to perform the generalization of the correlation function [Eq. (3.11) of Ref. (5)] between the members of the meson octet, but technically very difficult to get any result. The leading singularity at $\omega_1 = \omega_2 = 0$ cancels

and the next contribution comes from the line $\omega_1 = -\omega_2$, $\omega_1 \neq 0$. A very rough approximation, rather a feeling from the inspection of the formula, gives that the correlation length is inversely proportional to the mass-square of the particles.

5. DISCUSSION

In this Section we discuss the experimental consequences of our calculation. We shall deal only with the one-particle distribution, since we do not have enough data on correlations. All the data, except Fig. 7, were taken from Ref. 6), Fig. 7 was taken from Ref. 7).

As we learn from Eq. (4.15), $F_{ij}^\alpha(x)$ gives the invariant cross-section:

$$F_{ij}^\alpha(x) = \int d^2p_\perp \frac{E}{\pi k_{\max}} \frac{d\sigma}{dk_\perp^2 dx} \sim \frac{2E}{\pi \sqrt{s}} \frac{d\sigma}{dx} = \frac{1}{\pi} \frac{d\sigma}{dy} \quad (5.1)$$

for the process $q + \bar{q} \rightarrow \alpha + \text{anything}$; where $(E, \underline{k})$ is the four-momentum of particle α . To connect it to real processes we suppose the underlying picture that when particles collide, one quark from each particle interacts, all the others are spectators. Hence in a given reaction the ratio of the inclusive particle production is given by the ratio of F .

According to our result (4.17) we predict the ratio $d\sigma/dx$ at $x \sim 0$ for π, K and η production to be

$$\left. \frac{d\sigma}{dx} \right|_{x \sim 0}^{(\pi)} : \left. \frac{d\sigma}{dx} \right|_{x \sim 0}^{(K)} : \left. \frac{d\sigma}{dx} \right|_{x \sim 0}^{(\eta)} = 30 : 1 : \frac{2}{3} \quad (5.2)$$

for any two incoming particles.

The experimental data for pp collisions at different energies can be found in Figs. 1-4. They contain data on the π/K ratio at fixed p^2 . Although our predictions relate to quantities integrated over p^2 , we hope that the ratio is roughly the same. It is not easy to extrapolate the data to $x \sim 0$, but it is clearly exhibited that at all energies $K/\pi < 1/10$ in good agreement with our prediction.

The experimental data for π^+p collisions can be found in Figs. 5-7. The K/π ratio is about 1/20, again in good agreement with (5.2). The η/π ratio at $y = 0$, as can be read off from Figs. 6 and 7. is about 1/100. The agreement is tolerable. However, to obtain this ratio we had to compare results from different experiments at different energies, but we expect the order of magnitude to be correct.

The result (4.18) shows that although in any quark collision the $\pi:K:\eta$ ratio is the same, the absolute magnitude differs if, e.g., a pion is created in the collision of two strange or two non-strange quarks. This fact results in a difference in $\pi^+p \rightarrow \pi^- + X$ and $K^+p \rightarrow \pi^- + X$. Supposing independent quark counting we predict

$$\frac{\pi^+p \rightarrow \pi^- X}{K^+p \rightarrow \pi^- X} \Big|_{X \sim 0} = \frac{2 \cdot 2,6}{2,6 + 1,6} \sim 1,24 \quad (5.3)$$

The experimental ratio, as can be read off from Fig. 6 (although at different p_{inc}) is about 1.4. Of course, we would predict the ratio (5.3) for total cross-sections too, on the basis of (4.18). The experimental ratio at $p_{lab} = 100 \text{ GeV/c}$ is:

$$\frac{\sigma_{\pi^+p}^T}{\sigma_{K^+p}^T} = \frac{24 \text{ mb}}{19 \text{ mb}} \sim 1,2.$$

Summarizing our discussion we conclude that our predictions agree quite well with the data. We are aware of the fact that with spinless quarks and without handling baryons the agreement is not a real proof in favour of the model, but these results are certainly very encouraging.

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APPENDIX A

The calculation of $K(X, M, \delta)$.

Corresponding to the graph

$$F_{ij}^{(kl)}(x) = \sum_X \int \frac{d^2 k_\perp}{(2\pi)^3} \frac{1}{S}$$

K is defined by an integral expression which can be obtained if we insert (2.6) and (2.4') with one important difference: there is no separate summation over the intermediate states in $\textcircled{1}$ and $\textcircled{2}$. After having performed some of the integration with the help of the δ functions and having done the summations, we are left with the expression [see Ref. 5)]:

$$K(x, M, \delta) = \int d^2 \ell_\perp d^2 \ell'_\perp d^2 k_\perp \delta_R^4(\sigma) \delta_R^4(\sigma') f(\ell_\perp^2) f(\ell'^2_\perp) f(\ell_\perp - \ell'_\perp)^2 \quad (\text{A.1})$$

where

$$\sigma = \frac{1-x}{x} M^2 + x \ell_\perp^2 + \frac{1}{x} k_\perp^2 + 2 \ell_\perp k_\perp + \delta$$

$$\sigma' = \sigma(\ell \leftrightarrow \ell')$$

$$f(\ell^2) = \frac{2 \mathcal{F}_1(R\sqrt{-\ell^2})}{R\sqrt{-\ell^2}}$$

The function f is sharply peaked around $\ell^2 = 0$ and it is essentially an element of a δ -convergent function series, since

$$\lim_{\nu \rightarrow \infty} \frac{1}{2} \frac{\mathcal{F}_1(\nu/x)}{x} = \delta(x) \quad (\text{A.2})$$

We use $f((l-l')^2)$ as a true δ function, whereas the others are approximated as

$$\delta_{R^2}^4(z) = \left(\frac{R^2}{\pi}\right)^4 \exp\left\{-\frac{2R^2}{\pi}|z|\right\},$$

(A.3)

$$f^2(\ell^2) = \frac{1}{4} e^{-\ell^2/4}$$

When doing the integration only that region of σ is kept which is not very far from zero because of the exponential dependence. These approximations together lead to the result

$$K(x, M, \delta) = \text{const.} \times \exp\left\{-\frac{4R^2}{\pi}\left(\frac{1-x}{x}M^2 + \delta^2\right)\right\}.$$

(A.4)

APPENDIX B

In this section we discuss some properties of the four-particle vertex. Here we are only interested in the kinematical allowed region of one particle decaying into three. Corresponding to (B.1),

$$\begin{aligned} q_1 &= P/2 + k \\ q_2 &= P/2 - k \\ q_3 &= P/2 + k - p_1 \\ q_4 &= -(P/2 - k - p_3) \end{aligned}$$

and the rules given in Section 2, in the case of scalar particles, the elementary vertex has the value

$$\begin{aligned} V_{1234} &\sim \int d^4k \, \delta_{R^2}(q_1^2 - q_2^2) \delta_{R^2}(q_1^2 + q_3^2) \delta_{R^2}(q_1^2 + q_2^2) \delta_{R^2}(q_1^2 - q_3^2) \cdot \\ &\cdot \delta_{R^2}(q_2^2 - q_4^2) \delta_{R^2}(q_2^2 + q_4^2) \delta_{R^2}(q_3^2 - q_4^2) \delta_{R^2}(q_3^2 + q_4^2) \end{aligned} \quad (B.2)$$

With the help of the first two δ -functions the integration over k_0 and $\underline{k}^2$ can easily be performed in the CM and we get

$$V_{1234} \sim \int d\Omega_k \, \delta_{R^2}^3(q_3^2) \delta_{R^2}^3(q_4^2) \quad (B.3)$$

The integration can be done further by using the approximation (A.3) for the δ -functions. However, one can read off even from (B.3) that V_{1234} has the maximal value if $p^2 \sim p_3^2$ and $p_1^2 \sim p_2^2 \sim 0$, or $p^2 \sim p_1^2$ and $p_2^2 \sim p_3^2 \sim 0$. This brings to mind the similar behaviour of the three-particle vertex.

The contribution to the imaginary part of the quasi Green functions is given by the graphical equation

$$q = (\alpha - x)p + (\beta - y)p' + l_\perp - k_\perp, q^2 = \sigma;$$

$$\bar{q} = (\alpha - x - u)p + (\beta - y - v)p' + l_\perp - k_\perp - k'_\perp, \bar{q}^2 = \bar{\sigma}$$

We use the same convention to parametrize the momenta as in Appendix A. Eq. (B.4) yields the following difference in the kernel (A.2):

$$\delta_{R^2}^4(\sigma) \leftrightarrow \delta_{R^2}^3(\sigma) \int d^2 k'_\perp \int_0^{1-x} du \delta_{R^2}^3(\bar{\sigma}) \Big|_{V = \frac{\mu^2 - k'^2_\perp}{us}} \quad (B.5)$$

and the similar charge for $\delta_{R^2}^4(\sigma')$. Using again (A.3) and performing the integration over $k_\perp$, an integrand is obtained which contains an exponential damping around $u \sim 0$. Exploiting this, one obtains

$$\delta_{R^2}^3(\sigma) \int d^2 k'_\perp \int_0^{1-x} du \delta_{R^2}^3(\bar{\sigma}) = \text{const.} \delta_{R^2}^4(\sigma) \quad (B.6)$$

Hence, in our calculation the contribution coming from the four-point interaction is proportional to that one we have taken into account. Accordingly, it does not change the ratios (4.17) and (4.18).

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Fig. 1

Fig. 2

Fig. 3

Fig. 4

(a)

(b)

Fig. 5

Fig. 6

Fig. 7