PION CONDENSATION AND STABILITY OF ABNORMAL NUCLEI

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A B S T R A C T

A possible existence of super-dense nuclei with various Z/A ratios is discussed. The binding energy and the equilibrium density of super-dense and neutron nuclei are calculated in terms of simple models. It is shown that the accuracy of the present-day theory is not sufficient to make a definite conclusion whether such nuclei exist, but such a possibility exists at reasonable values of nuclear constants.

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The existence of super-dense nuclei due to the pion condensation was first discussed in Ref. 1). Later this problem was considered more elaborately in Ref. 2), where the possibility of the existence of neutron nuclei was also discussed. In the present paper, using recent results in the theory of pion condensation, the properties of abnormal nuclei will be considered in more detail.

Recent model calculations of the pion condensation energy made it possible to describe weak condensate fields³⁾ and strong fields⁴⁻⁶⁾. At the same time, it will be shown below that the accuracy of the condensation energy calculation (and also that of the nucleon energy) is not sufficient to make a definite theoretical conclusion whether such nuclei exist.

The binding energy, i.e., the difference between the total energy and the sum of nucleon masses $\Sigma m_N = mA$, of the system of A baryons with the net charge Z has the form

$$E(n, A, \nu) = \varepsilon(n, \nu)A + a_s(n, \nu)A^{2/3} + a_c \nu^2 A^{5/3} \quad (1)$$

$$\nu \equiv Z/A$$

The first term is the energy per particle in the infinite system. The terms proportional to $A^{2/3}$ and $A^{5/3}$ represent the surface and the Coulomb energy, respectively. For the sake of simplicity the terms accounting for pairing, deformation and shell effects are omitted in Eq. (1). Let us first consider the first term in Eq. (1). The energy $\varepsilon(n, \nu)$ includes the condensation energy and the energy of the baryon subsystem:

$$\varepsilon(n, \nu) = \varepsilon_\pi(n, \nu) + \varepsilon_B(n, \nu) \quad (2)$$

The condensation energy has been calculated using various models in Refs. 3)-10). Near the critical point one can obtain an analytical expression for $\varepsilon_\pi(n, \nu)$ after developing the perturbation theory up to the fourth order with respect to the condensate field amplitude³⁾

$$\varepsilon_\pi(n, \nu) = -\beta \frac{(n - n_c)^2}{2n}, \quad n > n_c \quad (3)$$

The values of n_c and β are presented in Refs. 3) and 7) for various isotopical concentrations of the medium and for various values of the parameter $\bar{g}$, which accounts for nuclear correlations^{11-13, 6)}.

The method for calculating the energy of the strong condensate field has been presented in Ref. 5), where an approach has been developed which allows one to obtain a simple analytical expression for the energy of the charged pion condensate in a sufficiently realistic model including the P- and S-wave πN -interactions, the $\pi\pi$ -interaction, the $N_{33}^*(1232)$ -resonance, and the effect of nuclear correlations⁶⁾. In the following the results of Refs. 5) and 6) will be used, but in our case, contrary to Refs. 5) and 6), where the infinite system was considered, the condition of electroneutrality should not be fulfilled.

At high densities ($n \gg n_c$) the model of the saturated condensate field ($\theta = \pi/2$), may be used with a sufficient accuracy. The condensation energy in this model takes the form⁶⁾

$$\mathcal{E}_\pi(n, \nu) = \mathcal{E}_\pi(n) + \alpha_\pi(n)(1-2\nu)^2 \quad (4)$$

$$\mathcal{E}_\pi(n) = - \left[\frac{81}{50} f'^2 (1-\gamma) n - \frac{\Delta}{3} \right] \quad (5a)$$

$$\alpha_\pi(n) = \frac{n}{2F^2} \quad (5b)$$

Here $\hbar = c = m_\pi = 1$, $\Delta = m_{N^*} - m_N = 294 \text{ MeV} = 2.1$, $F = 1.35 m_\pi$ is the pion decay constant, the πN -interaction constant f is determined by the relation $f = G_A/F$, where $G_A = 1.36$ is the axial coupling constant. To take into account the small renormalization of the πN -interaction constant in nuclear matter¹¹⁾, the value $f' = 0.9 f$ will be used for calculating the binding energy of abnormal nuclei. The value of γ , which takes into account the nucleon-nucleon correlations, is related to the local amplitude of spin-isospin nucleon-nucleon interaction g^{-11} as

$$g^- = (f')^2 \frac{2mP_0}{\pi^2} \gamma \quad (6)$$

where $P_0 = 1.92$ is the Fermi momentum in nuclear matter at normal nuclear density $n_0 = 0.17 \text{ fm}^{-3} = 0.48$.

In deriving Eq. (5a), it was supposed in Ref. 6) that apart from Clebsch-Gordan factors, the local amplitudes of NN , NN^* and N^*N^* interactions are identical. Quite probably, the local NN^* -interaction is weaker than the NN -interaction,

as may be concluded from the (pp, N^*n) scattering data with high transferred momenta¹⁴⁾. If this conclusion is correct, the condensation energy should significantly exceed Eq. (5a). On the other hand, relativistic corrections to vertex functions were not taken into account in our consideration; this would result in a reduction of the condensation energy gain. Due to the difficulties in taking these effects into account, while dealing with other unknown parameters, Eqs. (5a) and (5b) may be taken as a reasonable estimate.

In the following, for all densities of interest, a simple interpolation formula will be used

$$\varepsilon_{\pi}(n, \nu) = -\beta(n) \frac{(n - n_c)^2}{2n} \quad (7)$$

where

$$\beta(n) = A + B \frac{n_c}{n} + C \frac{n_c^2}{n^2} \quad (7a)$$

The coefficients A, B, and C, depending on ν , are chosen to give the values $\beta(n_c)$ from work⁷⁾, and to reproduce asymptotically the value ε_{π} given by Eqs. (5a) and (5b) at $n/n_c \rightarrow \infty$.

Let us consider now the baryon energy $\varepsilon_B(n, \nu)$. As will be seen from the following, only the cases $1-2\nu \ll 1$ and $\nu \ll 1$ are of interest to us. The baryon energy in the first case at $n - n_0 \ll n_0$ may be expressed in terms of the nuclear compressibility K

$$\varepsilon_B(n, 1/2) = -\varepsilon_0 + \frac{K}{2} \left(1 - \frac{n}{n_0}\right)^2 \quad (8)$$

where $\varepsilon_0 = 15.7 \text{ MeV} = 0.11$. The Fermi liquid theory gives $K = (2/3)\varepsilon_F(1+2f_0)$, where f_0 is the local amplitude of scalar nucleon interaction¹¹⁾. Comparing with experiment, the value $f_0 = 0.25 \pm 0.1$ is obtained¹⁵⁾. For estimation, we take $f_0 = 0.25$, which gives $K = 40 \text{ MeV} = 0.29$.

At high densities, the baryon subsystem is significantly rearranged. Instead of two Fermi seas of protons and neutrons, the baryon quasi-particles, which are mixtures of P , n , N^{*++} , N^{*+} , N^{*0} and N^{*-} , fill a single Fermi sea. The energy ε_B corresponding to such a configuration includes the kinetic energy and the interactions of baryon quasi-particles

$$\varepsilon_B(n) = \frac{3(3\pi^2 n)^{2/3}}{10 m} + U(n) \quad (9)$$

At high densities ε_B is determined by the short-range repulsion. Assuming this repulsion to be identical for all baryons, which form baryon quasi-particles, the value ε_B may be estimated, if one takes $U(n)$ from neutron matter calculation (for example see Ref. 16)).

In the following a simple interpolation formula will be used to describe the baryon energy at all densities:

$$\varepsilon_B(x = \frac{n-n_0}{n_0}) = -0.11 + \frac{0.14x^2}{0.37x+1} \quad (10)$$

At $x \ll 1$, Eq. (10) coincides with Eq. (8). At $n = 7.35$, $n_0 = 1.25 \text{ fm}^{-3}$. $\varepsilon_B(n)$ and $d\varepsilon_B/dn$ are equal to that calculated for the neutron matter. At higher densities ε_B is taken from the neutron matter calculations¹⁶⁾.

At present there are no reliable calculations of the nuclear surface energy at high densities. We estimate a_s in Eq. (1) assuming that the surface thickness coincides with the range of nuclear forces, i.e., it does not depend on the density

$$a_s(n, \nu) = 0.13 \left(\frac{n}{n_0} \right)^{1/3} \frac{\varepsilon(n, \nu)}{\varepsilon(n_0, 1/2)} \quad (11)$$

The Coulomb coefficient a_Q may be written as

$$a_Q = 0.005 \left(\frac{n}{n_0} \right)^{1/3} \quad (12)$$

The numerical coefficients in Eqs. (10) and (11) have been taken to give at $n = n_0$ their values for normal nuclei.

Let us now formulate the equilibrium conditions which should be fulfilled for a finite system at zero external pressure:

1) Positive mass defect

$$-E(n, A, \nu) > 0 \quad (13)$$

- 2) β equilibrium (electrons leave the nucleus freely)

$$\left(\frac{\partial E}{\partial Z}\right)_A = \mu_p - \mu_n = 0 \quad (14)$$

- 3) Stability with respect to fission is determined by the well-known condition

$$\frac{Z^2}{A} < 2 \frac{a_s(n, \nu)}{a_c(n)} \quad (15)$$

From Eqs. (11), (12), and (15) one has:

$$\frac{Z^2}{A} < 50 f(n, \nu) \quad (16)$$

where $f(n, \nu) = \varepsilon(n, \nu) / \varepsilon(n_0, 1/2)$. At $n = n_0$, Eq. (16) represents the well-known stability criterion for normal nuclei. From Eqs. (1), (5), and (14) the equilibrium value of ν may easily be found:

$$\nu = \frac{1}{2} \left[1 + \frac{a_c(n)}{4\alpha_\pi(n)} A^{2/3} \right]^{-1} \quad (17)$$

It follows from Eqs. (16) and (17) that there may exist two regions of stability:

- a) $A < A_1 \approx 200 f(n, 1/2)$ and $\nu \approx \frac{1}{2}$ (super-dense nuclei);
- b) $A > A_2 \approx 2 \times 10^5 (n/n_0)^4 f^{-3}(n, 0)$ and $\nu \approx 50 (n/n_0)^{2/3} A^{-2/3} \ll 1$ (neutron nuclei). Such nuclei have sufficiently small ratio Z^2/A for fission to be impossible, but their charge is great enough for the β decay to be forbidden by the Coulomb energy.

Nuclei with ratios Z/A different from the equilibrium one would be β active. From Eqs. (5) and (3) the maximum energy of β decay may easily be estimated

$$\varepsilon_\beta = -\left(\frac{\partial E}{\partial \nu}\right)_A \sim -\frac{\partial E_\pi}{\partial \nu} = 4\alpha_\pi(n)(1-2\nu) \quad (18)$$

At $n = 5n_0 = 2.4$ and $\nu = 0$ $\varepsilon_\beta = 2.6 \approx 400$ MeV.

Let us now consider the binding energy of abnormal nuclei. First of all one should note that at high densities the total energy is the sum of two big numbers; the positive baryon energy and the negative energy of condensation,

nearly compensating each other, and consequently the results for total energy should be considered only as an illustration of various possibilities.

The results of the calculation of $\varepsilon(n)$ for super-dense nuclei ($v = 1/2$) are presented in the figure. One must note that, in calculating the curves (a), (b), and (c) in the figure, the values of nuclear parameters were used for which $n_c > n_0$. It is possible that $n_c < n_0$; then the pion condensate should exist in normal nuclei¹¹⁻¹³, and all parameters characterising normal nuclei already involve the condensate contribution. Most probably, abnormal nuclei do not exist in this case. We have also calculated the corresponding curves for neutron nuclei with analogous results.

Thus, our analysis shows that parameters of abnormal nuclei depend significantly on the nuclear constants, which are not known to a high accuracy. It should also be mentioned that the model of Refs. 5) and 6) (used here) allows one to take into account only the energy of the condensate of charged pions. The condensate of neutral pions, which should appear as was shown in Refs. 12) and 13), should lower the energy of the system in nucleon matter at $n \sim n_0$. Moreover, the minimum energy may correspond to a space structure of condensate field different from a simple plane wave configuration^{3,17,18}). Both effects favour an existence of abnormal nuclei. On the other hand, if a more rigid equation of state for a baryon subsystem is chosen or the relativistic correction to form factors is taken into account, the total energy would increase. At present, it is impossible to take all these effects into account with the necessary accuracy; and our main conclusion is that a possibility of the existence of abnormal nuclei is not excluded theoretically. Evidently, the case should be solved experimentally.

In principle, higher binding energy may correspond to super-dense nuclei, i.e., the normal nuclei are metastable. Regarding this possibility, it would be interesting to put an experimental limit on the probability of spontaneous transitions of normal nuclei in a super-dense state. Up to now the search for nuclei with abnormally high binding energy gave no result¹⁹⁻²¹).

It would also be interesting to search for stable or short-lived β active abnormal nuclei of medium weight ($A \sim 100$) in the products of fission of normal nuclei.

Quite probably, super-dense nuclei may be created in heavy ion collisions at the energy of the order of some hundred MeV per nucleon. The nuclear shock wave arising at the head-on collision may create a region of high density. At a sufficiently high value of β (see Eq. (2)) the compressibility becomes negative

just at $n = n_c$. Thus, to start a process of creation of a super-dense phase, it is sufficient to compress the system to the critical density n_c . Whether stable super-dense nuclei exist or not, the dynamics of the collision will be governed by pion condensation.

Finally, the abnormal nuclei should be searched for in cosmic rays, as has first been proposed in Ref. 1). The possibility of observing the stable abnormal nuclei or their β active fragments, emerging as a result of collisions with atmospheric nuclei, must be taken into account in the planning and interpretation of cosmic-ray experiments. For instance, it may not be excluded that the highly unusual track assigned primarily to a magnetic monopole²²⁾ is, in fact, the trace of the abnormal ("neutron") nuclei with $Z \sim 10^2$ and $A \sim 10^4$. It would also be interesting to search for super-dense nuclei of cosmic origin accumulated in meteorites or in the surface layer of lunar soil.

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The energy per baryon of nuclear matter ($\nu = 1/2$). The dashed line is the baryon energy (Eq. (10)). The curves (a), (b), and (c) correspond to $\gamma = 0.45$, $\gamma = 0.5$ and $\gamma = 0.55$, and $n_c = 0.54$, 0.65 and 0.79 , respectively.