ULTIMATE TEMPERATURE AND THE STRUCTURE
OF ELEMENTARY PARTICLES

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ABSTRACT

From 30 April to 7 May 1975 a workshop on Theoretical Physics under the title: "Is there an ultimate temperature in hadron physics?" was held in the Centre for Scientific Culture "Ettore Majorana" in Erice, Sicily, Italy.

The present article tries to explain in a non-theoretical language the physical ideas which underly and motivate the research discussed in that workshop.

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What have these to do with each other? What, indeed, has temperature to do with how elementary particles look, and what does ultimate temperature mean?

Temperature is a measure of the vehemence of the irregular "heat motion" of the billions of billions of millions of molecules, or atoms, of matter. It is obvious that there must be an absolute zero point of temperature, corresponding to the situation where all motion ceases; you cannot have less than no motion, and therefore you cannot cool down matter below the temperature corresponding to absolutely motionless atoms: -273°C (centigrade) or zero K (kelvin). It is even impossible to reach this absolute zero.

But why should there be any upper limit to the temperature? Nobody has ever claimed that there is an upper limit of vehemence of motion, technically called "kinetic energy"; thus if we concentrate more and more energy in the form of irregular motion, why should we not be able to push the temperature as high as we like? One might guess I would claim that the total energy in the universe is limited and cannot be compressed indefinitely; but no, it is irrelevant whether the universe has a finite energy content or not. Even if I take only one litre of air and compress it with the dimension of the proton (the nucleus of the hydrogen atom), I would heat it up to something like $3 \times 10^{16}$ K -- provided that it would behave like an ideal gas, and provided that I could, during the compression, isolate it thermally. What I claim, however, is that it would never reach this fantastic temperature -- isolated or not -- but remain relatively cool whatever I do; namely it would reach only about $2 \times 10^{12}$ K. I further claim that this is true for all matter -- apples, stones, protons, neutrons, electrons, neutrinos, quarks, and whatever might still be discovered -- and that it has to do with the inner structure of one class of elementary particles: those which participate in the "strong interaction" (the interaction which keeps nuclei together, but can do a lot more). Before I try to explain this, I will illustrate what $2 \times 10^{12}$ K means.

Imagine an oven which grows hotter and hotter at the rate of 1 degree per second; after a little more than one minute it would be hot enough to boil an egg, after one hour or two it would have reached the surface temperature of the sun. It would take a year to reach the core temperature of the sun, but only after 640,000 years it would arrive at our limiting temperature which I said was relatively cool. It is indeed near absolute zero compared to the $3 \times 10^{16}$ K of our compressed litre of air, but it is still so fantastically high compared to all known temperatures on earth that it seems out of the question to ever reach it in a laboratory. And yet, this temperature
reigns during a short time when two elementary particles of the strongly interacting sort -- we call them hadrons -- collide at high energy. Thus, it is a common event in a high-energy laboratory where it happens billions of billions of times every day. The volume filled with this hot hadron matter is as small as the colliding particles, about $10^{-13}$ cm in radius, and it keeps together only for about $10^{-23}$ seconds (a second, compared to this time is as long as a million ages of our universe as compared to a second!). These numbers sound fantastic, but in the daily life in a high-energy laboratory they no longer astonish us; after so many years of examination in so many research centres, we take them for granted and meanwhile get used to them.

However, the mysterious temperature of $2 \times 10^{12}$ K (which we shall call $T_0$ from now on) made us uneasy: it was a real puzzle why, with higher and higher energies from larger and larger particle accelerators, the collision temperature rose first with the collision energy rather fast to $T_0$ and then, with energies still higher, refused to increase further; had it followed the trend at low energy, it should have reached a value three times larger than $T_0$ at the highest collision energy currently available in a laboratory (CERN's Intersecting Storage Rings).

The critical reader, if he is not acquainted with elementary particle physics, will by now have become angry with me: did I not say that temperature is a measure for the kinetic energy of the irregular motion of an inimaginable number of atoms of matter? And did I not, in the same breath, claim that the limiting temperature $T_0$ is reached when two elementary particles collide? Where is the "inimaginable number of atoms" if only two elementary (1) particles collide? Yes, something is wrong: the word elementary. What physicists nowadays call elementary particles (and this is a tradition hard to break), is no longer the same as only fifty years ago. The discovery that matter and energy are only different currencies of the same thing and can be freely exchanged at the rate $E=mc^2$ (Einstein), plus the discovery that nature does make jumps (quantum mechanics) and, in particular, that forces and interactions are generated by particles called field quanta, which jump to and fro between the interaction objects: all this makes it impossible to think any longer of an elementary particle as a single, peaceful point in space; it is a complicated -- almost I had said: living -- system of an undetermined number of field quanta and constituents filling a small but finite volume. One might be tempted to say: then the former elementary particles do not deserve this name; let us give it to their components and field quanta. This would be exactly in line with what has always been done: if we learn to decompose something,
considered so far as elementary, we no longer call it so but give that name to its constituents. However, here we run into difficulties: the procedure works fine if we decompose matter into molecules, these into atoms, these into nuclei and electrons, and nuclei into protons and neutrons. Each time the decomposition is unique, and one can say: the He^4 nucleus always consists of four nucleons, two protons, and two neutrons. Everything changes, however, if we try to decompose a proton: if we shoot protons on each other they decompose without being destroyed! Namely, as long as the energy is high enough, they decompose into many, but not a fixed number of, particles (mostly mesons, their field quanta) without disappearing themselves; we will, after such a collision, not only find in addition to the mesons the two original protons (or neutrons) without any scratch, but maybe even a few more of them together with their antiparticles. All this is a simple consequence of E=mc^2 and the quantum jumps. Imagine that such things would happen in our daily life: two trucks run into each other at full speed and decompose into a dozen Volkswagens plus a truck and an anti-truck, plus the two original trucks undamaged, except that perhaps they now have a different paint. Of course, the laws of relativity and quantum theory do not allow that this happens with macroscopic objects, but they do so for elementary particles. One actually can see such events on bubble chamber pictures (Fig. 1).

What is the lesson? This: if one tries to decompose a proton, one finds that part of the collision energy is transformed into other elementary particles while the proton remains what it is; the number and kind of debris is different from collision to collision and follows statistical laws.

The debris, however, have every right to be called elementary particles themselves, not only because among them particles identical to the initial ones appear, but also because it seems that the definition of an elementary particle should, under the given circumstances, read something like this:

an elementary particle is a particle which cannot, in a unique and reproducible way, be decomposed into elementary particles.

The reader should not stumble over this circular definition; such definitions may make sense, and this one does: the He^4 nucleus and the H_2 molecule are not elementary, the proton and all those particles we intuitively (as high energy physicists) would like to call elementary respond to the above definition which says: either it cannot be decomposed at all
or if it can, then it is into an undetermined number of other elementary particles; we also could say

 elementary particles consist of

While so far no particle has been seen in the laboratory which could not be decomposed, by shock, into others, one might say: it is not really decomposed, you have only transformed collision energy into new particles, which therefore are not composites of the original one. This argument is wrong, because we may let a particle annihilate with its antiparticle, without adding kinetic energy: what happens is very much the same.

But, asks the reader, is it not an established fact that elementary particles consist of more elementary units, called quarks: nucleons of three and mesons of two quarks (better: a quark and an antiquark)? Certainly; yet I would say it differently: under some circumstances elementary particles behave as if they were composed of more fundamental units, called quarks, but if one tries to decompose them into these, they behave as if they were composed of many other elementary particles (which again may consist of quarks under certain circumstances) and so far nobody has seen a proton splitting into three quarks; indeed nobody has ever seen a quark! It is somehow like a house built of red bricks bound together with white cement, where the cement is much harder than the bricks: if you look at the house, you see the bricks, but if you try to decompose it into bricks you do not get whole bricks because the cement is too hard and the first thing to break is the brick (this picture is not entirely correct).

The next important point is that the number of different kinds of elementary particles is large; we know today about 200 of them and call most of them "resonances". These are high-mass particles which decay, after a very short life ($10^{-23}$ s), into other, similar particles; and these decays are ruled by statistical laws. Each of these 200 sorts may appear as debris in a high energy collision.

Where does temperature come in? If temperature is a measure of the kinetic energy of a large number of randomly (= statistically) moving particles, it may be a natural parameter in the description of the statistical decay properties of highly excited resonances. However, not every statistical distribution lends itself to a description in terms of temperature. There is an infallible criterion for "thermal radiation":

namely, if the momentum distribution (momentum is the product of mass times velocity for particles with mass and it is equal to the energy for massless particles) of the emitted debris is of the type of Planck's law for black body radiation. Now, what is that? I am sorry to confront the reader with another item he may not know, but it is most important. Black body radiation is the radiation coming out of a tiny hole in a large box kept in a temperature bath; it is called black body radiation because if the box is cold, the hole looks black and if it is hot, it radiates exactly like an absolutely non-reflecting and totally absorbing body (black body). Suppose the box is red-glowing, then the little hole radiates a whole spectrum of light with the maximum in the red, but other colours being present, too. If you then draw a curve, which shows for each colour how much it contributes to the total radiation, then this curve is called the black-body spectrum of photons (photons are massless; colour and photon energy are the same). This curve, however, has a particular shape and this shape depends on the temperature $T$ of the box (Fig. 2). Now here we have two results: you measure the momentum distribution of photons coming from some source and draw this curve, then (and that is also true for massive particles)

- you can speak of thermal radiation, if and only if a Planck curve can be found, which coincides with the measured curve;

- if that is the case, the temperature value belonging to the fitting Planck curve is the temperature of the source.

This gives us a tool to determine whether a certain radiation is thermal and if so, to measure its temperature. In this way astronomers can tell the surface temperature of a distant star, in this way the $3\, \text{K}$ radiation, coming in isotropically from outer space and giving one of the most important hints as to the history of our universe some $10^{10}$ years ago, has been discovered and interpreted and in this way we also measure the temperature of hadronic matter compressed to $10^{-13}$ cm and existing for $10^{-23}$ seconds. How? Consider Fig. 1 and measure (we know methods for that) the momentum component transverse to the direction of the incident particle of each of the outgoing debris (Fig. 3); then draw a curve which shows how often each possible transverse momentum is actually realized. Do this for many thousand such photographs taken under identical conditions and you will obtain a rather smooth curve. It then turns out that this curve is definitely a Planck curve (for massive particles) and thus a temperature can be

*) We must take the transverse momentum because the sources off radiating particles move fast in the longitudinal direction; only the transverse momentum is unaffected by that.
assigned to what was going on during the collision. It is this temperature which shows the strange behaviour of tending rapidly to a limit $T_0$ and staying there fixed as the collision energy grows up to the highest known cosmic ray energies. Intuitively everyone had expected the temperature to grow with the fourth root of the energy density (energy per cm$^3$) as it does for ordinary thermal radiation.

I said that the transverse momentum distribution is definitely a Planck curve; this is not exactly true: less than 1% of the particles produced show a deviation whose origin is not yet understood; maybe they are ejected before the equilibrium necessary for a thermal radiation is reached, maybe they are produced by rapidly spinning sources of radiation; there are many conjectures but no definite conclusion. This problem is very actively worked on at present.

We shall forget about this 1% of disobedient particles and ask: is there an explanation for the limit of the temperature? It seems so, and it runs as follows.

Remember that we know more than 200 elementary particles, most of them being highly unstable resonances, i.e., decaying in about $10^{-23}$ seconds into other particles of lower mass, which in turn may decay further. Resonances of large mass (we know such objects up to several proton masses) decay statistically; the larger the mass, the more difficult it becomes to produce it in a collision and the more difficult it also becomes to identify it. Therefore we know such resonances only up to some proton masses, but nothing prevents us from assuming that there are resonances of all possible masses; we only cannot observe them because of practical difficulties. Let us call very heavy resonances "fireballs" and let us assume they also decay statistically into other particles, resonances and fireballs; no known law of physics forbids this extrapolation and therefore we must even assume it. If somebody says "there must be an end to it" he makes a much more daring proposition and has to say why. Thus (except for mass) particles, resonances, fireballs, are all the same: hadronic matter compressed to about a proton volume. Each of them may be produced in a high energy collision and the heavier ones decay into, or consist of, the lighter ones which, in turn, decay into, or consist of, still lighter ones, and so on, until we end up with stable particles. For the heavy ones, the decay is statistical. We therefore may say that our hypothetical extrapolation of known things: particles and resonances, into fireballs, entails the following definition of fireballs:
a fireball is:

a statistical equilibrium (hadronic black-body radiation)
of undetermined numbers of all kinds of fireballs, each
of which, in turn, is considered to be

If we add to this the postulate: all hadrons (including fireballs) have
approximately the same size (proton volume) then these two, the above cir-
cular definition and the volume postulate, are sufficient to set up an
equation for the "density of states" of hadronic matter obeying the above
conditions. This equation has a unique solution.

Before discussing that, I have to soothe the angry reader who remarks
that a circular definition is a logical nonsense and that, if fireballs
consist of fireballs, they cannot have all the same volume. To the first,
I answer that almost every non-trivial equation in mathematics is some sort
of circular definition of its unknown; take

\[ x = \frac{4}{3} + \frac{1}{6} x^2 \]

here the unknown \( x \) is defined by some manipulation (square it, divide by
six and add \( \frac{4}{3} \)) performed upon itself and giving \( x = 2 \) and \( x = 4 \) as the
only solutions. In our fireball case the equation is much more complicated;
it is what mathematicians call an "integral equation" which is, indeed, the
most beautiful example of a circular definition which does make sense. Here
the solution is not (as for \( x \) above) a number, but a function, that is:
an infinite set of pairs of numbers; we will come back to that in a moment.
To the second objection, I answer that our fireballs are quantum mechanical
objects and thus are in principle to be represented by wave functions con-
centrated in the said small volume. Such wave functions can be superim-
posed and thus many hadrons, all of the same size, can be put into a box of
again that size. It sounds strange, but it is true (of course it is not
true for apples).

Back to our integral equation for the density of states. What is the
density of states? For a gas in a box we may visualize different config-
urations of the atoms; for instance: all in one quarter of the box, or
all moving up, or those in the right half moving up, in the left half moving
down, or all in a random distribution. There are very many possible random
distributions which, for a superficial observer, all look alike (think of
the crowd in the shopping centers just before Xmas; each minute it looks
the same, but the individuals have changed). Now, quantum mechanics gives us rules to decide how different two similar configurations must be, in order to become distinguishable. In our case, the number of possible distinguishable configurations or, which is the same, the number of available states, depends only on the energy (= mass) of our fireball. The higher the mass, the more possible states it can assume, and each of these states has to be interpreted as another particle. We then call "density of states \( \rho(m) \)" that function which tells us, for each mass (= energy) \( m \), how many different states lie in the unit interval around \( m \) (less precise: how many different sorts of particles have mass \( m \)). This function, if known, allows us to calculate many things, in particular the temperature at given energy density. If we wish to calculate this function from our definition of a fireball, then it is clear that it enters once for "a fireball is..." and then once for each of the "undetermined numbers of all kinds of fireballs" which make up the first one; it is exactly this which leads to the integral equation for \( \rho(m) \), which thus is the mathematical formulation of the above circular definition.

The great surprise is that this equation has a unique solution, which says:

- the number of possible kinds of fireballs is unlimited;
- fireballs may have any mass;
- the density of states, that is the number of different sorts of fireballs, grows exponentially with the mass at the rate

\[
\rho(m) \sim \frac{\text{const}}{m^3} \exp \left( \frac{m}{T_0} \right)
\]

The constant as well as the value of \( T_0 \) follow uniquely from the single free parameter in the theory: the mass of the \( \pi \) meson, which is the lowest existing hadron mass. In fact, this should not be called a free parameter but a fundamental natural constant; if one agrees on that, we have a zero parameter theory giving a definite numerical prediction of the density of states of hadrons or, in other words, of the "hadronic mass spectrum". Now one goes to the library and checks whether the known particles, sorted into a mass spectrum, follow the above law. They do, although the above formula was rather meant to describe the high mass behaviour, where observation is lacking. Most important: the rate of increase of the number of different known particles with mass is well accounted for by the numerical value of \( T_0 \), which the theory predicts to be nearly equal to the pion mass. This is an extraordinary result, so
extraordinary that many physicists are uneasy with it. There is another
surprise waiting: the laws of statistical thermodynamics tell us that with
an exponentially rising mass spectrum there must be an upper limit to the
temperature, namely just the $T_0$ appearing in the mass spectrum (as usual,
we have elementary particle units, in which the Boltzmann constant $k$ and
the velocity of light, $c$, are equal to 1 and Planck's constant $= 2\pi$).
We remember $T_0 \approx m_\pi$ ($\approx 140$ MeV) which, in temperature units, is just
about $2 \times 10^{12}$K, the limiting temperature found experimentally.

A reader who is acquainted with particle physics and/or statistical
mechanics will now ask: how can one obtain such results, which obviously
have to do with strong interaction dynamics, without having mentioned
interaction at all? We have mentioned it: the resonances are a mani-
ifestation of strong interactions and just by admitting them as particles
in their own right, and by doing this in a self-consistent way (a fireball
is...), we actually have introduced interaction; its mathematical form
is the integral equation for $\rho(m)$. In the particle physicist's jargon it
is a "non-perturbative approach to strong interaction dynamics".

In ordinary statistical mechanics one does since long something similar
(without aiming at self-consistency): a dilute He$^4$ gas is described as
an ideal Bose gas, while it should be described as a gas of protons, neutrons,
electrons with unmanageable interaction; one just introduces He$^4$ - and
this works !

What is the result? We have learnt that the three statements:

- fireballs consist of fireballs;
- the mass spectrum grows exponentially with mass;
- there is a limiting temperature;

are three different wordings of the same fundamental property of matter.
The beauty and consistency of this theory and its capability for making
detailed predictions (with no free parameter), which agree so far with
experiments, make it convincing enough to conclude that the idea that ele-
mentary particles consist of elementary particles, is fruitful and cannot
be wrong. The statistical approach does not, however, allow the prediction
of individual properties of elementary particles.
And the quarks? If they exist, they are in some sense more fundamental than our other particles, resonances and fireballs, but not more elementary; for they also are surrounded by field quanta and they also, if we would try to decompose them, would decay into other elementary particles without being destroyed themselves.

Why has nobody ever seen a quark? We do not yet know; but as we are so convinced that they exist, we look for explanations of this strange fact. Our above theory gives a simple explanation: suppose quarks do exist, are very heavy (say 10 or more proton masses) and strongly bound to each other to form mesons and nucleons. Then if you hit very hard, although the collision energy may be far greater than necessary to unbind them, you still would not find them. Why? Because with the same energy so many other things can be done, that the single process of unbinding the two quarks, being in competition with all the other possibilities, becomes an extremely improbable event (but one which is in principle possible).

The above theory allows us to calculate the frequency with which this event would occur; it decreases exponentially with the quark mass like $\exp(-2m_q/T_0)$. For $m_q \geq 10 m_{\text{proton}}$ it is rather safe to say that quarks will never be seen, even if they exist and in principle can be unbound. Imagine a Chinese vase with a beautifully and finely drawn dragon painted on it. Smash it with all your force on the floor; it will go into a thousand pieces but it will never happen that one of the pieces is that dragon, exactly, cut out and without damage (although in principle it could happen). Maybe that is why we do not see quarks emerging as debris from collision experiments, but see them by less violent observation.

Needless to say, the theory which I have tried to describe in plain words, is technically and mathematically complicated enough; I have given you only the flavour of it.

Its significance, which lies not only in elementary particle physics but also in astronomy and astrophysics, is twofold: it often provides an easy tool to calculate things which other theories cannot attain and it gives us a new philosophical insight into the fundamental structure of matter: it offers a solution to the very old and puzzling problem of philosophy, namely that neither continuous matter nor indivisible, true "atoms" (in the old sense) are acceptable, as both pictures lead to insurmountable internal logical contradictions.
Figure 1  Tracks of high energy particles penetrating through liquid hydrogen. The straight parallel lines are high energy protons; one of them has hit a hydrogen nucleus (also a proton) and from the point of collision emerge the iebris, mostly pions, but also the initial two protons, undamaged.
Figure 2 Planck curves representing the number of particles having a given momentum; different curves belong to different temperatures $T_1$ such that $T_1 < T_2 < T_3 < T_4$. We do not give scales, as the figure shall show only the typical form of a Planck curve and its variation with the temperature $T$. 