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S CHANNEL UNITARITY AND INCLUSIVE DISTRIBUTIONS

IN CUT REGGEON FIELD THEORY

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A B S T R A C T

Unitarity constraints on generalized eikonal diagrams with Pomeron interactions are shown to hold in cut Reggeon field theory. A non-perturbative proof of the AGK cancellation in this class of diagrams is given for the single particle inclusive distribution. However, the two-particle distribution turns out to be modified by Pomeron interactions in a non-trivial way. An approximate calculation shows that these modifications are crucial for consistency with momentum sum rules.

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Recent theoretical investigations on high energy hadronic scattering have raised the question of whether Reggeon field theory (RFT) ^{1),2)} which is based on t channel discontinuity equations for Reggeons ³⁾, is also consistent with direct channel unitarity ⁴⁾. In this context it has been found ⁵⁾ that a class of unitary theories for which the hadronic topological expansion ⁶⁾ is valid also have RFT as an asymptotic limit. For these theories the s channel discontinuities can be evaluated at high energy ⁵⁾ and satisfy a (new) cut Reggeon field theory (CRFT) involving three Reggeon fields : one (ψ_c) corresponding to the cut Pomeron P_c and two ($\psi_{\pm}$) corresponding to uncut Pomerons $P_{\pm}$ left above (below) the discontinuity plane.

Unitarity constraints imply relations between the two field theories, giving rise to the Lagrangians ^{5),7)}

$$\mathcal{L} = \psi^\dagger K \psi + i \frac{g}{2} \psi^\dagger \psi (\psi + \psi^\dagger) , \quad (1)$$

$$\begin{aligned} \mathcal{L}_c &= \sum_i \psi_i^\dagger K \psi_i + i \frac{g}{2} \sum_{ijk} R_{jk}^i \left(\psi_i^\dagger \psi_j \psi_k + \psi_i \psi_j^\dagger \psi_k^\dagger \right) = \\ &= \sum_i \psi_i^\dagger K \psi_i + i g \psi_c^\dagger \psi_c (\psi_+ + \psi_- + h.c.) - \frac{g}{\sqrt{2} r} (\psi_c^\dagger \psi_+ \psi_- + h.c.) + \\ &+ \frac{g r}{\sqrt{2}} \psi_c^\dagger \psi_c (\psi_c + \psi_c^\dagger) + i \frac{g}{2} (\psi_+^\dagger \psi_+^2 + \psi_-^\dagger \psi_-^2 + h.c.) , \end{aligned} \quad (2)$$

where

$$K = -\frac{1}{2} \frac{\overleftrightarrow{\partial}}{\partial y} - \alpha' \nabla_b^2 + \Delta$$

is the kinetic energy operator, $(y, \underline{b}) \equiv v$ are rapidity and impact parameter, $\Delta = \alpha'_0(0) - 1$ and $i = c, +, -$.

The coupling ratios R_{jk}^i occurring in Eq. (2) are determined by unitarity in the enhanced graph approximation, i.e., by requiring that the uncut and cut Pomeron propagators (properly normalized) be identically equal :

$$\langle 0 | \psi(0) \psi^\dagger(v) | 0 \rangle = \langle 0 | \psi_c(0) \psi_c^\dagger(v) | 0 \rangle , \quad (3)$$

It turns out ^{5),7)} that (3) leaves still room for two possible values of the parameter r in (2), i.e.,

$$r_A = 1, \quad r_B = 1/\sqrt{2}. \quad (4)$$

Only the first solution (A) gives the AGK ⁸⁾ weight of the absorptive, diffractive and polyperipheral contributions to the two-Pomeron cut $(-4:1:2)$ coming from the first three interaction terms in the Lagrangian (2). Solution (B) changes this ratio to $(-4:2:1)$ and considerably modifies the structure of multiplicity distributions. We will argue that this ambiguity can be resolved on the basis of multiparticle unitarity and shall principally refer to the case $r=1$.

In this note we further investigate unitarity constraints and inclusive distributions in the context of CRFT, by considering non-enhanced Pomeron graphs where two-particle n Pomeron couplings $\beta^{(0)}(n)$ are added. The high energy scattering amplitude is given (see Fig. 1) by :

$$\text{Im } A = \sum_{n,m=1}^{\infty} \frac{\beta^{(0)}(n)}{n!} G^{(n,m)} \frac{\bar{\beta}^{(0)}(m)}{m!}, \quad (5)$$

where $G^{(n,m)}$ is the $n \rightarrow m$ Pomeron Green's function in RFT.

This analysis was initiated by Abramovskii, Gribov and Kancheli (AGK) ⁸⁾ who studied the contributions of eikonal type (i.e., those corresponding to a completely disconnected $G^{(n,m)}$, Fig. 1b). On the basis of ϕ^3 Feynman diagrams they found that the $\beta^0(n)$ are essentially unchanged, except for phase factors, when taking the various possible s channel discontinuities. More precisely ^{*)}

$$\beta^{(0)}(n_c, n_+, n_-) = (-i\sqrt{2})^{n_c-1} \beta^{(0)}(n), \quad n = \sum n_i, \quad (6)$$

where n_c and $n_{\pm}$ are the number of P_c 's and $P_{\pm}$'s, and no discontinuity is possible involving only P_{+} 's (P_{-} 's). Combining Eq. (6) with the relevant conjugate relation

^{*)} The $\sqrt{2}$ factors come in because P_c is normalized to $\text{Im } P$ rather than to $\text{Disc } P = 2i \text{Im } P$.

$$\bar{\beta}^{(0)}(n_c, n_+, n_-) = (i\sqrt{2})^{n_c-1} (-)^{n_++n_-} \beta^{(0)}(n)^* , \quad n = \sum n_i, \quad (6')$$

they found :

- a) the n Pomeron cut contributes a well-defined weight to the various possible s channel discontinuities, i.e.,

$$w(n_c, n_+, n_-) = 2^{n_c-1} (-)^{n_++n_-} \frac{n!}{n_c! n_+! n_-!} , \quad n = \sum n_i, \quad (7)$$

This weight ensures that the s channel discontinuity equation is satisfied for eikonal-type graphs (Fig. 1b) ;

- b) peculiar cancellations occur when single and many-particle inclusive distributions are computed from (7) ; in particular, the single particle distribution is not renormalized at all by absorptive corrections, and the only graph which contributes is given in Fig. 2a ;
- c) the two-particle distribution for the eikonal graphs takes contributions with only $n_c = 1$ and $n_c = 2$ with $n_{\pm} = 0$ (Figs. 2b,c).

Since CRFT is a general method for evaluating discontinuities when Pomeron interactions take place, we have re-examined these results by taking into account the full amplitude in Eq. (5) (Fig. 1). We are then able to give a non-perturbative proof ^{*)} that results a) and b) above are not changed by Pomeron interactions. On the other hand, the form of the two- (and many-) particle distributions turns out to be modified, with respect to the result c) above, by Pomeron interaction effects. This modification is shown to be crucial for assuring that inclusive momentum sum rules be satisfied.

The central rôle in this study is the relation we deduce among the Green's function $G^{(n,m)}$ of RFT and the corresponding ones of CRFT, $G^{(n_c, n_+, n_-; m_c, m_+, m_-)}$. The key point in obtaining this relation is a re-definition of the fields ψ_i and conjugate momenta ψ_i^+ obtained by a similarity transformation ⁶⁾ and leading to a partial diagonalization of the CRFT Lagrangian. Let us in fact define

^{*)} The possibility of such generalization was suggested by AGK and proved for some two-loop diagrams by Lehman and Winbow ⁹⁾.

$$\varphi_\alpha = \sum_i D_{\alpha i} \psi_i, \quad \pi_\alpha = \sum_i \psi_i^+ (D^{-1})_{i\alpha} \quad (8)$$

where

$$D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & -i\sqrt{2}r \end{pmatrix}, \quad r=1 \quad \text{or} \quad r=1/\sqrt{2}. \quad (9)$$

By substituting Eqs. (8) and (9) into Eq. (2) we get a new Lagrangian

$$\mathcal{L}' = \sum_\alpha \pi_\alpha K \varphi_\alpha + i \frac{g}{2} \sum_\alpha (\pi_\alpha \varphi_\alpha^2 - \bar{\varphi}_\alpha^2 \bar{\pi}_\alpha), \quad (10)$$

where

$$\bar{\varphi} = -D D^T \pi, \quad \bar{\pi} = -\varphi (D D^T)^{-1}, \quad (11)$$

are the "adjoint" fields and momenta. For the particular values of r in (9) the interaction Lagrangian takes the form

$$\begin{aligned} \mathcal{L}'_{int} = & ig \left[\left(1 - \frac{1}{2r^2}\right) \pi_0 (\pi_+ + \pi_-) - \frac{1}{2r^2} \pi_+ \pi_- \right] (\varphi_0 - \varphi_+ - \varphi_-) + \\ & + i g \pi_0 (\pi_+ \varphi_+ + \pi_- \varphi_-) + i \frac{g}{2} \sum_\alpha (\pi_\alpha \varphi_\alpha^2 + \pi_\alpha^2 \varphi_\alpha). \end{aligned} \quad (12)$$

It is important to note that the Feynman rules for the φ fields involve off-diagonal vertices only in a definite time-rapidity direction. For instance, for the case $r=1$ the vertices involved are represented in Fig. 3. From this property we obtained a simple relation among RFT and CRFT Green's functions :

$$\langle 0 | \varphi_\alpha \dots \varphi_\alpha \pi_{\beta_1} \dots \pi_{\beta_m} | 0 \rangle = \delta_{\alpha\beta_1} \dots \delta_{\alpha\beta_m} G^{(n,m)}, \quad (13)$$

where the n fields on the left have all the same α (not summed), and $G^{(n,m)}$ is the usual Green's function of RFT. In terms of the ψ field Green's functions we have, by (8)

$$\sum_{i_1 \dots i_n} D_{\alpha i_1} \dots D_{\alpha i_n} \langle 0 | \psi_{i_1} \dots \psi_{i_n} \psi_{j_1}^+ \dots \psi_{j_m}^+ | 0 \rangle = D_{\alpha j_1} \dots D_{\alpha j_m} G^{(n, m)} \quad (14)$$

for any given α and $j_1 \dots j_m$. Particular cases of Eq. (14) are the relations between propagators [see Eq. (3)] and vertex functions of CRFT and RFT. More precisely

$$\begin{aligned} \langle \psi_i \psi_j^+ \rangle &= \delta_{ij} G^{(1,1)}, \\ \langle \psi_i \psi_j^+ \psi_k^+ \rangle &= (-)^{n_+ + n_-} \langle \psi_i^+ \psi_j \psi_k \rangle^* = R_{jk}^i G^{(1,2)}. \end{aligned} \quad (15)$$

For $n \geq 2, m \geq 2$, Eq. (14) gives in general a linear relation (not a proportionality one) among Green's functions of RFT and CRFT. A relation similar to Eq. (14), where the rôle of the indices i and j is exchanged, can be obtained from the conjugate relation for $G^{(n, m)}$ and $G^{(n_i, m_j)}$. By using the phase properties of the interaction Lagrangians, one can prove in fact that

$$\begin{aligned} G^{(n, m)}(y_1, y_2) &= \langle 0 | \psi(y_1)^n \psi(y_2)^m | 0 \rangle = (-)^{n+m} \langle 0 | \psi(-y_2)^m \psi(-y_1)^n | 0 \rangle^*, \\ G^{(n_i, m_j)}(y_1, y_2) &= (-)^{n_+ + y_+ + m_+ + m_-} \left(G^{(m_j, n_i)}(-y_2, -y_1) \right)^*. \end{aligned} \quad (16)$$

Weight of the n Pomeron cut discontinuities

In order to prove Eq. (7) we need to show that the full two-particle n Pomeron vertex function $\beta(n)$ (including Pomeron interactions) and its counter part $\beta(n_+, n_-, n_-)$ are proportional and in the same ratio as the bare couplings [Eq. (6)]. By definition, we have

$$\begin{aligned} \beta_m(v_1 \dots v_m) &= \sum_n \beta_n^{(0)} \frac{1}{n!} \langle 0 | \psi(0)^n \psi^+(v_1) \dots \psi^+(v_m) | 0 \rangle, \\ \beta_{m_+, m_+, m_-}(v_1 \dots v_m) &= \sum_n \sum_{n_i}' \beta_{n_+, n_+, n_-}^{(0)} \frac{1}{n_+! n_+! n_-!} G^{(n_i, m_j)}(0, \dots, v_1, \dots, v_m), \end{aligned} \quad (17)$$

where Σ' runs over the n_i 's such that $\Sigma n_i = n$, but excluding the terms with $n_{\pm} = n$.

On the other hand, if all ψ 's are evaluated at $v=0$, Eq. (14) for the case $\alpha=0$ can be written, by (9), as

$$\begin{aligned} \sum_{n_i}' (-i\sqrt{2}r)^{n_c-1} \frac{n!}{n_c!n_+!n_-!} G^{(n_i, m_j)}(0, \dots, v_1, \dots, v_m) = \\ = (-i\sqrt{2}r)^{m_c-1} G^{(n, m)}(0, \dots, v_1, \dots, v_m), \quad \sum m_i = m, \quad m_{\pm} \neq m. \end{aligned} \quad (18)$$

It is therefore clear that if the relation (6) is assumed for the bare couplings $\beta_n^{(0)}$ and $\beta_{n_i}^{(0)}$, and r is taken to be $r=1$, by (17) and (18) the full vertex functions β_n and β_{n_c, n_+, n_-} are proportional and related as in Eq. (6). Analogously, by using Eq. (16) one can prove the conjugate relation, Eq. (6') for the full vertex functions.

Unitarity relation

The physical content of Eqs. (14) and (18), which relate the Green's functions of RFT and CRFT, is essentially given by the discontinuity formula for the n, m contribution to the amplitude in Eq. (5). The unitarity equation for A can be written (see Fig. 4)

$$\begin{aligned} \text{Im } A = \sum_{n, m} \frac{\beta_n^{(0)}}{n!} G^{(n, m)}(v_1; v_2) \frac{\bar{\beta}_m^{(0)}}{m!} \\ = \sum_{n, m} \sum_{n_i}' \sum_{m_j}' \frac{\beta_{n_i}^{(0)}}{n_c!n_+!n_-!} G^{(n_i, m_j)}(v_1; v_2) \frac{\bar{\beta}_{m_j}^{(0)}}{m_c!m_+!m_-!}, \end{aligned} \quad (19)$$

where in Σ' one has $\Sigma n_i = n$, $\Sigma m_j = m$ and $n_{\pm} \neq n$.

This equation can be proved by using the relation (6) among coupling of external particles with Pomerons and the relation (18) among Green's functions in the case $r=1$. In fact, we get :

$$\begin{aligned} \frac{1}{n!} \beta_n^{(0)} G^{(n,m)}(v_1; v_2) \sum_{m_i} ' 2^{m_c-1} (-)^{m_+ + m_-} \frac{1}{m_c! m_+! m_-!} \beta_m^{(0)*} = \\ = \frac{1}{2} \left[(2-1-1)^{m_c} - 2(-)^{m_c} \right] \left(\frac{\beta_n^{(0)}}{n!} \frac{\beta_m^{(0)*}}{m!} G^{(n,m)}(v_1; v_2) \right) = \frac{\beta_n^{(0)}}{n!} \frac{\beta_m^{(0)}}{m!} G^{(n,m)}(v_1; v_2). \quad (20) \end{aligned}$$

This proves that unitarity holds for each term of the amplitude in Fig. 1, also when Pomeron interactions are included. Note that the ambiguity in Eq. (4) for the couplings in CRFT is solved if one assumes ⁸⁾ that the couplings of Pomerons with external particles are not changed by various s channel discontinuity, i.e., Eq. (6) holds. This property is a consequence of many-particle unitarity ^{*}) since, for instance, the coupling $\beta^0(n=2)$ is related to the six-point amplitude. Equation (6) can be proved on the basis of simple φ^3 Feynman diagram ⁸⁾ and it is in general consistent with many-particle unitarity, if the phase of the amplitude is simply given by a product of signature factors.

This result clarifies in part how to introduce non-enhanced many-Pomeron interactions in RFT so as to preserve the discontinuity formulas. If $g_{n,m}$ denote the couplings among n incoming and m outgoing Pomerons, and $g_{n_c, n_+, n_-; m_c, m_+, m_-}$ the corresponding ones in CRFT, they must satisfy the relation, Eq. (18)

$$\sum_{n_i} ' (-i\sqrt{2})^{n_c-1} \frac{n!}{n_c! n_+! n_-!} g_{n_i, m_i} = (-i\sqrt{2})^{m_c-1} g_{n, m}. \quad (21)$$

^{*}) The fact that the ambiguity in Eq. (4) should be solved by many-particle unitarity has been pointed out to us by G. Veneziano.

These equations do not completely determine, however, the $\mathcal{E}\{n_i, m_j\}$ in terms of $g_{n,m}$ if $n \geq 2$ and $m \geq 2$. In such cases other (non-linear) constraints have to be imposed in order to satisfy Eq. (19) to all orders of perturbation theory.

Even if we have argued that the ambiguity in Eq. (4) is solved and $r=1$, it is still amusing to note that Eq. (19) can be valid also for the case $r=1/\sqrt{2}$, provided Eq. (5) is replaced by

$$\beta_{n_+, n_+, n_-}^0 = (-i\sqrt{2} r)^{n_+ - 1} \beta_n^0. \quad (22)$$

In fact, the quantity occurring in the second line of Eq. (20) is in this case

$$(2r^2)^{-1} \left[(2r^2 - 2)^n - 2(-)^n \right] = (-)^{n+1}$$

for both $r=1$ and $r=1/\sqrt{2}$.

Inclusive distributions

These distributions in CRFT are obtained by singling out the observed particles among the intermediate states of cut Pomeron propagators, to obtain a Mueller-type inclusive diagram. For instance, the one-particle distribution for a single Pomeron exchange in Fig. 1, is given by Fig. 5a, where the P_c propagator with the inclusive particle is indicated. In the field theoretical language, this corresponds to study the Green's functions $G^{(n_i, m_j)}$ with mass insertions of ψ_c fields, $\rho \psi_c \psi_c^+$, where ρ is the inclusive coupling. It is clear in this way that we obtain only distributions in the v space. Let us consider first the single particle distribution for the exchange of a single (renormalized) Pomeron in Fig. 1, which is given by (Fig. 5a)

$$\frac{d\sigma(v)}{dv} = \beta^2 \rho \langle \psi_c(v) \psi_c^+(v) \psi_c(v) \psi_c^+(v) \rangle, \quad (23)$$

with $v=(y, \underline{b})$, $V=(Y, \underline{B})$, rapidity and impact parameters of observed and external particles. By inserting a complete set of states of fixed y we have :

$$\frac{d\sigma}{dv} = \beta^2 \rho \sum_{n=0} \sum_{n_i}' \langle 0 | \psi_c(0) \psi_c^+(v) | n_i \rangle \langle n_i | \psi_c(v) \psi_c^+(v) | 0 \rangle, \quad (24)$$

with ^{*})

$$|n_i\rangle = \left(\frac{1}{n_c! n_+! n_-!} \right)^{1/2} (\psi_c^+)^{n_c} (\psi_+^+)^{n_+} (\psi_-^+)^{n_-} |0\rangle, \quad \sum n_i = n \quad (25)$$

We now use Eq. (14) to get in particular the vertex

$$\langle \psi_c (\psi_c^+)^{n_c+1} (\psi_+^+)^{n_+} (\psi_-^+)^{n_-} \rangle = (-i\sqrt{2}r)^{n_c} G^{(1, n+1)} \quad (26)$$

together with its conjugate one

$$\langle (\psi_c)^{n_c+1} (\psi_+)^{n_+} (\psi_-)^{n_-} \psi_c^+ \rangle = (-i\sqrt{2}r)^{n_c} G^{(n+1, 1)} \quad (27)$$

From Eqs. (26) and (27), we can now evaluate Eq. (24) :

$$\frac{d\sigma}{dv} = \beta^2 \rho \sum_{n=0} \left(\sum_{n_i} (-2r^2)^{n_c} \frac{n!}{n_c! n_+! n_-!} \right) \langle 0 | \psi(0) \psi^+(v) | n \rangle \langle n | \psi(v) \psi^+(v) | 0 \rangle \quad (28)$$

The factor in parenthesis is $(-2r^2+2)^n$ and therefore vanishes for $r=1$, unless $n=0$. We get therefore in this case

$$\frac{d\sigma}{dv} = \beta^2 \rho \langle 0 | \psi(0) \psi^+(v) \rangle \langle 0 | \psi(v) \psi^+(v) | 0 \rangle. \quad (29)$$

This equation generalizes the famous AGK cancellation (Fig. 5b). By the same method one can prove that all other contributions of Fig. 1, with $n, m \neq 1$ do not contribute to the single particle density. In this case one can use, instead of Eq. (27) the full Eq. (18) in order to sum over the possible cuts of $\beta_n^{(0)}$. However, the result can also be guessed from the fact, proved above, that the weight of the n_i Pomeron cut is the same as in AGK.

^{*}) Integrations in the b phase space is understood.

This general property confirms the result ^{8),9)} that the mass insertion operator has no anomalous dimension, so that the asymptotic behaviour of multiplicity moments $n_k(s)$ is determined by the behaviour of the cross-section. In particular, if the final Pomeron intercept is at $\alpha_P=1$, we have ^{*}) $n_k(s) \approx c_k (\ln s)^{k(1+\eta)}$ where $\sigma_T \sim (\ln s)^\eta$. Let us stress that our algebraic proof of Eq. (29) is independent of the values of the renormalized Pomeron intercept, provided, of course, that the inclusive distribution is defined in Eq. (23), i.e., external particles are coupled to the ψ and ψ^+ fields. For a different definition in the case of $\alpha_P > 1$, see Ref. 10).

Let us make a remark on the amusing case $r=1/\sqrt{2}$ for which the factor in parentheses in Eq. (28) is $(2-2r)^n=1$. We have, also in this case, an exact formula, i.e.,

$$\frac{d\sigma}{dv} = \beta^2 \int \langle 0 | \psi(0) \psi^+(v) \psi(v) \psi^+(v) | 0 \rangle, \quad r = 1/\sqrt{2}. \quad (30)$$

However, in this case, renormalization effects are present. In particular when $\alpha_P=1$ the anomaly in the dimensions of $\psi^+(v)\psi(v)$ is to the leading order in the ϵ expansion ¹¹⁾ given by -2η .

Let us consider now the double inclusive distribution which is given by two mass insertions ; for the single Pomeron exchange

$$\frac{d^2\sigma}{dv_1 dv_2} = \langle 0 | \psi_c(0) \psi_c^+(v_1) \psi_c(v_1) \psi_c^+(v_2) \psi_c(v_2) \psi_c^+(v) | 0 \rangle, \quad (31)$$

corresponding to Fig. 6. The two terms correspond to the case in which the Pomeron lines with the mass insertion overlap in rapidity (Fig. 6a) or not (Fig. 6b). In the first case the AGK cancellation mechanism holds unchanged. In the second case, we will see that Pomeron interaction plays a crucial rôle. More formally we can write :

^{*}) Note that, according to the positivity properties proved in Ref. 6), the exact values of c_K have to preserve the positivity of σ_n .

$$\begin{aligned} \frac{d^2\sigma}{dv_1 dv_2} = & \sum_{n_i} \langle 0 | \psi_c(0) \psi_c^+(v_1) \psi_c^+(v_2) | n_i \rangle \langle n_i | \psi_c(v_1) \psi_c(v_2) \psi_c^+(V) | 0 \rangle + \\ & + \sum_{n_i, m_j} \langle 0 | \psi_c(0) \psi_c^+(v_1) | n_i \rangle \langle n_i | [\psi_c(v_1), \psi_c^+(v_2)] | m_j \rangle \langle m_j | \psi_c(v_2) \psi_c^+(V) | 0 \rangle \end{aligned} \quad (32)$$

The generalization for the full amplitude of Fig. 1 is trivial. The first sum can be manipulated exactly as in Eq. (24) so that only the vacuum intermediate state contributes yielding for $d^2\sigma/dv_1 dv_2$ the term in Fig. 7a, i.e., the term

$$2 \langle 0 | \psi(0) \psi^+(v_1) \psi^+(v_2) | 0 \rangle \langle 0 | \psi(v_1) \psi(v_2) \psi(V) | 0 \rangle. \quad (33)$$

This, for what we said before, is the only relevant contribution to the correlation function when $y_1 \sim y_2$.

The second sum in (32) contains the disconnected term, when $|n_i\rangle = |0\rangle$, which is represented in Fig. 7b, i.e.,

$$\langle 0 | \psi(0) \psi^+(v_1) | 0 \rangle \langle 0 | \psi(v_1) \psi^+(v_2) | 0 \rangle \langle 0 | \psi(v_2) \psi^+(V) | 0 \rangle. \quad (34)$$

When only eikonal-type of diagrams are considered, Eqs. (33) and (34) give all the contributions to $d\sigma/dy_1 dy_2$ as AGK have proved (see Figs. 2b,c). However, in general, other contributions come from the connected diagrams in Fig. 6b, i.e., from the second sum in Eq. (32) when $|n_i\rangle \neq |0\rangle$. For instance, the first non-vanishing connected contribution is given in Fig. 8. In general these contributions can be computed from the skeleton expansion of Fig. 6b. The weights of CRFT skeleton diagrams with respect to the RFT ones can be computed by using the relations among the three Pomeron vertex functions given in Eq. (15). The weights of the diagram of Fig. 8, for instance, is

$$\sum_{ijk} R_{ci}^c R_{ij}^c R_{jk}^c R_{kc}^c = [(R^c)^4]_{cc} = -1. \quad (35)$$

Similarly, one can verify that all other two-loop skeleton diagrams with two ψ_c mass insertions do instead vanish. In general, it seems that no simple sum rule exists to give a close expression of Fig. 6b in terms of RFT Green's functions like in Eqs. (33) and (34).

In order to check the importance of higher order graphs of the type in Fig. 8 [i.e., $|n_1\rangle \neq |0\rangle$ in the second sum in Eq. (32)] we have computed the contributions to the inclusive distributions of the amplitude of Fig. 1 [Eq. (5)] in the limit in which the Green's functions $G^{n,m}$ do not contain loop diagrams (tree approximation). By studying the classical field equations of RFT in the approximation $\alpha' \approx 0$, it has been shown by Amati, Caneschi and Jengo¹²⁾ that this contribution to the S matrix is bounded below one even if the Pomeron intercept is $\alpha_P = 1 + \Delta > 1$. We have done a similar calculation in CRFT with a mass insertion term of the form $\rho J(y) \psi_c(y) \psi_c^+(y)$ to compute inclusive distributions. It turns out that even if $\alpha_P > 1$ the inclusive distributions (in the tree approximation) are consistent with energy momentum sum rules, and that the higher order corrections we have considered are crucial to this. The CRFT classical equations can be solved exactly for $J(y) = 0$ to give the classical fields $\langle \psi_c(y) \rangle$, $\langle \psi_c^+(y) \rangle$ which, in the high energy limit, are independent of the external coupling¹³⁾ and

$$\langle \psi_c(y) \rangle \simeq \langle \psi_c^+(y) \rangle \simeq \sqrt{2} \left(\frac{2\Delta}{g} \right) . \quad (36)$$

The single particle distribution is then easily computed to give (cf. Fig. 9a) at high energy

$$\frac{d\sigma}{dy} = \frac{1}{2} \rho \langle \psi_c \rangle \langle \psi_c^+ \rangle \simeq \rho \left(\frac{2\Delta}{g} \right)^2 . \quad (37)$$

The two-particle distribution turns out to be

$$\frac{d\sigma}{dy_1 dy_2} \simeq \frac{1}{2} \left\{ \left(\rho \langle \psi_c \rangle \langle \psi_c^+ \rangle \right)^2 + \rho^2 \langle \psi_c \rangle \langle \psi_c^+ \rangle \left(5 + 2|y_1 - y_2| \Delta \right) e^{-\Delta|y_1 - y_2|} \right\} , \quad (38)$$

where the first term is essentially the square of Eq. (37). Note that the second term in (38) is of short-range type, i.e., exponentially decreasing with $|y_1 - y_2|$ so that the distribution is consistent with inclusive

momentum sum rules. This effect depends crucially on the extra terms in the second sum of Eq. (32) of connected type, i.e., $|n_i\rangle \neq |0\rangle$. In fact for $|n_i\rangle = |0\rangle$ one obtains for $do/dy_1, dy_2$ the contribution in Fig. 9b which diverges as $e^{\Delta|y_1-y_2|}$ to violate the momentum sum rules. This divergent contribution is, however, cancelled at the end in Eq. (38) from the other contributions with $|n_i\rangle \neq |0\rangle$ (the first is given in Fig. 9c). A detailed discussion of the method and results of this calculation will appear in a forthcoming paper.

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FIGURE CAPTIONS

- Figure 1 Generalized eikonal diagram with Pomeron interactions. $\beta^o(n)$ is the n Pomeron coupling with external particles and $\bar{\beta}^o(m)$ its conjugate one. G represents the full Pomeron Green's function and G' its not completely disconnected part.
- Figure 2 The Mueller diagrams for the single (2a) and double (2b,c) particle distribution for the eikonal type of Pomeron graph in Fig. 1b.
- Figure 3 Interaction vertices of CRFT for the fields φ_α and π_α defined in Eq. (8).
- Figure 4 Representation of the unitarity relation in Eq. (19). v_1 and v_2 represent the impact parameters and rapidities of external particles ; solid lines represent cut Pomerons.
- Figure 5 Mueller diagram for the single particle inclusive distribution for the exchange of a single renormalized Pomeron in Fig. 1.
- Figure 6 Two-particle inclusive distribution in Eq. (31). The two contributions correspond to the case in which the two-cut propagator with the observed particles (mass insertion) are overlapping in rapidity (6a) or not (6b).
- Figure 7 Contributions to the two-particle inclusive distribution. The diagram 7a is the only contribution from Fig. 6a ; therefore it is the only relevant one when the two particles are close in rapidity. Diagram 7b is the simplest contribution of type of Fig. 6b.
- Figure 8 Example of another contribution to the two-particle inclusive distribution of the class of Fig. 6b. The weight of the contribution with respect to a RFT diagram (8b) is computed from Eq. (15).

- Figure 9
- a) Single-particle distribution for the amplitude in Fig. 1 in the tree approximation.
 - b) A contribution to the two-particle distribution (in the tree approximation) which, for $\alpha_P > 1$ would violate inclusive momentum sum rules.
 - c) Example of contributions to the two-particle distribution which cancels the graph in b) to give at the end the short-range term in Eq. (38).

FIG.1

FIG. 2

FIG.3

FIG.4

FIG.5

FIG.6

FIG.7

FIG.8

FIG.9