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SUM RULES FOR ELASTIC $\gamma\gamma$ SCATTERING
AND MESON DECAYS IN TWO PHOTONS

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A B S T R A C T

Using very general assumptions we derive superconvergence type sum rules for physical absorptive parts in elastic real $\gamma\gamma$ scattering. They are analogous to the crossing sum rules studied extensively in $\pi\pi$ scattering. Our main result is the general proof that tensor meson decays into two photons must involve predominantly photons with opposite helicities. Estimates for the $f_{\gamma\gamma}$ width are also given.

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1. INTRODUCTION

Although electron storage rings are at present mainly used to study new phenomena, the more conventional problem of $\gamma\gamma$ scattering remains an interesting subject.

In this paper we shall study what restrictions on absorptive parts, and in particular on resonance couplings to two photons, follow from general principles. These principles are essentially only (a) a good high-energy behaviour, as found in renormalizable perturbation theory and as proven rigorously for massive particle scattering from axiomatic field theory, and (b) crossing symmetry. It is well known that these two requirements together give stringent constraints in processes involving the exchange of spinning particles. One solution to these constraints is the Veneziano model with its many modifications. Another solution, within the framework of field theory, is gauge theory. Indeed, it has been shown that any field theory involving spin-1 particles and satisfying the above principles on the tree level, must be a gauge theory¹⁾.

Both these examples are "global" approaches in the sense that one searches a solution for the general n-point function, which implies among others that it is not at all trivial to go beyond the tree approximation. In the present paper we shall be much more modest. We shall derive a set of sum rules by S-matrix methods, and try a purely phenomenological saturation of the most important ones. Our results will seem less spectacular, but on the other hand we will have little trouble to include, for example, the Pomeron.

The method used is a straightforward generalization of work done for pion-pion scattering²⁾ to (external) particles involving spin. Take, for instance, an amplitude for which one can write unsubtracted dispersion relations (DR) but no superconvergence relations. Sum rules of the superconvergence type will, nevertheless, be obtained by writing for the same amplitude fixed-t and fixed-s DRs. In general, the two integrands will be different even for crossing-symmetric reactions, and demanding the integrals to be equal will result in a non-trivial sum rule.

2. AMPLITUDES FOR PHOTON-PHOTON SCATTERING

For simplicity, we shall restrict ourselves here to the elastic scattering of real photons, assuming P and T invariance. We have then five independent helicity amplitudes

$$\begin{aligned}
T_1(s,t,u) &= T_{++,++}(s,t,u) \\
T_2(s,t,u) &= T_{++,--}(s,t,u) \\
T_3(s,t,u) &= T_{+-,+-}(s,t,u) \\
T_4(s,t,u) &= T_{+-,+ -}(s,t,u) \\
T_5(s,t,u) &= T_{+-,-+}(s,t,u) ,
\end{aligned} \tag{1}$$

with $s + t + u = 0$. The first three amplitudes have no helicity flip, and thus they are the only ones surviving at $t = 0$. The amplitude T_4 is proportional to t at $t \rightarrow 0$, and $T_5 \sim t^2$ for small t .

Since all external particles are massless, the crossing properties are extremely simple. Indeed, we have under (s,t) resp. (s,u) crossing

$$\begin{aligned}
& \begin{array}{c} \nearrow \\ \searrow \end{array} \begin{array}{l} (T_5, T_2, T_3, T_4, T_1) \\ (T_3, T_2, T_1, T_4, T_5) \end{array} \begin{array}{l} (s \leftrightarrow t) \\ (s \leftrightarrow u) , \end{array} \tag{2a} \\
(T_1, T_2, T_3, T_4, T_5) & \tag{2b}
\end{aligned}$$

while the exchange of the two outgoing photons (Bose symmetry) gives

$$(T_1, T_2, T_3, T_4, T_5) \rightarrow (T_1, T_2, T_5, T_4, T_3) \quad (t \leftrightarrow u) . \tag{2c}$$

Summarizing, we see that T_2 and T_4 are completely symmetric under the exchange of any two Mandelstam variables, while T_1 , T_3 , and T_5 get simply interchanged by such an exchange. Together with the behaviour at $t \approx 0$, this means that we can define reduced amplitudes τ_i without kinematic singularities by

$$\begin{aligned}
\tau_1(s,t,u) &= \frac{1}{s^2} T_1(s,t,u) \\
\tau_2(s,t,u) &= T_2(s,t,u) \\
\tau_3(s,t,u) &= \frac{1}{u^2} T_3(s,t,u) \\
\tau_4(s,t,u) &= \frac{1}{stu} T_4(s,t,u) \\
\tau_5(s,t,u) &= \frac{1}{t^2} T_5(s,t,u) .
\end{aligned} \tag{3}$$

Indeed, these amplitudes are also free of kinematical zeros, except for τ_2 which has a zero at $s = t = u = 0$ ^{3,4)}. However, the latter cannot be split off by a polynomial factor, unless we keep $s = 0$ or $t = 0$ or $u = 0$ fixed⁴⁾.

3. THE BASIC SUM RULES

From Eq. (3) one can estimate the high-energy behaviour of the τ_i at fixed t . At first we assume only a Jin-Martin⁵⁾ type bound $|\tau_i| < s^2$ for $|t| < \varepsilon$. Though such a behaviour cannot be proven rigorously in the present case, it should hold in any renormalizable theory order by order. On a more phenomenological level, it would also follow from vector dominance, which connects the photonic amplitude to a hadronic one for which the Jin-Martin arguments work. Equation (3) tells us then, that τ_1 , τ_3 and τ_4 should satisfy unsubtracted fixed- t DRs, while two subtractions might be necessary for τ_2 and τ_5 .

The simplest sum rule is obtained if one compares the fixed- t DRs for $\tau_4(s, t, u)$ and for $\tau_4(s, u, t)$. The result is the infinite set of sum rules

$$\delta_4(s, t, u) \equiv \int_{m_\pi^2}^{\infty} \frac{dx}{(x-s)(x-t)(x-u)} \left\{ (x-t)(2x+t) \operatorname{Im} \tau_4(x, t) - (x-u)(2x+u) \operatorname{Im} \tau_4(x, u) \right\} = 0. \quad (4)$$

For $t = u = 0$, this gives a trivial identity. But, if we take $u = 0$ and then consider the derivative with respect to t at $t = 0$, we find

$$\left. \frac{d}{dt} \delta_4(-t, t, 0) \right|_{t=0} = \int_{m_\pi^2}^{\infty} \frac{dx}{x^2} \left\{ 2x \frac{\partial}{\partial t} \operatorname{Im} \tau_4(x, t) \right|_{t=0} - \operatorname{Im} \tau_4(x, 0) \right\} = 0. \quad (5)$$

In order to discuss the physical content of this sum rule, let us insert the partial wave decomposition

$$T_i(s, t) = \sum_J (2J+1) f_{\lambda_a \lambda_b; \lambda_c \lambda_d}^J(s) d_{\lambda_i \mu}^J(\theta) \quad i=1, \dots, 5 \quad (6)$$

with $\lambda = \lambda_a - \lambda_b$, $\mu = \lambda_c - \lambda_d$. Furthermore, let us for the moment assume a saturation with narrow resonances, i.e.

$$\operatorname{Im} f_{\lambda_a \lambda_b; \lambda_c \lambda_d}^J(s) = \pi \sum_A g_{A, \lambda_a \lambda_b} g_{A, \lambda_c \lambda_d} \delta(m_A^2 - s). \quad (7)$$

The sum here extends over all resonances with spin J ; the $g_{A, \lambda_a \lambda_b}$ are the coupling constants for the particle A to two photons with helicities λ_a and λ_b (in the A -rest frame). Parity conservation and Bose symmetry imply

$$g_{A, \lambda_a \lambda_b} = P_A g_{A, -\lambda_a -\lambda_b} \quad (8a)$$

and

$$g_{A, \lambda_a \lambda_b} = (-)^{J_A} g_{A, \lambda_b \lambda_a}, \quad (8b)$$

respectively.

Thus there are at most two independent couplings $g_{A,++}$ and $g_{A,+ -}$ for each particle. The possible couplings are listed in the following⁶⁾:

$$\begin{aligned} J^P &= 1^+, 3^+, 5^+, \dots & : & \quad g_{A,++} = g_{A,+ -} = 0 \\ J^P &= 3^-, 5^-, \dots & : & \quad g_{A,++} = 0 ; \quad g_{A,+ -} \neq 0 \quad (9) \\ J^P &= 0^\pm, 2^-, 4^-, \dots & : & \quad g_{A,++} \neq 0 ; \quad g_{A,+ -} = 0 \\ J^P &= 2^+, 4^+, \dots & : & \quad g_{A,++} \neq 0 ; \quad g_{A,+ -} \neq 0 \end{aligned}$$

Inserting Eqs. (6) and (7) into Eq. (5), and keeping all resonances with $J \leq 4$, we find

$$\sum_{T=f, f', A_2} \frac{g_{T,++} g_{T,+ -}}{m_T^{10}} = 15 \sqrt{15} \sum_{4^+ = h, \dots} \frac{g_{4^+,++} g_{4^+,+ -}}{m_{4^+}^{10}} + \quad (10)$$

+ high energy contributions .

This sum rule shows explicitly the drawbacks of Eq. (5) and, indeed, of most other sum rules which one can derive in a similar way. First of all, low-spin particles (with generally better known couplings) do not appear. This can be traced back to our weak assumptions which are identically fulfilled for the exchange of low-spin particles. Secondly, even the sign of the high-energy contribution is unknown unless one is willing to make rather strong assumptions.

There exists, however, one set of sum rules where the sign of the high-energy contribution can be controlled by the help of unitarity, and in the following we shall mainly concentrate on the simplest of these.

Comparing fixed- t DRs for $\tau_1(s, t, u)$ and $\tau_1(s, u, t)$, we find, in a way analogous to the one leading to Eq. (4), the sum rule

$$\delta_1(s, t, u) \equiv \int_{m_\pi^2}^{\infty} dx \left\{ \frac{\text{Im} [\tau_1(x, t) - \tau_1(x, u)]}{x-s} + \frac{\text{Im} \tau_3(x, t)}{x-u} - \frac{\text{Im} \tau_3(x, u)}{x-t} \right\} = 0 \quad (11)$$

Taking again, as before, $u = 0$ and t small, we find

$$\left. \frac{d}{dt} \delta_1(-t, t, 0) \right|_{t=0} = \int_{m_\pi^2}^{\infty} \frac{dx}{x^2} \left\{ x \frac{\partial}{\partial t} \text{Im} [\tau_1(x, t) + \tau_3(x, t)] \right|_{t=0} - \text{Im} \tau_3(x, 0) \right\} = 0 \quad (12)$$

which we can write in terms of the partial waves defined in Eq. (6) as

$$\int_{m_\pi^2}^{\infty} \frac{ds}{s^4} \sum_{J \geq 2} (2J+1) \left\{ J(J+1) \text{Im} f_{++ , ++}^J(s) + (J^2 + J - 7) \text{Im} f_{+ - , + -}^J(s) \right\} = 0 \quad (13)$$

Unitarity now implies

$$\text{Im } f_{++}^J(s) \geq 0 \quad (14a)$$

and

$$\text{Im } f_{+-}^J(s) \geq 0, \quad (14b)$$

and we see that the $J = 2$ contribution to the second term is the only negative contribution to Eq. (13). If we assume now again a saturation by narrow resonances only, in the spirit of Freund-Harari duality⁷⁾, we find the exact inequality

$$\sum_{T=f, \dots} \frac{g_{T,+-}^2}{m_T^8} \geq 6 \sum_T \frac{g_{T,++}^2}{m_T^8} + \sum_{4^+ = \dots} (36 g_{4^+,++}^2 + \frac{117}{5} g_{4^+,+-}^2) \frac{1}{m_{4^+}^8} \quad (15)$$

In particular, this implies that the coupling of tensor mesons must be mainly to two photons of opposite helicity,

$$|g_{T,+-}| \gg |g_{T,++}| \quad (16)$$

A method of testing this inequality experimentally is to measure the angular distribution in the process $\gamma\gamma \rightarrow T \rightarrow \pi\pi$ with unpolarized photons. Following from formula (16) one should expect

$$\frac{d\sigma}{dt} \propto \sin^4 \theta + (\cos^2 \theta - \frac{1}{3})^2 \cdot 6 \left| \frac{g_{T,++}}{g_{T,+-}} \right|^2 \approx \sin^4 \theta \quad (17)$$

In the last two equations, the index "T" means the sum over all tensor mesons. Since the contribution of daughters should be completely negligible owing to the factors m_T^8 , these are only the f , f' , and A_2^0 . To single out the individual tensor meson couplings we assume exact SU(3). Even then the relative coupling strengths are not fully determined because they still depend on the unknown singlet/octet coupling ratio. Assuming for this the value predicted by the U(3) quark model⁸⁾ and assuming a quadratic mixing angle of 31° , we get, for the dimensionless couplings:

$$\frac{g_{f,\lambda\mu}}{m_f} : \frac{g_{f',\lambda\mu}}{m_{f'}} : \frac{g_{A_2,\lambda\mu}}{m_{A_2}} = 1 : 0,20 : 0,59 \quad (18)$$

Thus the main contribution should stem from the f meson. This further shows that formula (16) is correct for the individual tensor mesons separately.

The inequality $|g_{T,+-}| \gg |g_{T,++}|$ has already been suggested using much more stringent assumptions, namely either on the basis of broken dynamical symmetries and VMD⁹⁾, or starting from finite energy sum rules for the process $\gamma\gamma \rightarrow \pi\pi$ ¹⁰⁾. Here we have derived it from very general assumptions. We have seen that it is a necessary condition in *any* dual resonance model, but it might hold even much more generally.

In Ref. 2, one of us has shown that for $\pi\pi$ scattering Harari-Freund duality seems to be broken in the sense that the Pomeron is dual to part of the f meson [see also Lovelace¹¹⁾]. Here we see exactly the same, unless we have in Eq. (13) a strong contribution from the low-energy (threshold region) background in the f_{+-}^2 partial wave. This is not very probable, because of the following reason: as it has been suggested by many authors, the dominant (non-resonant) contribution to the absorptive parts at low energy stems from two-pion production calculated in spin-0 QED (cf. Fig. 1). The latter is, however, a renormalizable theory, and thus all sum rules derived from the above assumptions must be identically fulfilled for the graphs of Fig. 1 *).

Taking this f -Pomeron "duality" seriously, we can even attempt a crude estimate of the $f \rightarrow \gamma\gamma$ decay width. Choosing the normalization of the amplitudes such that the width is given by

$$\Gamma_{f,\lambda\mu} = \frac{g_{f,\lambda\mu}^2}{m_f} \quad , \quad (19)$$

we make for the Pomeron the ansatz

$$\text{Im } \tau_i^P(s,t) \approx \text{Im } \tau_3^P(s,t) \approx \frac{1}{32 \pi s} \sigma_{tot}^{rr} e^{\frac{1}{2}bt} \theta(s-s_0).$$

For the slope parameter we choose $b = 5 \text{ GeV}^{-2}$. For the total cross-section we use factorization**) to predict $\sigma_{tot}^{\gamma\gamma} = (\sigma_{tot}^{p\gamma})^2 / \sigma_{tot}^{pp} = 0.25 \text{ } \mu\text{b}$, and for the cut-off we use $s_0 = (2 \text{ GeV})^2$. For simplicity we neglect the contributions from high-spin mesons and normal Regge exchanges, and obtain [using Eq. (18)]

$$\begin{aligned} \Gamma_{f,+-} - 6 \Gamma_{f,++} &= \frac{1}{128} \cdot \frac{\sigma_{tot}^{rr} b m_f^2}{160 \pi^2 s_0} \\ &= 2.4 \text{ keV} \end{aligned} \quad (20)$$

For comparison, the authors of Ref. 10 found $\Gamma_{f,+-} = 5.6 \text{ keV}$ and $6\Gamma_{f,++} = 0.55 \text{ keV}$. In view of the crudeness of both calculations, we consider this as good agreement.

4. FURTHER SUM RULES

The sum rules we have considered so far resulted from the Jin-Martin type estimate $|T_i(s,t)| < s^2$ for fixed $t \approx 0$ and $s \rightarrow \infty$. Let us now discuss what further sum rules one might expect using more stringent hypotheses. The weakest additional assumption is

*) The same argument has already been used implicitly to reject contributions from quark loops or leptonic QED.

**) The validity of the factorization property in $\gamma\gamma$ -scattering has been proven (with detailed consideration of fixed poles) by Evans et al.¹²⁾.

$$|T_i(s, t)| < \frac{s}{(\ln s)^{1+\epsilon}} \quad \text{for } s \rightarrow \infty, -\epsilon < t < 0. \quad (21)$$

For hadronic interactions, this holds unless $\alpha'_p(0) = 0$, and using vector meson dominance we can expect the same for $\gamma\gamma$ scattering. One should, however, be careful: there is no general proof that this estimate is true order by order in renormalizable perturbation theories. Therefore, sum rules derived from it might, in general, have contributions from the graphs in Fig. 1 as well as from quark loops. The same remark will hold for sum rules deduced from even stronger assumptions [Roy¹³⁾ and Ref. 3].

To derive a set of sum rules following from Eq. (21), we write fixed- t dispersion relation for $\tau_1(s, t, u) - \tau_3(s, t, u)$. Since $\tau_1 - \tau_3$ is crossing-odd [see Eq. (2b)], we have

$$\tau_1(s, t, u) - \tau_3(s, t, u) = \frac{s-u}{\pi} \int_0^\infty \frac{dx}{(x-s)(x-u)} \text{Im} [\tau_1(x, t) - \tau_3(x, u)] \quad (22)$$

Taking the limit $s \rightarrow \infty$, we obtain the one-parameter family of superconvergence sum rules

$$\int_0^\infty ds \text{Im} [\tau_1(s, t) - \tau_3(s, t)] = 0 \quad (23)$$

For $t = 0$, this has already been derived in Ref. 13 and by Budnev et al.¹⁴⁾. The assumption entering is actually somewhat weaker than Eq. (21) since the Pomeranchuk theorem tells us that $|(\tau_1 - \tau_3)/\tau_1| \rightarrow 0$, and Eq. (23) should therefore also hold at $t = 0$. Gerasimov and Moulin¹⁵⁾ show that the lowest-order graphs in scalar (Fig. 1) and spinor QED do not contribute at $t = 0$ *). Therefore, we try a resonance saturation with pseudoscalar, scalar, and tensor mesons [using again Eq. (18)]. The f -meson couplings are taken from Ref. 10, and for π and η we take the experimental values $\Gamma_\pi = 7.8$ eV, $\Gamma_\eta = 0.32$ keV¹⁶⁾. We find

$$\begin{aligned} \frac{\Gamma_{\eta'}}{m_{\eta'}^3} + \frac{\Gamma_s}{m_s^3} &= -\frac{\Gamma_\pi}{m_\pi^3} - \frac{\Gamma_\eta}{m_\eta^3} - \\ &\quad - \frac{6.78}{m_f^3} \left[\left(1 + \frac{6t}{m_f^2} + \frac{6t^2}{m_f^4} \right) \Gamma_{f,++} - \Gamma_{f,+} \right] \\ &= \begin{cases} 13.36 \text{ keV/GeV}^3 & (\text{at } t=0) \\ 13.76 \text{ keV/GeV}^3 & (\text{at } t=-0.5 \text{ GeV}^2) \end{cases} \end{aligned} \quad (24)$$

Here, Γ_s/m_s^3 denotes sum over all scalar meson (ϵ, δ, S^*) contributions. If we assume ideal mixing among these scalar mesons (the fact that S^* decays mostly in $K\bar{K}$ seems to support this assumption), this sum can be replaced by $1.3(\Gamma_\epsilon/m_\epsilon^3)$.

*) This seems in contradiction to a note added in print in Ref. 12.

Fig. 1

Scalar QED graphs which should dominate the threshold discontinuities of elastic $\gamma\gamma$ amplitudes.