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Archives

NEW MODEL INDEPENDENT RESULTS ON THE DIFFRACTION PEAK

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1. - INTRODUCTION

In this session we have heard very interesting talks on possible dynamical schemes which could describe high energy collisions and especially the diffraction peak, i.e., the small t large s Reggeon for elastic scattering. It may also be of some usefulness to try to obtain as much as possible rigorous information from general principles, if there is any rigorous information still to be found. It happens that during the last few months Henri Cornille and I have been able to obtain some new results, some of which, concerning the real part effects in the forward elastic peak, are already in the process of being published ¹⁾ while the study of scaling of elastic differential cross-sections, stimulated by the experimental observation of the general trend of the elastic pp diffraction peak to shrink at all values of t ²⁾, is still unpublished. Finally, the extension of the old inequality obtained by MacDowell and myself ³⁾ to particles with spins is still, as you will see, unfinished. We shall, however, start with the latter item.

2. - AN INEQUALITY ON THE SLOPE OF THE DIFFRACTION PEAK VALID FOR ARBITRARY SPINS

In 1964, Sam MacDowell and I obtained the following inequality on the slope of the diffraction peak, for particles with spin 0 :

$$R = \frac{36\pi \frac{d}{dt} \ln A(s,t) \big|_{t=0}}{\frac{(\sigma_t)^2}{\sigma_{el}}} \geq 1 \quad (1)$$

σ_t total cross-section,

σ_{el} elastic cross-section,

$A(s,t)$ absorptive part of the scattering amplitude.

Terms of order $1/s$ have been neglected.

To obtain this inequality we used explicitly the fact that the imaginary parts of the partial wave amplitudes were positive. The question is now to generalize this to particles with spins. The two-body elastic cross-section for particles with spins is of the form

$$\frac{d\sigma}{d\Omega} = N(s) \sum_i |F_i|^2 \quad (2)$$

where the F 's are amplitudes in a convenient basis (for instance, the helicity basis or the transversity basis) such that no cross terms are present. Thus one can define

$$\langle A \rangle^2 = \sum_i |A_i|^2 \quad (3)$$

where the A 's are the absorptive parts of the various amplitudes.

We have proved from $\text{Im } T = T^+ T$ that $\langle A \rangle^2$ possesses the following positivity property

$$\langle A \rangle^2 = \sum_n c_n \cos(n\theta), \quad (4)$$

$c_n \geq 0$, θ scattering angle.

From this property alone we have been able to find a generalization of inequality (1) :

$$R = \frac{18 \pi \frac{d}{dt} \ln \langle A \rangle^2 \big|_{t=0}}{(\sigma_t)^2 / \sigma_{el}} > C \quad (5)$$

The proof uses general properties of sums of the type (4) which hold, in fact, as shown by Henri Cornille, for a very large class of sums of orthogonal polynomials with positive coefficients ⁴⁾.

At present we know with certainty that $C > 0.8$. However, a calculation in collaboration with Guy Auberson is in progress and I believe with 90% confidence that, except for terms of order $1/s$ we shall get the same answer as in the scalar case, i.e.,

$$C = 1 \quad (6)$$

To reach this goal it may be necessary to use a property which is stronger than (4).

Assume $C = 1$. When can we get $R = 1$? Only when

- i) the imaginary part of the double flip terms vanish in the forward direction ;
- ii) total cross-sections are independent of the spins on the initial particles.

R can be estimated from experiment if one neglects the real part of the scattering amplitude. Then we find that R is never much bigger than unity. Typical values are 1.2, 1.4 depending on the process. If the differential cross-section could be fitted by a pure exponential of t , R would necessarily be equal to $9/8$, if i) and ii) are satisfied well enough.

3. - THE QUESTION OF SCALING

3.1. - The case $\sigma_t \sim \text{const.}(\log s)^2$

This part is historical, in fact almost prehistorical. In 1970, Auberson, Kinoshita and I⁵⁾ proved the following theorem.

Assume that the Froissart bound is qualitatively saturated, i.e.,

$$\sigma_{\text{Total}} \sim \text{const} (\log s)^2 \quad (7)$$

where the constant in front need not be the largest possible constant. Then define

$$\tau = t(\log s)^2 \sim t\sigma_t \sim t \frac{(\sigma_t)^2}{\sigma_{el}}. \quad (8)$$

Then there exists a sequence of energies $s_1, s_2, s_N \rightarrow \infty$ such that, for any fixed τ ,

$$\frac{\frac{d\sigma}{dt}(s_N, t)}{\frac{d\sigma}{dt}(s_N, 0)} \rightarrow f(\tau). \quad (9)$$

$f(\tau)$ is an entire function of τ of order $\frac{1}{2}$ which has infinitely many zeros, some of which may be very close to the physical region.

All models which saturate the Froissart bound, such as the Cheng-Wu-Walker model ⁶⁾ automatically possess these properties if they satisfy s channel unitarity and have good analyticity properties in t .

Perhaps you will wonder why we did this in 1970, at a time where there were no indications that cross-sections were rising. Our paper was devoted to the analysis of the violation of the Pomeranchuk theorem. However, we were wise to indicate in the last paragraph of our paper, that all the results we had obtained on the odd signature amplitude could be applied to the even signature amplitude in a situation where the cross-section would rise like $(\log s)^2$. We know now that total cross-sections are rising. However, though they are compatible with an asymptotic $(\log s)^2$ rise they may be not rising as fast as that. A fit with $\sigma_t \sim \log s$ is perfectly compatible with the data, for instance. This is the motivation for the next subsection.

3.2. - The case where $\sigma_t/(\log s)^2 \rightarrow 0$

In that case we cannot establish the scaling of differential cross-sections from nothing. First one has to choose a proper scaling variable. Our choice is

$$\tau = t \frac{(\sigma_t)^2}{\sigma_{ee}} \quad (10)$$

(this does not always coincide with what is called geometric scaling, where $\tau = t\sigma_t$).

In Section 2, we have defined the quantity R , and established $R \geq \text{const}$ (probably unity). Experimentally R is never much larger than unity. Then let us make the following assumptions :

- i) the real parts of the amplitudes are negligible ;
- ii) $R < \text{const}$, independent of energy.

Under these assumptions we can prove again that there is a sequence $s_1, s_2, s_N \dots \rightarrow \infty$, such that

$$\lim_{N \rightarrow \infty} \frac{d\sigma}{dt}(s_N, t) / \frac{d\sigma}{dt}(s_N, 0) \rightarrow f(\tau)$$

for any fixed real $\tau < 0$. $f(\tau)$ is a continuous, differentiable function such that $f(0) = 1$, but not identically equal to unity. $f(\tau)$ is defined only on the negative real axis. It may be impossible to continue it for complex τ .

Let us sketch the proof. Define

$$f(s, \tau) = \frac{\frac{d\sigma}{dt}(s, t)}{\frac{d\sigma}{dt}(s, 0)} \quad (11)$$

- i) for $\tau < 0$, physical, we have $|f(s, \tau)| \leq 1$, because of the positivity property (4) :

$$\left| \sum C_n \cos n\theta \right| \leq \sum |C_n| = \sum C_n .$$

ii) the derivative of a sum of the type (4) is again of the same type ; therefore

$$\left| \frac{d}{d\tau} f(s, \tau) \right| < \frac{d}{d\tau} f(s, 0) = \frac{R}{18\pi} < \text{const}$$

If we take a sequence $s_1, s_2 \dots s_N \dots \infty$ the set of all $f(s_i, \tau)$ is a set of uniformly bounded, equicontinuous functions [such that $|f(s_i, \tau) - f(s_i, \tau')| < C|\tau - \tau'|$, C , independent of s_i]. The theorem of Ascoli-Arzelà states that out of this set we can find a subset of functions approaching a limit when $s_i \rightarrow \infty$. The limit function will be called $f(\tau)$.

What can we say about $f(\tau)$? It is obviously continuous, with a bounded derivative, and it is non trivial. To prove the latter point, consider $\tau_0(s)$ such that for $\tau = -\tau_0(s)$, $f(s, -\tau_0(s)) = \frac{1}{2}$; $\tau_0(s)$ defines the half width. Then we have

$$\sigma_{el} > \frac{1}{2} \frac{d\sigma}{dt}(s, t=0) \times \frac{\sigma_{el}}{(\sigma_t)^2} \tau_0(s)$$

and using the optical theorem

$$\tau_0(s) < \text{const} \quad (12)$$

This shows that $f(\tau)$ cannot be identically equal to unity.

Other conditions for scaling can be found. One of them is particularly appealing. Suppose we fit the small t region, for a given energy, by

$$\frac{d\sigma}{dt}(s, t) / \frac{d\sigma}{dt}(s, 0) = \exp [b(s)t + c(s)t^2]$$

take as a scaling variable

$$\tau = b(s)t \quad (13)$$

assume now

$$\frac{c(s)}{(b(s))^2} < \text{const.} \quad (14)$$

Then we get scaling in the same sense as previously.

Let me make a few final remarks on this section. We have not been able to prove the uniqueness of the limit function $f(\tau)$. There could exist another set of energies for which we get a different limit. Then, as we increase the energy, the amplitude would oscillate between the two limits. This could even happen if R approaches a limit as the energy goes to infinity. This incidentally is also the case when $\sigma_t \sim (\log s)^2$. Let us notice also that $\sigma_t \sim (\log s)^2$ can also be incorporated in (3.2). Indeed, it is established that

$$\frac{d}{dt} \log A(s,t) \Big|_{t=0} < (\log s)^2$$

Hence if $\sigma_t \sim (\log s)^2$ we also have $\sigma_{el} \sim (\log s)^2$ and $R < \text{const.}$ follows.

4. - REAL PART EFFECTS

Here we have some results which are being published ¹⁾, but these results are not yet sufficient to justify the neglect of the real part in Section 3. Let me state what we have for the scalar case.

The only control we have on the real part comes from analyticity with respect to s , i.e., dispersion relations. To use them in an efficient way we must assume that odd signature terms are negligible [this excludes in particular terms of the form $s f(t)$ in the amplitude]. Then, if we assume, in addition

$$\sigma_t^- \sim s^{\frac{1}{2}} \rightarrow \infty, \quad (15)$$

a very reasonable assumption, we can prove

$$(\log s)^2 > \eta = \frac{d}{dt} \log \left(\frac{d\sigma^-}{d\Omega} \right)_{t=0} > 0 \quad (16)$$

at least for a sequence of energies $s_1, s_2 \dots s_N \rightarrow \infty$. This proves the existence of a forward peak of the elastic differential cross-section. If

$$\eta_{\text{abs}} = 2 \frac{d}{dt} \log A(s, t) \Big|_{t=0}$$

is sufficiently smooth [example $\eta_{\text{abs}} \sim C(\log s)^Y$],

$$|\eta / \eta_{\text{abs}}| < C \quad |\eta_{\text{abs}} / \eta| < C$$

for a sequence of energies. This holds also if η (instead of η_{abs}) is sufficiently smooth.

These results are not sufficient to treat the problem of scaling with real parts. There we would have to show, for instance, in the case of $\sigma_t^- \sim (\log s)^Y$, that there is a sequence of energies for which, for all $-T \leq t \leq 0$, the real part is small, compared to the imaginary part. What is relatively easy to do is to show that for one given value of t there is a sequence of s_i 's for which the real part is small. If we change t the new sequence may have no common point with the previous one. This is the problem.

The assumption that the odd signature amplitude is negligible is unavoidable. Otherwise it is easy to build counter-examples with a dip instead of a peak in the forward direction. I do not believe that odd signature amplitudes are important at high energies ; I even made a bet of a *bottle of champagne* about this with Elliott Leader. However, I must frankly admit that there is no fundamental objection against the presence of such terms.

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