Spin of Stellar-Mass Black Holes Estimated by a Model of Quasi-Periodic Oscillations

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Abstract

We have proposed in previous papers that the high-frequency pair QPOs observed in black-hole binaries with frequency ratio 3:2 are inertial-acoustic oscillations (nearly horizontal oscillations with no node in the vertical direction) or g-mode oscillations, which are resonantly excited on warped relativistic disks. The resonance occurs through horizontal motions. In this model the dimensionless spin parameter $a_*$ of the central sources can be estimated when their masses are known from other observations. This estimate is done for three sources (GRO J1665-40, XTE J1550-564, GRS 1915+105). For all of them we have $a_* \leq 0.45$.

Key words: accretion, accretion disks — quasi-periodic oscillations — resonance — stars: individual (GRO J1665-40, XTE J1550-564, GRS 1915+105) — X-rays; stars

1. Introduction

High-frequency quasi-periodic oscillations (HF QPOs), whose frequencies are in the range of 100 to 450 Hz, have been observed in some black-hole binaries and black-hole candidates. One of characteristics of these HF QPOs is that they often appear in a pair and their frequencies change little with time \(^1\), keeping the frequency ratio close to 3:2. These sources are GRO J1665-40 (300, 450 Hz), XTE J1550-564 (92, 184, 276 Hz) and GRS 1915+105 (41, 67, 113, 168 Hz).

\(^1\) In the kHz QPOs of neutron-star X-ray binaries, the frequencies (and their ratio) of the pair oscillations change with time. This is a definite difference between HF QPOs in black holes binaries and kHz QPOs in neutron star binaries. In our warp models, kHz QPOs of neutron stars are interpreted as disk oscillations with vertical resonance (see Kato 2005b), or as the case in which warp has precession (Kato 2005a).
Hz) (e.g., a review by McClintock and Remillard 2006). Importance of commensurability of pair QPO frequencies on understanding the disk structure in the innermost region was emphasized by Abramowicz and Kluźniak (2001) and Kluźniak and Abramowicz (2001).

It is known that oscillations can be excited in a deformed disk by resonant processes. One well-known example is superhumps in tidally-deformed dwarf-novae disks (Whitehurst 1988; Hirose, Osaki 1990; Lubow 1991). Another example is a spiral pattern on ram-pressure deformed galactic disks (Tosa 1994; Kato and Tosa 1994). In black-hole X-ray binaries, similar types of resonant oscillations should occur when the disks are deformed. We think that one of the most probable deformations of disks in the innermost region is a warp. Based on this idea, we examined excitation of disk oscillations on warped disks (Kato 2003b, 2004b), and proposed a resonant excitation model of QPOs (Kato 2004a,b, 2005a,b; Kluźniak et al. 2004).

In this warped-disk model, the high-frequency QPOs in black-hole binaries are g-mode oscillations or inertial-acoustic oscillations$^2$ or their combination, excited by a horizontal resonance. If this resonance model is correct, it gives a way to estimate the spin of the central source from observed pair frequencies of QPOs, if the mass of the source is observationally known.

Recently, Shafee et al. (2006) evaluated the spins of two black-hole sources (GRO J1655-40 and 4U 1543-47) whose masses are observationally known, by fitting their spectra with model spectra derived from current disk models. Since one of the sources (i.e., GRO J1655-40) which they adopted has 3:2 pair frequencies, we can independently estimate the spin of the source. The purpose of this paper is to present the frequency-spin relation based on the warped-disk model and to estimate spins of some black-hole X-ray binaries, including GRO J1655-40.

2. Horizontal Resonances of G-Mode Oscillations and Inertial-Acoustic Oscillations in Warped Disks

Here, we outline the essence of our resonance model in warped disks [see figure 1 in Kato (2004a) and the similar figures in his subsequent papers]. Let us consider a wave specified by $(\omega, m, n)$, where $\omega$ is the frequency of the wave, $m$ is the wavenumber in the azimuthal direction, and $n$ is a number specifying the node number in the vertical direction (for details,

$^2$ In this paper inertial-acoustic oscillations represent the fundamental p-mode oscillations, in which oscillations are nearly horizontal and horizontal velocity has no node in the vertical direction. In some recent papers by Kato, however, inertial-acoustic oscillations are treated together with g-mode oscillations, since in mathematical analyses of the present resonance problem both of them can be treated together in a pack without making distinction (see Kato 2004b). Hence, when we used the term of g-mode oscillations, inertial-acoustic oscillations were implicitly included there. This was misleading. Hence, in this paper we explicitly mention inertial-acoustic oscillations without including them in g-mode oscillations. Here, it is noted that we use the following terminology for disk oscillations. The modes in which horizontal velocity has no node in the vertical direction ($n = 0$) is called inertial-acoustic oscillations (p-modes). The modes with $n = 1$ are g-modes and corrugation modes (c-modes). The modes with $n \geq 2$ are g-modes and vertical p-modes
see, for example, Kato et al. 1998; Kato 2001). A warp with no precession is described by
(0, 1, 1), since it is a kind of global, one-armed deformation. Nonlinear interaction between
the wave of \((\omega, m, n)\) and the warp \((1, 1, 0)\) produces oscillations specified by \((\omega, m \pm 1,\
n \pm 1)\), which are called here the intermediate oscillations. If the amplitude of the wave mode
of \((\omega, m, n)\) is fixed, the disk experiences forced oscillations due to the intermediate oscillations.
The disk then resonantly responds to the intermediate oscillations at some particular radius
where the dispersion relation for these intermediate oscillations is satisfied. At this radius
energy exchange between the disk rotation and the intermediate oscillations is realized. After
this resonant interaction, the intermediate oscillations feedback to the original oscillation of \((\omega,\
m, n)\) by making again nonlinear interaction with the warp. This nonlinear feedback process
amplifies or dampens the original oscillation \((\omega, m, n)\), since a resonant process is involved in
the feedback process (Kato 2003b, 2004b).

There are two kinds of resonances. One is horizontal, and the other is vertical [see Kato
2004b for details]. Detailed examinations on resonant processes (Kato 2004b) show that the
case in which resonance excites oscillations is the case in which oscillations are inertial-acoustic
oscillations and/or g-mode oscillations and the resonance is horizontal. Hence, in the followings,
we restrict our attention only to the case.

First, we remember that g-mode oscillations and inertial-acoustic oscillations with frequency \(\omega\) and azimuthal wavenumber \(m\) predominantly exist around the radius specified by
\((\omega - m\Omega)^2 \sim \kappa^2\) by the following reasons. Here, \(\Omega\) and \(\kappa\) are Keplerian and (radial) epicyclic
frequencies, respectively. In both cases of inertial-acoustic and g-mode waves, the group velocity
of these waves vanishes at the radius where \((\omega - m\Omega)^2 = \kappa^2\). That is, if we consider wave
packets, they stay there for a long time compared with in other places. Hence, we think that
the waves exist mainly around the radius specified by
\[
(\omega - m\Omega)^2 = \kappa^2. \tag{1}
\]

The nonlinear interaction of the above oscillations with a warp gives rise to intermediate

3 In geometrically thin disks, the local dispersion relation of oscillations is given by
\[
[(\omega - m\Omega)^2 - \kappa^2][(\omega - m\Omega)^2 - n\Omega_{\perp}^2] = c_s^2 k^2 (\omega - m\Omega)^2,
\]
where \(\Omega_{\perp}\), \(c_s\), and \(k\) are, respectively, the vertical epicyclic frequency, the acoustic speed, and the radial
wavenumber. This dispersion relation gives the group velocity \((= \partial \omega / \partial k)\) as
\[
\frac{\partial \omega}{\partial k} = \pm c_s \frac{(\omega - m\Omega)^2[(\omega - m\Omega)^2 - \kappa^2]^{1/2}}{(\omega - m\Omega)^4 - n\kappa^2\Omega_{\perp}^2}.
\]

4 The places of \((\omega - m\Omega)^2 = \kappa^2\) are also particular places in the sense that they are boundaries between the
propagation and evanescent regions of waves. In the case of the inertial-acoustic waves, the propagation
region is described by \((\omega - m\Omega)^2 > \kappa^2\), and the region of \((\omega - m\Omega)^2 < \kappa^2\) is the evanescent region. In the
case of the g-mode oscillations, the situation is changed. That is, \((\omega - m\Omega)^2 > \kappa^2\) is the evanescent region
and \((\omega - m\Omega)^2 < \kappa^2\) is the propagation region.
oscillations of \((\omega, m \pm 1)\). These intermediate oscillations have resonant interaction with the disk at the radii where the dispersion relation of the intermediate oscillations is satisfied (Kato 2003b, 2004b). In the case of the horizontal resonances the radius is close to the radii specified by
\[
[\omega - (m \pm 1)\Omega]^2 = \kappa^2.
\]

It is important to note that the resonant radii are independent of the vertical structure of the oscillations, i.e., independent of \(n\).

The resonant radii and the radii where the oscillations predominantly exist must be the same for resonant interactions to occur efficiently. That is, equations (1) and (2) must be satisfied simultaneously, which gives
\[
\kappa = \frac{\Omega}{2}.
\]

This is the condition determining the resonant radius. From equation (1), we then see that the frequencies of resonant oscillations are \(m\Omega \pm \kappa\) at the resonant radius. The above argument is free from the metric. That is, the above resonant condition is valid even in the case of the Kerr metric, if the angular velocity of the Keplerian rotation, \(\Omega\), and the epicyclic frequency, \(\kappa\), in the Kerr metric are adopted.

3. Resonant Radius and Frequencies of Resonant Oscillations

In the limit of non-rotating central source (i.e., the metric is the Schwarzschild one), the condition, \(\kappa = \Omega/2\), is realized at \(4.0r_g\), which is just the radius where \(\kappa\) becomes the maximum. Here, \(r_g\) is the Schwarzschild radius defined by \(r_g = 2GM/c^2\), \(M\) being the mass of the central source. As the spin parameter, \(a^*\), increases the resonant radius, \(r_c\), decreases. The \(r_c - a^*\) relation derived from the resonant condition, \(\kappa = \Omega/2\), is shown in figure 1.

Next, we calculate frequencies of inertial-acoustic and/or g-mode oscillations which have resonance at \(\kappa = \Omega/2\). As mentioned before, they are \(m\Omega \pm \kappa\) at the resonant radius. They are a set of frequencies, since there are various \(m\). Among them the most observable ones will be those with small number of \(m\). The axially symmetric oscillations, \(m = 0\), however, will be less observable by the very nature of symmetry. Hence, the oscillations which will be most interesting in relation to observed QPO frequencies are those with \(m = 1\) or \(m = 2\). Considering this situation, we introduce, for convenience, symbols given by
\[
\omega_H = (\Omega + \kappa)_c, \quad \omega_L = (2\Omega - \kappa)_c, \quad \omega_{LL} = (\Omega - \kappa)_c,
\]
where the subscript \(c\) denotes the values at the resonant radius, \(\kappa = \Omega/2\). It is noted that \(\omega_H\) and \(\omega_L\) are equal, i.e., \(\omega_H = \omega_L\). Outside the resonant radius (i.e., \(r > r_c\)), \(\Omega + \kappa\) is larger than \(2\Omega - \kappa\) since \(\kappa > \Omega/2\) there. Inside the resonant radius, \(2\Omega - \kappa\) is larger than \(\Omega + \kappa\).
3.1. **Source State and QPO Frequencies**

The next problem to be examined is the relation between the frequencies of disk oscillations mentioned above and the observed QPOs frequencies. For simplicity, let us neglect effects of disk rotation (such as Doppler boosting) and geometrical effects (such as gravitational bending of light rays and occultation). Then, no luminosity variation is observed in geometrically thin, no warped disks, even if rotating non-axisymmetric oscillations are superposed on the disks. This means that some careful consideration on geometrical states of disks is necessary. Observations show (Remillard 2005) that all high-frequency QPOs are associated with the steep power-law state of sources. They are not observed in the thermal state (i.e., the soft/high state with no corona), nor in the hard state (i.e., hard/low state with no thermal disk component). It is noted that in the steep power-law state a compact hot torus (corona) and a thermal disk coexist in the innermost region. Observations further show that the QPOs are observed in the high energy photons of the power-law component, not in the soft photons of the thermal disk component.

This observational evidence suggests that a thermal disk is necessary as a place where oscillations are generated, but the observed QPO photons are those Comptonized in the hot compact corona (a hot torus). If this picture is adopted, one-armed oscillations are observed as time variations with twofold frequency, as described below.

Let us consider one-armed disk oscillations propagating in the azimuthal direction with angular frequency $\omega$. The hot disk region associated with the disk oscillations is assumed to be inside a torus. Now, we consider the path of observed photons which are originally emitted from the hot region of the disk as soft photons and are observed as high energy photons by Comptonization in the torus. The path length of the photons in the torus depends on the phase relation between the hot region and the observer, as shown in figures 2 and 3. In the phase shown in figure 2, the path length of photons in torus is short. (This phase is called hereafter phase 0.) In the phase shown in figure 3, however, the path length within the torus is long. The latter occurs when the phase is close to 0.75 as well as 0.25. In the phase 0.5 the path in the torus is shorter than that in the phase of figure 3, but longer than that in the phase of figure 2 (phase 0). Hence, observed Comptonized photon numbers will vary as shown in figure 4. That is, we have two peaks during one cycle of the oscillations.

Here, a brief comment is made on depths of the primary minimum (phase 0) and secondary minimum (phase 0.5) in figure 4. In the phase of the primary minimum, the path length of photons in the torus is short, but they pass through an inner hot and dense region of the torus (see figure 2). This will increase the Comptonized photon flux, compared with that in the case in which photons pass an outer cool and less dense region. The phase of the secondary minimum (phase 0.5) corresponds to the latter case. This consideration suggests that the difference between the Comptonized photon fluxes in phases 0 and 0.5 is smaller than that simply estimated from the difference of geometrical path lengths. Figure 4 should be regarded...
as results in which the above effects are already taken into account.

Figure 4 shows that one-armed oscillations with frequency $\omega$ bring about two time-varying components with $\omega$ and $2\omega$. Let us now roughly estimate the amplitude ratio of the two components from the light curve in figure 4. The flux, $f(t)$, shown in figure 4 will be approximated by

$$f(t) = (1 + A) - \cos(4\pi t) - A\cos(2\pi t),$$  \hspace{1cm} (5)

with $A$ moderately smaller than unity, where $t$ represents the phase of the light curve (i.e., $t = 0$ at phase 0 and $t = 1$ at phase 1). The amplitude of the $2\omega$-oscillation is normalized to unity and that of the $\omega$-oscillation is $A$. The total flux is normalized to become zero at phase 0. Then, the maximum of the flux is realized near $t = 1/4$ and $3/4$ as far as $A$ is moderately smaller than unity, and is about $2 + A$. The flux at the secondary minimum ($t = 0.5$) is $2A$. That is, the flux ratio of the secondary minimum to the maximum is roughly $2A/(2 + A)$. The case where the secondary minimum is as deep as the primary minimum, i.e., $2A/(2 + A) = 0$, is realized if $A = 0$. That is, in this case we have only $2\omega$ oscillation, as expected. If, for example, the flux ratio is 0.3, the amplitude ratio is found to be roughly 0.35. That is, the amplitude of the oscillation with $\omega$ is smaller than that with $2\omega$ by about factor 3.

In the case in which the observer is in a direction close to the edge-on, a light ray leaving the torus to go to the observer may enter again into another part of the torus on the way of the path. Furthermore, the Doppler effects are not negligible on light variation. Such cases of high inclination angle, however, will not be the major cases in which QPOs are observed, since outer parts of disk will screen QPO photons from the observer.

In the case of two-armed oscillations, we can easily find that the main frequency of observed luminosity variation is the same as that of the oscillations.

These considerations suggest that the resonant oscillations with frequency $\omega_{LL}$ mainly give rise to QPOs whose frequencies are $2\omega_{LL}$, since they are one-armed oscillations, i.e., $m = 1$. Hence, we think that the observed main frequencies of QPOs, i.e., the frequencies of the pair QPOs, are $\omega_{L} (= \omega_{H})$ and $2\omega_{LL}$. Their frequency ratio is just 3:2, i.e.,

$$\omega_{L} (= \omega_{H}) : 2\omega_{LL} = 3 : 2.$$  \hspace{1cm} (6)

In the present disk-oscillation model there is no reason why oscillations with $\omega_{LL}$ are not observed, although their amplitude may be small. We think that these oscillations are really observed in some sources. In XTE J1550-564 three QPOs are observed whose frequencies are 276Hz, 184Hz, and 92Hz. Their frequency ratios are just 3:2:1, suggesting that $\omega_{LL}$ has been observed. Furthermore, in a black-hole X-ray transient XTE J1650-500, QPO frequencies vary with time, but their frequencies are consistent with being 1:2:3 harmonics (Homan et al. 2003), suggesting that $\omega_{LL}$ has been also observed in this source.

One may think why QPOs with frequency $2\omega_{H}$ are not observed. (It is noted that the oscillations with $\omega_{H}$ are one-armed.) We think that they should be observed, but there is still
Table 1. Estimated spin parameter $a_\ast$.

<table>
<thead>
<tr>
<th>Sources</th>
<th>$3\nu_0$(Hz)</th>
<th>$M/M_\odot$</th>
<th>$a_\ast$</th>
</tr>
</thead>
<tbody>
<tr>
<td>GRS 1915+105</td>
<td>168</td>
<td>10.0 – 18.0</td>
<td>negative – 0.44</td>
</tr>
<tr>
<td>XTE 1550-564</td>
<td>276</td>
<td>8.4 – 10.8</td>
<td>0.11 – 0.42</td>
</tr>
<tr>
<td>GRO 1655-40</td>
<td>450</td>
<td>6.0 – 6.6</td>
<td>0.31 – 0.42</td>
</tr>
</tbody>
</table>

no serious attempt to detect such high frequency QPOs, since the frequency is higher than the Keplerian frequency in the innermost region of disks.

4. Estimate of Spin from Pair QPO Frequencies

In the case in which the central source is non-rotating, the resonance occurs at $4.0r_g$, and $\omega_L (= \omega_H)$ can be easily expressed as

$$\omega_L = 2.14 \times 10^3 \left( \frac{M}{M_\odot} \right)^{-1} \text{Hz.} \quad (a_\ast = 0) \quad (7)$$

Masses of three sources (GRO J1665-40, XTE J1550-564, GRS 1915+105) which display a pair of HF QPOs have been obtained from spectroscopic observations. Using the data, McClintock and Remillard (2005) derived an interpolation formula giving a relation between observed frequencies of HF QPOs and $M$, which is

$$3\nu_0 = 2.79 \times 10^3 \left( \frac{M}{M_\odot} \right)^{-1} \text{Hz}, \quad (8)$$

where $\nu_0$ is the fundamental frequency of 3:2:1, and thus $3\nu_0$ corresponds to $\omega_L$ in our model. The frequency $\omega_L$ for $a_\ast = 0$ is smaller than $3\nu_0$, suggesting that the central sources are certainly rotating.

The dependence of $\omega_L$ on the spin parameter $a_\ast$ is numerically obtained by substituting $r_c$ obtained by solving equation (3) into the expression for $\omega_L$ [equation (4)]. The results are shown in figure 5. For the three sources, where $M$ and $3\nu_0$ are known, the spin parameter $a_\ast$ can be calculated, assuming that the observed $3\nu_0$ is $\omega_L (= \omega_H)$. The results are shown in table 1 (see also table 3 of Kato 2004b). As shown in table 1, the value of spin parameter $a_\ast$ derived for GRO J1665-40 is $a_\ast = 0.31 – 0.42$, which is somewhat smaller than $a_\ast = 0.65 – 0.75$ derived by Shafee et al. (2006) from a spectrum fitting. In the case of GRS 1915+105, the value of $a_\ast$ is negative if $M \sim 10.0M_\odot$ is adopted. This suggests that the mass is much larger than $10M_\odot$, closer to $18.0M_\odot$.

5. Propagation Regions

It is worthwhile to note that the propagation region of inertial-acoustic oscillations and that of g-mode ones are different, even when their frequencies are the same. They are inside or outside of the resonant radius, depending on the modes. The propagation regions of the
inertial-acoustic oscillations with frequency $\omega$ and azimuthal wavenumber $m$ are in the region described by $(\omega - m\Omega)^2 > \kappa^2$, which is $\omega > m\Omega + \kappa$ or $\omega < m\Omega - \kappa$. In the case of g-mode oscillations, the region is $(\omega - m\Omega)^2 < \kappa^2$, which is $m\Omega - \kappa < \omega < m\Omega + \kappa$. To demonstrate these situations, we show in figure 6 the propagation regions of inertial-acoustic oscillations and those of g-mode oscillations whose frequencies are $\omega_H$, $\omega_L$, and $\omega_{LL}$.

As shown in figure 6 and mentioned above, the propagation regions of inertial-acoustic oscillations and those of g-mode oscillations are in the opposite sides of the resonant radius, when their frequencies are the same. In the propagation regions of the g-mode oscillations, there is a corotation radius, i.e., the radius where $\omega - m\Omega = 0$. At the corotation radius the g-mode oscillations are damped (Kato 2003a, Li et al. 2003). The inertial-acoustic oscillations which propagate inward from the resonant radius will be partially reflected back near the inner edge of the disk, which may lead to quasi-trapped oscillations. These considerations may suggest that the main contributor to the HF QPO may be inertial-acoustic oscillations, rather than g-mode oscillations.

6. Discussion

The basic idea of our model is that the high-frequency QPOs are disk oscillations and a deformation of the disk is the essential cause of their excitation. As the cause of disk deformation we consider warp. This is because warp will be one of the most conceivable deformation of disks in the innermost region. As mentioned before, the QPOs are associated with the steep power-law (SPL) state and are certainly not in the thermal state where the disk consists only of a thermal disk component (Remillard 2005). In the SPL state a compact corona and a thermal disk coexist. We suppose that triggers forming a compact high-temperature torus in the innermost region will not generally axisymmetric since the disks are highly turbulent, and deform the disk as well as formation of a torus. This will be one of possible causes of formation of warped disk.

Let us denote the observed upper and the lower frequencies of the pair QPOs by $\nu_u$ and $\nu_l$. Then, as mentioned before, our present model predicts the presence of QPOs with frequency of $2\nu_u$. Analysis of observational data to see whether QPOs of $2\nu_u$ are present or not is a crucial check of the present model.

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References

Fig. 1. Resonant radius as a function of the spin parameter $a_*$. The resonant radius is given by $\kappa = \Omega/2$.

Kato, S. 2004a, PASJ, 56, 559
Kato, S. 2004b, PASJ, 56, 905
Fig. 2. A schematic picture showing the light path from a hot region in disks to an observer in the phase in which the hot region is just in the opposite side of the central source to the observer. The path within the torus is shown by dashed line. It is noticed that the path length in the torus is short, and the observed QPO photons are not many. This phase is referred to phase 0, and the phase in which the hot region of the disk is between the central source and the observer is phase 0.5

Fig. 3. A schematic picture showing a straight light path from the hot region in disks to an observer in a phase close to 3/4. The part of the pass within the torus is shown by dashed line. The pass within the torus is the longest in this phase as well as in a phase close to 1/4, compared with in other phases.
Fig. 4. A schematical light-curve during one revolution of an one-armed oscillation around a central source. We have two peaks during one cycle of oscillations around phases of 0.25 and 0.75.

Fig. 5. The frequency $\omega_L$ of the upper HF QPO as a function of the spin parameter $a_*$. 
Fig. 6. Propagation regions of the resonant oscillations whose frequencies are $\omega_H$, $\omega_L$, and $\omega_{LL}$. For each, the region, which is shown by arrow, depends on whether the oscillations are inertial-acoustic modes or g-modes. The symbols attached to arrow show frequency and mode. For example, $\omega_{H,p}$ denotes the inertial-acoustic oscillations of resonant frequency $\omega_H$. The case of g-mode oscillations, the subscript g is attached instead of p. To make clear the propagation regions, curves representing radial distributions of $\kappa$, $\Omega$, $\Omega \pm \kappa$, $2\Omega$ and $2\Omega \pm \kappa$ are shown. The vertical line shows the radius where the resonance occurs. This figure is drawn for $M = 10M_\odot$ and $a_*=0.2$. 