Soft Leptogenesis in Warped Extra Dimensions

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Abstract

We implement soft leptogenesis in a warped five dimensional scenario with two branes on the orbifold boundaries coming from an $S^1/Z_2$ symmetry, and supersymmetry broken on the IR brane. The SM hypermultiplet fields (fermions and Higgs) live in the UV brane and we allow the vector supermultiplets corresponding to the gauge bosons and a hypermultiplet corresponding to the right hand ed neutrino to live in the bulk. We assume that there are Majorana mass terms for the right handed neutrino superfield fixed on each brane and that there is a Yukawa term involving the right handed neutrino, the left handed neutrino and the Higgs fixed on the UV brane. Supersymmetry is broken by a constant “superpotential” on the IR brane, which induces an F-term for the radion hypermultiplet. This F-term leads to a B-term for the right handed sneutrinos as well as a soft SUSY breaking gaugino mass in the 4D theory for the zero modes. The gaugino mass naturally induces an A-term for the right handed sneutrino, left handed sneutrino and the Higgs to be formed through gaugino mediation with a non-trivial CP violating phase. Moreover, we show that within the context of extra dimensions, the condition of out-of-equilibrium decay and the phenomenological constraints on the neutrino mass are both satisfied in a natural way, for UV Majorana masses of the order of the fundamental scale of the theory. Thus all necessary elements for soft leptogenesis are at hand and we are able to predict a correct value for the baryon asymmetry.
1 Introduction

The idea of “soft leptogenesis” [1], [2] which can explain the baryon asymmetry in the universe is very attractive because of its simplicity. The soft parameters provide the source of CP violation, not relying on flavor physics like regular leptogenesis does. The presence of these terms allows oscillations between right handed sneutrinos and anti-sneutrinos which induce significant CP violation in sneutrino-decay processes. In [2], [3] a study of the parameter space was done, but no compelling model was addressed which could explain the values obtained. A study of this sort was made in [3], where gauge mediation and SUGRA effects were used to explain the parameters. Our idea is to extend these results to supersymmetric warped extra dimensional theories with one additional spatial extra dimension as in RS1 [5] [6]. Working in a supersymmetric scenario grant us the opportunity to change the size of the extra dimension without worrying about the hierarchy problem which is solved by supersymmetry. Part of the motivation for such an extension of the RS1 scenario comes from the point of view of string theory where naturally supersymmetry and compact extra dimensions are related, even though still there hasn’t been found any connection between this model and string theory. Furthermore, not many models of leptogenesis in extra dimensions have been worked out in the literature [7].

In our framework, the Standard Model fermions and Higgs superfields live in what we call the UV brane, the unwarped brane. The right handed sterile neutrino and the gauge superfields are in the bulk of the extra dimension and the radion superfield acquires a non-vanishing F-term on the IR brane (warped brane) which breaks supersymmetry. In this context, the location of the fields in the fifth dimension provides a natural way to explain the lepton asymmetry as well as to satisfy the constraints necessary for leptogenesis to succeed. Further constraints from neutrino masses and gravitino relic energy densities are satisfied too. This leads to a strong predictive model which adjusts fairly well to all constraints required for specific locations of the fields in the extra dimension. The coupling constants and masses have natural values except for the 5D right handed neutrino mass parameter $M_2$ which must be some orders of magnitude smaller than the GUT scale. One of the attractive points of this scenario is that the condition of out of equilibrium decay of the right handed sneutrinos is automatically obtained for values of the mass parameter $M_1$ fixed on the UV brane of the order of the GUT scale. This, in turn, leads to a neutrino mass of the order of $10^{-3}$ eV, consistent with phenomenological constraints. As was shown in [8], the GUT scale for our model coincides with the normal 4D GUT scale. This is related to the fact that though bare masses can be high, the KK masses always start at the order of the compactification scale $k e^{-k \pi R}$; something that doesn’t happen in flat extra dimensions.

The paper is organized as follows: in section 2 we introduce the model in 5D superspace and we show how the F-term of the radion superfield induces a gaugino mass. We write the Lagrangian in term of the component fields and we give an interpretation to the $\delta^2$ terms we find. To simplify the presentation, we work in a one-generation model, discarding flavor
indices, and we calculate the 4D effective soft supersymmetry breaking terms we will be working with. In section 3 we show where specifically CP violation comes from, calculate the lepton asymmetry and discuss the different constraints on the model. We arrive at the conclusions in section 4.

2 Superfield action in a warped 5D space

Let us consider a 5D theory where the extra dimension is warped. The extra dimension which we will denote with the letter $y$ is compactified on an orbifold $S^1/Z_2$ of radius $R$, with $0 \leq y \leq \pi$ the angular coordinate. The metric is given by

$$ds^2 = e^{-2R\sigma} \eta_{\mu\nu} dx^\mu dx^\nu + R^2 dy^2$$

(1)

where $\sigma = k|y|$ and $1/k$ is the curvature radius. This space corresponds to a slice of AdS$_5$. We promote $R$ to a superfield which corresponds to the 4D chiral radion superfield $T$ that, together with $R$, it is known to contain the fifth-component of the graviphoton $B_5$, the fifth-component of the right handed gravitino $\Psi^R_5$ and a complex auxiliary field $F_T$. Higher-dimensional supersymmetric theories contain 4D supersymmetry and therefore it is always possible to write them using $N=1$ superfields. We will write $T$ as

$$T = R + iB_5 + \theta \Psi^R_5 + \theta^2 F_T$$

(2)

and we will take $<B_5> = <\Psi^R_5> = 0$.

The five dimensional action in superspace for a hypermultiplet corresponding to the right handed neutrino is given by,

$$S_5 = \int d^5x \left( \int d^4\theta \frac{1}{2}(T + T^\dagger)e^{-(T + T^\dagger)\sigma}(N^\dagger N + N^c N^{c\dagger}) + \int d^2\theta e^{-3T\sigma} N^c \left[ \partial_y - (\frac{3}{2} - c_{\nu R}) T\sigma' \right] N + h.c. 
+ \frac{1}{2} \int d^2\theta e^{-3T\sigma} N^c (-M_1 T) N^c \delta(y) + h.c. + \frac{1}{2} \int d^2\theta e^{-3T\sigma} N^c (-M_2 T) N^c \delta(y - \pi) 
+ h.c. - \int d^2\theta e^{-3T\sigma} \lambda L N^c HT \delta(y) + h.c. \right)$$

(3)

where $N^c$ is the right handed neutrino chiral $N = 1$ hypermultiplet which together with $N$ forms the 5D off-shell right handed neutrino hypermultiplet. $N^c$ is even and $N$ odd under $S^1/Z_2$ respectively. H is the Higgs hypermultiplet of the up-type, L is the left handed neutrino hypermultiplet, M1 and M2 are the Majorana masses in 5D, $\lambda$ is the Yukawa constant in 5D and we parameterized the hypermultiplet mass as $c\sigma'$. In our conventions, $d^5x = d^4xdy$. In
components these fields can be written as

\begin{align*}
L & = \tilde{\nu} + \sqrt{2} \nu \theta + F_\nu \theta^2 \\
N^c & = \tilde{\nu}_R + \sqrt{2} \nu R \theta + F_{N^c} \theta^2 \\
H & = H + \sqrt{2} h \theta + F_H \theta^2 \\
N & = \tilde{N} + \sqrt{2} \psi \theta + F_N \theta^2
\end{align*}
(4)

The auxiliary field \( F_T \) which will be responsible for breaking supersymmetry on the \( y = \pi \) boundary, comes from the effective Lagrangian \([9]\)

\[ \mathcal{L}_{4D} = -\frac{6M_P^3}{k} \int d^4 \theta \phi^\dagger \phi (1 - e^{-(T+T^\dagger)k\pi}) + \int d^2 \theta \phi^3 [W_0 + e^{-3Tk\pi} W] + h.c. \]
(8)

where \( W \) and \( W_0 \) are superpotentials at the orbifold positions \( y = 0 \) and \( y = \pi \), \( \phi \) is the compensator field and \( M_P^2 = M_5^3 (1 - e^{-2k\pi R})/k \), with \( M_P \) being the 4D Planck mass. \( W_0 \) was introduced to cancel the cosmological constant in the 4D theory, and \( |W_0|^2 = e^{-4k\pi R} |W|^2 \).

This implies that SUSY breaking is heavily suppressed on the Planck brane; therefore we can assume that \( F_T \) is localized on the IR brane and its effective 4D form is

\[ F_T = e^{-k\pi R} \frac{W}{2\pi M_5^3}. \]
(9)

So far, we have not introduced the gauge field components. These, however, play an essential role in the model analyzed in this article, since the soft supersymmetry breaking parameters of the Higgs and left-handed lepton chiral fields are generated via the mechanism of gaugino mediation. The 5D vector superfield includes two gauginos. One of these gauginos, which we will denote as \( \lambda_1 \), transforms in a vector supermultiplet together with the gauge fields while the other, \( \lambda_2 \), forms the fermion component of a scalar superfield transforming in the adjoint representation of the group. The kinetic action for the vector supermultiplet can be parameterized in terms of the fermion chiral superfield \( W^\alpha \simeq \lambda_1^\alpha + \ldots \), and has the form

\[ S_5 = \int d^5 x \left[ \frac{1}{4g_5^2} \int d^2 \theta T W^\alpha W_\alpha + h.c. + \ldots \right] \]
(10)

The radion F-term breaks SUSY inducing a localized gaugino mass given by

\[ \mathcal{L}_{soft} = \frac{\delta(y - \pi) e^{-k\pi R} W \lambda_1 \lambda_1}{RM_5^3} + h.c. \]
(11)

Redefining \( \lambda_i \rightarrow e^{-2R\sigma} \lambda_i \), \( i = 1, 2 \) to absorb the spin connection term, the equations of motion for the gauginos are given by

\[ ie^{R\sigma} \bar{\sigma}_\mu \partial_\mu \lambda_2 + \frac{1}{R} (\partial_5 + \frac{1}{2} R \sigma') \bar{\lambda}_1 = 0, \]
\[ ie^{R\sigma} \bar{\sigma}_\mu \partial_\mu \lambda_1 - \frac{1}{R} (\partial_5 - \frac{1}{2} R \sigma') \bar{\lambda}_2 - \frac{1}{2M_5^3 R} \delta(y - \pi) \bar{\lambda}_1 = 0. \]
(12)
We solve these equations in the bulk, ignoring boundary effects which will only play a role when imposing boundary conditions. Looking for solutions of the form $\lambda_i(x, y) = \sum \lambda^{(n)} f_i^{(n)}(y)$ and using the orthogonality condition of the modes, Eq. (12) leads to the second order differential equations:

$$\left[ \frac{1}{R^2} e^{R_x} \partial_y (e^{-R_y} \partial_y) - \left( \frac{1}{4} \pm \frac{1}{2} \right) k^2 \right] f_{1,2}^{(n)}(y) = e^{2R_x} m_n^2 f_{1,2}^{(n)}$$

with solutions

$$f_1^{(n)}(y) = e^{R_x/2} N_n \left[ J_1 \left( \frac{m_n}{k} e^{R_x} \right) + b_1(m_n) Y_1 \left( \frac{m_n}{k} e^{R_x} \right) \right]$$

$$f_2^{(n)}(y) = \frac{\sigma' e^{R_x/2}}{k N_n} \left[ J_0 \left( \frac{m_n}{k} e^{R_x} \right) + b_2(m_n) Y_0 \left( \frac{m_n}{k} e^{R_x} \right) \right]$$

where $b_i$ and $m_n$ will be determined by the boundary conditions, and $N_n$ are normalization constants.

Taking into account the $Z_2$ assignment, $f_i^{(n)}$ must fulfill the following conditions on the $y = 0$ boundary

$$f_2^{(n)}|_{y=0} = 0$$

$$\left( \frac{d}{dy} + \frac{R}{2} \sigma' \right) f_1^{(n)}|_{y=0} = 0$$

which imply $b_1(m_n) = b_2(m_n) = -J_0(m_n/k)/Y_0(m_n/k)$. On the other hand, the presence of the Majorana gaugino mass on the $y = \pi$ boundary in Eq. (12) implies

$$f_2^{(n)}(\pi) = \frac{W}{4M_5^3} f_1^{(n)}(\pi)$$

These conditions yield that the following determinant should vanish

$$\det \left( \begin{array}{cc} J_0(x_n e^{k\pi R}) & Y_0(x_n e^{k\pi R}) \\ \frac{W}{4M_5^3} J_1(x_n e^{k\pi R}) & Y_1(x_n e^{k\pi R}) \end{array} \right),$$

where $x_n = m_n/k$. From here we get the KK gaugino mass spectrum. Solving this equation we find a non-zero value for the zero mode gaugino mass. In the case that $\eta \equiv \frac{W}{4M_5^3} \ll 1$ (small SUSY breaking) and $x_n e^{k\pi R} \ll 1$ (for the zero mode) we find

$$m_{\lambda_1} \approx -\frac{\eta}{\pi R} e^{-k\pi R}$$

Multiplying numerator and denominator by $k$, we see that under this conditions, the zero mode gaugino mass will be much smaller than the KK mass scale, parameterized by $ke^{-k\pi R}$.
It is important to stress again that, contrary to the standard warped extra dimension scenarios, the hierarchy is stabilized by supersymmetry and therefore there is no need for the KK mode masses to be close to the weak scale. In general the KK masses will be out of the reach of the LHC. As we will show, the phenomenological properties of this model will be similar to those of low energy supersymmetry breaking with a light gravitino.

We are interested in obtaining the effective action for the right-handed neutrinos. In order to do that, we need to calculate the auxiliary field for $N^c$ and $N$. From Eq. (3), we obtain,

\[
F_{N^c}^\dagger = -\frac{e^{-R\sigma}}{R} \left[ \partial_5 - \left( \frac{3}{2} - c_{\nu_R} \right) R\sigma' \right] \tilde{N} - M_1 \tilde{\nu}_R \delta(y) - M_2 \tilde{\nu}_R \delta(y - \pi) \\
- \lambda \tilde{\nu}_H R \delta(y) - \frac{1}{2R} \tilde{\nu}_R F_T (1 - 2R\sigma) \\
F_N^\dagger = \frac{e^{-R\sigma}}{R} \left[ \partial_5 - \left( \frac{3}{2} + c_{\nu_R} \right) R\sigma' \right] \tilde{\nu}_R - \frac{1}{2R} \tilde{N}^* F_T (1 - 2R\sigma)
\]

Replacing these F-terms in Eq. (3) and integrating over superspace we get the following 5D Lagrangian for $\tilde{\nu}_R$ and $\tilde{N}$

\[
\mathcal{L}_{5D} = \sqrt{-g} \left( -|\partial_M \tilde{N}|^2 - |\partial_M \tilde{\nu}_R|^2 - m^{2}_{\tilde{N} c} \tilde{N} \tilde{N}^* - m^{2}_{\tilde{N} c} \tilde{\nu}_R \tilde{\nu}_R^* \\
+ \frac{e^{R\sigma}}{2R^2} \tilde{N} F_T (1 - 2R\sigma) (\partial_5 \tilde{\nu}_R - (3/2 + c_{\nu_R}) \sigma' \tilde{\nu}_R) + h.c. - \frac{e^{R\sigma}}{2R^2} 2(3/2 - c_{\nu_R}) \sigma' R F_T \tilde{\nu}_R \tilde{N} \\
+ h.c. - \frac{e^{R\sigma}}{2R^2} F_T \tilde{\nu}_R (1 + 4R\sigma) (\partial_5 \tilde{N} - (3/2 - c_{\nu_R}) \sigma' R \tilde{N}) + h.c. - M_1^2 \tilde{\nu}_R \tilde{\nu}_R^* \delta(y)^2 \\
- \lambda \tilde{\nu}_H \tilde{\nu}_H^* \delta(y)^2 - M_1 \tilde{\nu}_R \lambda \tilde{\nu}_H \delta(y)^2 + h.c. \\
+ \frac{e^{2R\sigma}}{2R} (2R\sigma - 2) F_T \tilde{\nu}_R \tilde{\nu}_R^* \tilde{N} \tilde{N}^* \\
- M_2^2 \tilde{\nu}_R \tilde{\nu}_R^* \delta(y - \pi) + h.c. \\
+ \tilde{\nu}_R M_1 \frac{1}{R} \partial_5 \tilde{N}^* - (3/2 - c_{\nu_R}) \sigma' R \tilde{N}^* \delta(y) + h.c. \\
+ \tilde{\nu}_R M_2 \frac{1}{R} \partial_5 \tilde{N}^* - (3/2 - c_{\nu_R}) \sigma' R \tilde{N}^* \delta(y - \pi) + h.c. + \\
\frac{\lambda \tilde{\nu}_H}{R} \partial_5 \tilde{N}^* - (3/2 - c_{\nu_R}) \sigma' R \tilde{N}^* \delta(y) + h.c. + \frac{M_2^2}{2} \sigma^r R \tilde{\nu}_R \tilde{\nu}_R \delta(y - \pi) + h.c.),
\]

where $m^{2}_{\tilde{N}, N^c} = (c_{\nu_R} \pm c_{\nu_R} - 15/4) k^2$. The delta-squared terms are similar to those found by Horava in [10]. They are related to the bulk-boundary coupling, and as pointed out in [11] they are necessary in order to have SUSY conserved. They can also be thought as parameterizing the effects induced by the sum over the KK towers. Let us show this in the explicit example of the relation between neutrino and sneutrino masses.

The Majorana mass term for the right handed neutrino localized on the UV brane is
given by
\[ \frac{1}{2} M_1 \nu_R(x,0) \nu_R(x,0) = M_1 \sum_{n,m} f_R^{(n)}(0) f_R^{(m)}(0) \nu_{R}^{(n)}(x) \nu_{R}^{(m)}(x) \] (24)
which can be interpreted in matrix form in the basis of \((\nu_{R}^{(0)}(x), \nu_{R}^{(1)}(x), \nu_{R}^{(2)}(x), \ldots)\) as
\[
S = \begin{pmatrix}
    f_R^{(1)}(0) & f_R^{(1)}(0) & f_R^{(1)}(0) & f_R^{(1)}(0) & f_R^{(1)}(0) & \\
    f_R^{(2)}(0) & f_R^{(1)}(0) & f_R^{(2)}(0) & f_R^{(2)}(0) & f_R^{(2)}(0) & \\
    f_R^{(3)}(0) & f_R^{(1)}(0) & f_R^{(2)}(0) & f_R^{(3)}(0) & f_R^{(3)}(0) & \\
    \vdots & \vdots & \vdots & \vdots & \vdots & \\
\end{pmatrix}
\] (25)

The same KK expansion can be done for the right handed sneutrino with functions \(g_R^{(n)}(y)\). Since the \(g_R^{(n)}\) form a complete orthonormal system we can expand the \(\delta(0)\) in this basis as
\[ \delta(0) = \sum_k g_R^{(k)}(0) g_R^{(k)}(0) \] (26)

Therefore if we look at the SUSY mass term
\[
\int dy \sqrt{-g} (M_1^2 \bar{\nu}_R \nu_R \delta(y)^2) = \\
\int dy \left[ \sqrt{-g} (M_1^2 \bar{\nu}_R \nu_R \delta(y)) \right] \times \delta(y)
\] (27)

After proper normalization, it will take the form
\[
M_1^2 \sum_{n,m} g_R^{(n)}(0) g_R^{(m)}(0) \delta(0) \bar{\nu}_R^{(n)}(x) \nu_R^{(m)}(x) = \\
M_1^2 \sum_{n,m,k} g_R^{(n)}(0) g_R^{(k)}(0) g_R^{(k)}(0) g_R^{(m)}(0) \bar{\nu}_R^{(n)}(x) \nu_R^{(m)}(x)
\] (28)

which can be interpreted in the basis \((\bar{\nu}_R^{(0)}(x), \bar{\nu}_R^{(1)}(x), \bar{\nu}_R^{(2)}(x), \ldots)\) as \(M_1^2 S' \times S'\) with \(S'\) the mass matrix formed with the \(g_R^{(n)}(0)\) functions. We remind the reader that in AdS\(_5\) background fields in the same supermultiplet must have different masses which will lead them to have different dependence on the fifth dimension. However, in the case of flat extra dimensions \(g_R^{(n)}(y) = f_R^{(n)}(y)\) and we see then that this leads to the conventional supersymmetric relations between the neutrino and sneutrino mass matrices.

### 2.1 Sneutrino bilinear and trilinear SUSY breaking terms

The mechanism of soft leptogenesis requires specific relations between the soft supersymmetry breaking bilinear and trilinear terms of the sneutrinos. From the Lagrangian, Eq. (23), we see that nor A-term, neither B-term can be formed on the UV brane at tree-level. This
has to do with the fact that SUSY breaking is localized on the IR brane and that the B-term is proportional to $\sigma$. However, since the right-handed neutrino superfield propagates into the extra dimension, a B-term will be naturally induced on the IR brane (we remind the reader that the terms proportional to $F_T$ are localized on the IR brane).

For the zero modes, Eq. (23) reduces to

$$L_{5D,0} = \sqrt{-g}(-|\partial_M \tilde{N}|^2 - |\partial_M \tilde{\nu}_R|^2 - M_1^2 \tilde{\nu}_R \tilde{\nu}_R^* \delta(y)^2$$

$$- \lambda \lambda^* \tilde{\nu} \tilde{\nu}^* HH^* \delta(y)^2 - M_i \tilde{\nu} \lambda \tilde{\nu} \delta(y)^2 + h.c.$$  

$$+ \frac{e^{2R\sigma}}{2R}(2R\sigma - 2)F_T F_T \sigma(\tilde{\nu}_R \tilde{\nu}_R^* + \tilde{N} \tilde{N}^*) - M_2^2 \tilde{\nu}_R \tilde{\nu}_R^* \delta(y - \pi)^2 +$$

$$\frac{1}{2} M_2 \sigma e^{\sigma R} F_T \tilde{\nu}_R \tilde{\nu}_R \delta(y - \pi) + h.c.)$$  

(29)

We know that the right-handed massless zero mode for $\tilde{\nu}_R$ satisfies the following equation

$$[\partial_5 - (3/2 + c_{\nu R})T\sigma']g^{(0)} = 0$$  

(30)

whose solution is $g^{(0)} = e^{(3/2 + c_{\nu R})T\sigma}/N_0$, where $N_0$ is a normalization constant. Analyzing the form of the kinetic term in Eq. (29), we can obtain the normalization factor for the zero mode. Canonically normalizing the right handed sneutrino field, we find that

$$\frac{1}{N_0^2} = \frac{2(1/2 + c_{\nu R})k}{e^{2(1/2 + c_{\nu R})k\pi R} - 1}.$$  

(31)

Similarly, the normalization condition for the zero mode fields fixed on the UV brane comes from

$$\int_0^\pi \frac{1}{N_0^2} \delta(y) R dy = 1$$  

(32)

Therefore $N_0 = \sqrt{R}$.

In order to derive the form of the $A$ and $B$ parameters of the sneutrino, we need to determine the size of the effective Yukawa and Majorana masses for the right-handed neutrino field. If we look at the fermionic interactions for the superfields, we obtain the following term

$$L_{\text{Yukawa}} \simeq \lambda R (\nu \nu_R H + \nu_R h \tilde{\nu} + h \nu_R) + h.c.$$  

(33)

After canonically normalizing, this term takes the form

$$L_{\text{Yukawa}} \simeq \frac{\lambda \sqrt{k(1 + 2c_{\nu R})}}{\sqrt{e^{2(1/2 + c_{\nu R})k\pi R} - 1}} (\nu \nu_R H + \nu_R h \tilde{\nu} + h \nu_R) + h.c.$$  

(34)

Therefore, we identify the 4D Yukawa coupling constant

$$\lambda_4 = \frac{\lambda \sqrt{k(1 + 2c_{\nu R})}}{\sqrt{e^{2(1/2 + c_{\nu R})k\pi R} - 1}}$$  

(35)
We can do the same in the case of the Majorana mass. Then we get for the fermionic part in the case of IR or UV Majorana term

\[ \mathcal{L}_M \simeq \frac{1}{2} M_2 e^{2c_{\nu_R} k\pi R} \nu_R \nu_R R \delta(y - \pi) + \frac{1}{2} M_1 R \nu_R \nu_R \delta(y). \]  

(36)

Canonically normalizing these terms, we obtain the values of the localized Majorana masses for the right-handed neutrino,

\[ M_{4, IR} = 2 \frac{(1/2 + c_{\nu_R}) k R e^{2c_{\nu_R} k\pi R}}{e^{2(1/2 + c_{\nu_R}) k\pi R} - 1} M_2, \]  

(37)

\[ M_{4, UV} = 2 \frac{(1/2 + c_{\nu_R}) k R}{e^{2(1/2 + c_{\nu_R}) k\pi R} - 1} M_1. \]  

(38)

We see that provided the right-handed neutrino zero mode is localized towards the IR brane, \( c_{\nu_R} > -1/2 \), as we will assume in our model, the localized Majorana mass in the ultraviolet will be much larger than the one in the infrared, \( M_{4, UV} \gg M_{4, IR} \). Thus the effective Majorana mass of the right-handed neutrino, \( M_4 \), is dominated by the ultraviolet term, \( M_4 \simeq M_{4, UV} \).

We are now prepared to compute the bilinear and trilinear soft supersymmetry breaking terms in the effective four dimensional effective theory. Replacing the zero-mode for \( \tilde{N}_c \) in Eq. (23), integrating on the fifth dimension and canonically normalizing we get the following 4D B-term (we define the bilinear term in the soft Lagrangian as \(-\mathcal{L}_{\text{soft,4D}} = \ldots + \frac{1}{2} B_4 \tilde{\nu}_R(x) \tilde{\nu}_R(x) + \ldots \), where \( M_4 \) is the Majorana mass of the right handed neutrinos),

\[ B_4 = k\pi F_T \frac{M_{4, IR}}{M_{4, UV}}. \]  

(39)

On the other hand, in the presence of phases in the gaugino mass terms, massive gauginos will naturally induce an \( A_4 \)-term with a CP violating phase. We define the A-term as \(-\mathcal{L}_{\text{soft,4D}} = \ldots + A_4 \lambda_4 \tilde{\nu}_R \tilde{\nu} H + \ldots \). This terms comes from a 1-loop triangle diagram (see Fig. II) involving \( \tilde{\nu}, \tilde{\nu}_R \) and \( H \) [12] [13] [14] [15].

From the diagram, Fig. II we see that the only possible meaningful contributions can come from the gaugino zero mode and the right handed sneutrino zero mode, since \( \tilde{\nu} \) and \( H \) live in the UV brane. Concentrating on the dominant wino contribution, we get

\[ A_4 \lambda_4 = 4 \lambda_4 g_4^2 C_2(N) \int \frac{d^4 p}{(2\pi)^4} \frac{1}{p^2 + m_W^2} \frac{m_W}{m_{\tilde{W}}} \]  

(40)

where \( g_4^2 = g_5^2 / R \). Integrating up to the compactification scale \( k e^{-k\pi R} \) (the scale at which SUSY breaking is transmitted) we get for \( A_4 \),
Figure 1: Feynman diagram for $A_4$ generation.

$$A_4 \simeq 3\alpha_2 \frac{m_{\tilde{W}} \log (k e^{-k\pi R} / m_{\tilde{W}})}{2\pi}. \quad (41)$$

where $\alpha_2 = g_{4,2}/(4\pi) \approx 0.03$. Although the proper result for $A_4$ can only be obtained after resummation, due to the presence of the weak gauge coupling and the relatively low scale for $ke^{-k\pi R}$, the above result provides a very good approximation for the Wino contribution to $A_4$. Also, since the beta function of $\alpha_2$ in the MSSM is small, the value of $\alpha_2$ may be approximately replaced by its weak scale value.

The mass spectrum of squarks and sleptons can be obtained through gaugino mediation as is done in [12] since the F-term radion contributions to these masses will be smaller.

### 3 Results

Now we have all the necessary ingredients to do soft leptogenesis in warped extra dimensions. We will constraint our model by obtaining reasonable values for the lepton asymmetry $\epsilon_L$, the left handed neutrino mass through the see saw mechanism $m_\nu$, requiring that the necessary conditions for leptogenesis are satisfied and that the lepton asymmetry is not erased because of the KK modes.

As was mentioned in the introduction, we will consider a single generation of right handed neutrinos, since the effect survives even in this limiting case. We will drop all flavor indices. Under these conditions, a CP-violating phase is still present. We can see that in the following way. Let us write the important terms for CP violation in the 4D Lagrangian,

$$- \Delta \mathcal{L}_{4D} = \ldots + \lambda_4 (\nu \nu_R H + \nu_R h \bar{\nu} + h \nu \bar{\nu}_R) + g_{4,2} \sqrt{2} (\bar{\nu}^* \tilde{W} \nu + H^* \tilde{W} h) +$$
\[
\frac{1}{2} \tilde{B}_4 \tilde{\nu}_R \tilde{\nu}_R + \frac{1}{2} (M_{4,IR} + M_{4,UV}) \nu_R \nu_R + \frac{1}{2} m_{\tilde{W}} \tilde{W}\tilde{W} + h.c. \tag{42}
\]

where \( \tilde{B}_4 = B_4 M_4 = k \pi F \tau M_{4,IR} \), and \( m_{\tilde{W}} \propto F \tau \). The conformal sector of the action is invariant under an R-symmetry \( U(1)_R \) and a Peccei-Quinn-type symmetry \( U(1)_Q \). In analogy with what was discussed in Ref. \[16\], under \( U(1)_R \) and \( U(1)_Q \) the fields have the corresponding charges listed in Table 1. Then if, for instance, we start with a single phase

<table>
<thead>
<tr>
<th>Field</th>
<th>R-Charge</th>
<th>PQ-Charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>h</td>
<td>-1</td>
<td>-2</td>
</tr>
<tr>
<td>( \tilde{\nu} )</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>( \nu )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \tilde{\nu}_R )</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>( \nu_R )</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>( \tilde{W} )</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 1: R-Charges

in the gaugino mass \( m_{\tilde{W}} \) and assume that the other parameters are real, since the first line of Eq. (42) and the Majorana masses for \( \nu_R \) are invariant under the R-symmetry, we can rotate away the phase in the gaugino mass parameter by doing a \( U(1)_R \) transformation, while generating a phase on \( \tilde{B}_4 \). We can also remove the phase in \( \tilde{B}_4 \) by means of a \( U(1)_Q \) superfield rotation of the right handed neutrino and the Higgs, but we generate a phase in the total Majorana mass \( M_4 = M_{4,IR} + M_{4,UV} \). So we see that there is no possible way to eliminate this CP-violating phase, which is therefore physical. In general, we can identify this phase with

\[
\phi = \arg \left( M_4 m_{\tilde{W}} \tilde{B}_4^* \right). \tag{43}
\]

Using the fact that \( B_4 \propto m_{\tilde{W}} M_{4,IR} \), one can easily see that \( \phi = \arg(M_4 M_{4,IR}^*) \). We will work in a basis in which the total Majorana mass \( M_4 \) and the bilinear mass term \( \tilde{B}_4 \) are real (\( B_4 \) real), and therefore \( \phi \) may be identified with the CP-violating phase associated with the gaugino mass term. From Eq. (43) it follows that the phase \( \phi \) is transferred to the trilinear term \( A_4 \) at the loop level. Following \[2\], in the limit \( |\lambda_4|^2 A_4/4\pi \ll B_4/2 \) the lepton asymmetry is approximately given by

\[
\epsilon_L \simeq \frac{4 \Gamma B_4}{4 B_4^2 + \Gamma^2} \frac{Im A_4}{M_4} \Delta_{BF} \tag{44}
\]

\[
\Delta_{BF} = \frac{c_B - c_F}{c_F + c_B} \tag{45}
\]

where \( c_B \) and \( c_F \) represent the fermionic and bosonic decay channel rates with final states \( f = h\nu \) and \( f = H\tilde{\nu} \) respectively, and \( \Delta_{BF} \approx 0.8 \) for \( T = 1.2 M_4 \).
Here an important point must be raised. What we measure from experiments is the ratio
\[
\frac{n_B}{n_\gamma} \approx 7 \frac{n_B}{s} \approx 6 \times 10^{-10}
\]  
(46)

where \( n_B \) is the baryon number density, \( n_\gamma \) the photon number density and \( s \) is the entropy density (for a more detailed discussion see section 3.2). After production, and after the action of weak sphaleron effects, the total baryon asymmetry is fixed and therefore the baryon to entropy ratio remain constant. In that sense, the present measurement of \( n_B/s \) reflects the primordially generated value.

As shown in [4], we can write the baryon to entropy ratio as
\[
\frac{n_B}{s} = -\left( \frac{24 + 4n_H}{66 + 13n_H} \right) Y_{\nu_R}^\text{eq} \xi \left[ \frac{4|B_4|}{4|B_4|^2 + \Gamma^2} \right] \frac{|A_4|}{M_4} \sin(\phi)
\]  
(47)

where the first factor takes into account the reprocessing of the B-L asymmetry by sphaleron transitions, \( n_H \) is the number of Higgs doublets which is equal to 2, \( Y_{\nu_R}^\text{eq} = 45\zeta(3)/(\pi^4g_*) \) where \( g_* \) is the number of thermalized degrees of freedom, \( \xi \) is an efficiency parameter that slightly depends on the production mechanism for the right handed neutrinos and \( \phi \) is the CP violating phase defined above (\( \sin(\phi) \approx 1 \)). Assuming thermal production, \( \xi \) is suppressed for small and large \( m_\nu \) because of insufficient \( \nu_R \) production and strong washout effect, respectively. The maximum value is \( \mathcal{O}(0.1) \) for \( m_\nu \approx 10^{-(3-4)} \) eV.

Now, as seen from Eq. (47), during the radiation dominated era the entropy density \( s \) is given by
\[
s \sim g_* T^3
\]  
(48)

So we see from here that the towers of KK modes, if thermalized, will contribute to \( g_* \). Since the mechanism of leptogenesis depends on the decay of the chiral right-handed (s)neutrino zero mode, the presence of the KK towers will not induce an extra contribution to the lepton asymmetry. Therefore, the effect of the KK modes will be in general to dilute the leptonic asymmetry. Such a dilution, if large, would make it difficult to obtain the experimentally observed baryon asymmetry. Therefore, for simplicity, we shall assume \( ke^{-kR} \gtrsim M_4 \) and therefore only the zero mode will be in thermal equilibrium at temperatures of the order of the Majorana neutrino mass, \( T \sim M_4 \). Values of \( kR \sim 10 \) satisfying \( ke^{-kR} \simeq M_4 \), are also consistent with the ones necessary to stabilize the vacuum expectation value of \( T(x) \) [17], and therefore from now on we shall assume that the latter is satisfied.

Under the above conditions, the light neutrino properties will be governed by the right-handed neutrino mass. Namely, through the implementation of the see-saw mechanism in warped extra dimensions [18], the left handed neutrino mass is given by
\[
m_\nu \sim \frac{v^2|\lambda_4|^2}{M_4}
\]  
(49)

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where \( v = \langle H(x) \rangle \simeq 174 \) GeV is the expectation value of Higgs field, and we are assuming \( \tan \beta \gg 1 \).

To satisfy the out of equilibrium condition in the decay of the right handed sneutrino, we should have a decay rate, \( \Gamma = M_4 |\lambda_4|^2 / 4\pi \) that is not much faster than the expansion rate of the universe \( H \). Since at \( T \sim M_4 \) only the zero modes are in thermal equilibrium, we can essentially use four dimensional cosmology, ignoring the extra dimensional contributions of the gauge and neutrino fields propagating in the bulk. Therefore, we shall use the conventional four dimensional expression \( H = \frac{1.66 g_\ast^{1/2} T^2}{M_P} \) at the time when \( T \sim M_4 \) \( (M_P = 4.2 \times 10^{18} \) GeV and \( g_\ast \) counts the number of degrees of freedom (d.o.f) in thermal equilibrium).

We can improve the above approximation by writing an expression for the number of thermalized degrees of freedom as \( g_\ast = N_{KK} \times g_{\ast,1} + g_{\ast,2} \), where \( N_{KK} \) counts the number of excited KK levels that are in thermal equilibrium.

The masses of the first KK excited levels of a vector superfield (the major contribution to the KK states comes from the gauge modes), in the limit \( KR \gg 1 \) and \( m_n \ll k \), are given by
\[
m_{n,V} \simeq (n - \frac{1}{4}) \pi k e^{-k\pi R}
\] (50)

where \( n = 1, 2, \ldots \) is the KK level.

In the UV brane, there are 45 chiral superfields (quarks and leptons), 2 Higgs doublets which include 4 chiral superfields. Therefore, there are 49 chiral superfields, each one containing 4 physical degrees of freedom (2 fermionic and 2 bosonic). The total number of effective degrees of freedom adds to \( g_\ast = 98 \times (1 + 7/8) \simeq 184 \). The zero mode fields of the right handed neutrino and gauge superfields contribute in the following way. There are 12 gauge fields (8 from QCD and 4 from electroweak), each one having two polarizations since they are massless. Therefore, the gauge superfields zero-modes contribute \( 12 \times 2 \times 15/8 = 45 \) effective degrees of freedom, where the last factor of 15/8 comes from SUSY. The right handed neutrino zero modes superfields belong to 3 families and they are Weyl fermions (2 degrees of freedom). Thus, they contribute as \( 3 \times 2 \times 15/8 \simeq 11 \) degrees of freedom. So we conclude that \( g_{\ast,2} = 240 \).

In the case of the KK towers we have to remember that fields then are part of \( \mathcal{N} = 2 \) SUSY. Therefore each tower (counting gauge and three right-handed neutrino superfields which are the only ones that contribute to them) will add \( 60 \times 15/8 \simeq 112 \) degrees of freedom and thus \( g_{\ast,1} = 112 \). To calculate \( N_{KK} \) we go through the following derivation. The entropy density \( s \) is given by the expression
\[
s = \frac{\rho_p + p}{T}
\] (51)
Now $\rho$ and $p$ can be written as

$$\rho = \frac{g}{2\pi^2} \int_m^\infty \frac{t^{3/2} \overline{e}^{\frac{E-m}{T}} - 1}{e^{(E-m)/T} - 1} dB$$

$$p = \frac{g}{6\pi^2} \int_m^\infty \frac{t^{3/2} \overline{e}^{\frac{E-m}{T}} - 1}{e^{(E-m)/T} - 1} dB$$

where we are doing the calculations for bosons. We will assume that $T \gg \mu$. In the
relativistic limit, $T \gg m$, $\rho = (\pi^2/30)gT^4$ and $p = \rho/3$. We define $N_{KK}|n$ as

$$N_{KK}|n = \frac{s|_{n,non-rel}}{s|_{n,rel}}$$

for each KK level parameterized by $n$ in Eq. (50); $s|_{n,rel} = g(2\pi^2/45)T^3$ is the entropy con-
tribution in the relativistic limit, and we use the full expressions, Eqs. (52)-(53) to calculate
the non-relativistic entropy contribution, $s|_{non-rel}$. Therefore $N_{KK}$ is given by

$$N_{KK} = \sum_{n=1}^{+\infty} N_{KK}|n$$

For values of $m \gg T$, the effective number of degrees of freedom associated with a given
specie is suppressed by a factor $(m/T)^{3/2} \exp(-m/T)$.

Using the above expression for the KK mode masses, with $m_{1,V} \approx 2.3k \exp(-k\pi R)$, we
can easily perform the sum. It is straightforward to prove that, provided $T < 2.3k \exp(-k\pi R)$
the value of $N_{KK}$ will remain lower than one, leading to only a small modification of the
effective number of degrees of freedom at the freezing temperature. We shall require that
this relation is fulfilled, in order to avoid a dilution of the baryon asymmetry. In addition,
since the freezing temperature $T \simeq M_4$, the above relation ensures that the zero right-handed
neutrino mode will have only small mixing with the heavier KK modes, increasing the va-
lidity the approximations used in this work to generate the Majorana mass contribution to
the right-handed neutrino zero mode.

The out of equilibrium condition then reads,

$$M_4/|\lambda_4|^2 \gtrsim \frac{M_P}{4\pi \times 1.66 \times (1.2)^2\sqrt{g_\ast}} \text{GeV}$$

On the other hand, the sneutrino decay should occur before the electroweak phase transition,
when sphalerons, responsible for the conversion of lepton asymmetry into baryon asymmetry
are still active, $\Gamma > H(T \sim 100\text{GeV})$

$$M_4|\lambda_4|^2 \gtrsim 3 \times 10^{-13} \text{GeV}$$

\footnote{the results are basically the same for fermions}
3.1 Analytical Estimates

Taking into account the above constraints, we can obtain information about the fundamental parameters of the theory from an analytical point of view. Assuming that the right handed neutrino is located away from the infrared brane,

\[ e^{1/2+c_{\nu R}} \gg 1 \]  

and that the five dimensional Yukawa coupling \( \lambda_5 \simeq a/\sqrt{k} \), with \( a \) a number of order one, we obtain,

\[ \lambda_4 = a \sqrt{1 + 2c_{\nu R}e^{-(1/2+c_{\nu R})k\pi R}} \]  

\[ M_{4,UV} = \frac{\lambda_4^2}{a^2}kRM_1 \simeq M_4 \] 

\[ M_4/\lambda_4^2 \simeq \frac{kRM_1}{a^2}. \]  

We can now see that the out of equilibrium condition, Eq. (56), sets the scale for \( M_1 \)

\[ M_4/\lambda_4^2 = \frac{v^2}{m_\nu} \]  

and therefore, for the above values of \( M_1 \), the neutrino mass parameter acquires phenomenologically acceptable values \( m_\nu \sim 10^{-3} \) eV.

In order to determine the value of the remaining parameters, we should take into account the conditions necessary for the realization of the soft leptogenesis scenario. As it is clear from Eq. (65), these depend on the specific values of the bilinear and trilinear parameters derived before, as well as the decay width. The relevant parameters are given by

\[ M_{4,IR} \simeq \frac{\lambda_4^2}{a^2}kRM_2e^{2c_{\nu R}k\pi R} \]  

\[ B_4 = 2k\eta e^{-k\pi R} \frac{M_{4,IR}}{M_{4,UV}} = 2k\eta e^{(2c_{\nu R}+1)k\pi R} \frac{M_2}{M_1} \] 

\[ m_W \simeq \frac{\eta k}{\pi kR} e^{-k\pi R} \] 

\[ \Gamma = \frac{M_{4,UV}\lambda_4^2}{4\pi} = \frac{M_1 kR a^2}{4\pi} \frac{(1 + 2c_{\nu R})^2 e^{-(2+4c_{\nu R})k\pi R}}{4\pi} \]
The primordial lepton asymmetry is maximized when the decay with is of the order of $B_4^4$. Requiring the parameters to be close to the resonance condition for $\epsilon_L (\Gamma = 2B_4)$ we obtain the following relationship

$$\frac{M_2k}{M_1^2a^2} = \frac{kR}{16\pi\eta}(1 + 2c_{\nu R})^2e^{-(1+6c_{\nu R})k\pi R}$$

(68)

Furthermore, now asking that at the specific temperature of soft leptogenesis there are very few KK excited states, $T \sim M_4 \sim k e^{-k\pi R}$, implies

$$k \sim (1 + 2c_{\nu R})kRe^{-2c_{\nu R}k\pi R}$$

(69)

Combining Eq. (68) with Eq. (69) we arrive at the following relation

$$M_2 \sim \frac{ka^2}{16\pi\eta kR}e^{-(1+2c_{\nu R})k\pi R}$$

(70)

A further constraint we need to impose on the model is that the mass of the NLSP which is the stau $\tilde{\tau}_1$, as is the case in [12], be in accordance with experimental constraints. We use the RGE at one loop [19],

$$\frac{d\tau}{dt} = \ldots + \frac{1}{8\pi^2} \left( -\frac{12}{5} g_{4,1}^2 m_B^2 \right) + \ldots$$

(71)

where $g_{4,1}$ is the U(1)$_Y$ 4D hypercharge, $m_B$ is the bino mass, $t = k e^{-k\pi R}/m_\tilde{\tau}$, and we only included the relevant term. To avoid experimental constraints, $m_{\tilde{\tau}_1} > \sim 100 \text{ GeV}$ or bigger, we need to have a gaugino mass $m_\lambda > \sim 500 \text{ GeV}$.

Taking the compactification scale $k e^{-k\pi R} \sim M_4$ but at the same time having a gaugino mass of $O(500) \text{ GeV}$ and furthermore having a small gravitino mass ($m_{3/2} < 16 \text{ eV}$ from cosmological constraints, see next section) fixes the values of $kR$ and $\eta$. With this requirements, we find that the value of the trilinear term $A_4 \sim O(60) \text{ GeV}$. Now, in the case of resonance, $\Gamma \simeq 2|B_4|$, this implies a maximum value for $M_4$ since $\epsilon_L \simeq A_4/M_4 \simeq 10^{-6}$. But from Eq. (63) and the discussion about the efficiency parameter $\xi$ following Eq. (47), we see that by fixing $m_\nu \simeq 10^{-3-4} \text{eV}$ we completely determine the value of $\lambda_4$. Thus in turn, from Eq. (62), draws us to fully fix the ratio $M_1/a^2$. Since $a \simeq O(1)$, we see that

$$M_1 \simeq O(10^{14-15}) \text{ GeV}, \text{ of the order of the unification scale, as stated in the introduction and previously on this section. Rewriting Eq. (70) as}$$

$$M_2 = \frac{k\lambda_4^2}{16\pi\eta kR(1 + 2c_{\nu R})}$$

(72)

we see that $M_2$ is also almost fully fixed by the choice of $m_\nu$ except for a mild dependence on $1/(1 + 2c_{\nu R})$. Thus, we have shown that given the conditions described above, all parameters are fixed by choosing values for $m_\nu$ and the parameter $a$. 

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3.2 Numerical Results

The numerical ratio of the baryon density to the entropy density may be obtained experimentally by two methods. Firstly, by the requirement of consistency between the observed and the predicted abundance of primordial elements by the Standard Big-Bang Nucleosynthesis model [20]. This leads to a value of the baryon to photon ratio [21],

$$4.5 \times 10^{-10} \lesssim \frac{n_B}{n_{\gamma}} \lesssim 6.5 \times 10^{-10}.$$  \hspace{1cm} (73)

The second method is related to the baryon energy density determination by the WMAP experiment [22],

$$\Omega_b h^2 = (2.233 \pm 0.072) \times 10^{-2}. \hspace{1cm} (74)$$

Considering the relation between the entropy and the photon density, \( s \simeq 7n_{\gamma}^2 \), the Big-Bang Nucleosynthesis results translate into a value of

$$6.5 \times 10^{-11} \lesssim \frac{n_B}{s} \lesssim 9.5 \times 10^{-11},$$  \hspace{1cm} (75)

with a narrower band of values, around \( 9 \times 10^{-11} \) being selected if only the WMAP values are considered. The WMAP result, Eq. (74), may be slightly modified (up to values of \( \Omega_b h^2 \simeq 0.019 \)) by assuming different shapes of the power spectrum [21, 23]. We shall require that the baryon number to entropy density ratio that we compute is within the broader range given above, (75). However, values within the WMAP allowed band, Eq. (74), may always be obtained by appropriate tuning of the parameters of the model.

In Tables 2 and 3 we give the results from numerical computations for two different acceptable points in parameter space. The input parameters are listed on the left column and the output on the right column.

From the tables we see that, as emphasized before, \( M_4 = M_{4,UV} + M_{4,IR} \approx M_{4,UV} \). We also notice that as we lower \( M_4 \) we don’t need to be so close to the resonance condition which, as said before, is fulfilled when \( \Gamma = 2|B_4| \). Moreover, \( M_4/\lambda_4^2 \) satisfies the out of equilibrium condition, Eq. (56). All necessary conditions for soft leptogenesis are satisfied and we get a left-handed neutrino mass which is of the order of the one associated with the values of the eigenstate mass differences implied by solar neutrino experiments.

To calculate the gravitino mass we used the approximate formula

$$m_{3/2} \approx \frac{\eta k e^{-2k\pi R}}{\sqrt{3} M_P} \hspace{1cm} (76)$$

and we have required that the gravitino mass obtained from this expression is lower than about 20 eV, to be consistent with astrophysical and cosmological bounds (see below).

\footnote{Although the addition of the gravitino increases the total entropy density, this increase is very small due to the large dilution factor associated with the gravitino decoupling temperature, \( T_D > 1 \text{GeV} \).}
\[
\begin{align*}
\lambda_4 & = 1.98 \times 10^{-3} \\
k e^{-k R} & = 1.216 \times 10^7 \text{ GeV} \\
M_{4,UV} & = 9.23 \times 10^6 \text{ GeV} \\
M_{4,IR} & = 0.73 \text{ GeV} \\
m_{\lambda_1} & = 484 \text{ GeV} \\
A_4 & = 70.11 \text{ GeV} \\
m_{\nu} & = 1.29 \times 10^{-3} \text{ eV} \\
B_4 & = 0.0019 \text{ GeV} \\
\Gamma_4 & = 0.00029 \text{ GeV} \\
\epsilon_L & = 1.12 \times 10^{-6} \\
m_{3/2} & \approx 20 \text{ eV} \\
M_4/\lambda_4^2 & = 2.44 \times 10^{16} \text{ GeV} \\
N_{KK} & = 0.55 \\
n_B/s & \approx 7.2 \times 10^{-11}
\end{align*}
\]

Table 2: Results

In Tables 2, 3 we have chosen a value of \( k \approx 10^{18} \text{ GeV} \), of the order of the fundamental Planck scale. As the value of \( k \) is lowered, we see that \( N_{KK} \gtrsim 1 \) and \( M_2 \lesssim 10^{10} \text{ GeV} \). We provide an example, with values of \( k \approx 2 \times 10^{17} \text{ GeV} \) in table 4.

In models of low energy supersymmetry breaking, like the one under consideration, the gravitino is the lightest SUSY particle and thus it is stable. Therefore gravitinos generated at high temperature contribute to the matter density of the universe. For masses \( m_{3/2} \lesssim 100 \text{ eV} \) the goldstino component of the gravitino has large interaction with the MSSM particles, and therefore the gravitino can thermalize at high temperature. The number density in this case is just the equilibrium value and, taking into account the diluting effect of the decoupling of heavy particles, the energy density is approximately given by \( \Omega_{3/2} h^2 \sim 0.1 (m_{3/2}/100 \text{ eV}) \), satisfying cosmological bounds. Since these gravitinos are warm, from Lyman-\( \alpha \) forest and WMAP data in order for them not to smear out the density perturbations on the matter power spectrum at small scales their masses are excluded from the region \( 16 \text{ eV} \lesssim m_{3/2} \lesssim 100 \text{ eV} \). The gravitino mass in our scenario falls naturally in the 1–100 eV range, and satisfies these constraints for a broad range of parameters, as shown by the specific examples above.

In the above, we have considered a model of gaugino mediation in warped extra dimensions, in which the matter fields are localized on the UV brane, while the dominant supersymmetry breaking contribution is localized on the IR brane. A question arises about the possible origin of the supersymmetry Higgsino mass term \( \mu \) within such scenario. Since the value of the gravitino mass is much lower than in supergravity mediated scenarios, the Giudice-Masiero mechanism \([24]\) won’t provide a sufficiently large mass. A logical possibil-
\begin{table}[h]
\centering
\begin{tabular}{|l|l|}
\hline
Input 2 & Output 2 \\
\hline
$\nu_R = -0.105$ & $\lambda_4 = 1.54 \times 10^{-3}$ \\
$kR = 8.42$ & $ke^{-k\pi R} = 9.75 \times 10^6$ GeV \\
$M_1 = 1.1 \times 10^{15}$ GeV & $M_{4,UV} = 6.148 \times 10^6$ GeV \\
$M_2 = 10^{10}$ GeV & $M_{4,IR} = 0.21$ GeV \\
$k = 3 \times 10^{18}$ GeV & $m_{\lambda_1} = 479$ GeV \\
$\lambda = 0.6/\sqrt{k}$ & $A_4 = 66$ GeV \\
$\eta = 1.3 \times 10^{-3}$ & $m_\nu = 1.176 \times 10^{-3}$ eV \\
\hline
\end{tabular}
\end{table}

Table 3: Results

ity is the addition of a singlet in the spectrum, which couples to the Higgs superfields and induces a $\mu$-term by acquiring a v.e.v. If this singlet is localized, it will acquire a negative supersymmetry breaking squared mass term by radiative corrections. Since supersymmetry is mediated by gaugino interactions, this is a higher-loop effect, and numerically the mass values are too small to lead to a phenomenologically acceptable $\mu$ parameter, for natural values of the Higgs and singlet couplings. However, as has been previously done in similar low energy supersymmetry breaking models [25], one can make use of supergravity induced tadpole contributions and the compensator field (8), whose F-term is $F_\phi \approx m_{3/2}$, to lead to an acceptable value of $\mu$. As an alternative to the localized singlet field, one can also consider the case of a singlet field propagating in the bulk of the warp extra dimension. Although in this case the result depends on the precise localization of the singlet zero mode in the bulk, an acceptable $\mu$-term may be obtained for reasonable values of the bulk mass parameters.

4 Conclusions

In this article, we have studied the possibility of realizing the mechanism of soft leptogenesis within the context of warped extra dimensions. We have assumed that all the quark and lepton fields are localized on the UV brane, while the gauge fields and the right-handed neutrinos propagate into the extra dimension. Assuming the presence of localized Majorana mass terms on the UV and IR branes, we have shown that the condition of out of equilibrium may be naturally fulfilled by assuming that the UV Majorana mass term is of the order of the GUT scale. Furthermore, for the same conditions, the neutrino mass acquires phenomenologically acceptable values, and of the order of the ones necessary to maximize the baryon asymmetry result.

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Soft supersymmetry breaking parameters for the gauginos and for the right-handed sneutrinos are generated by the auxiliary component of the radion field, which acquires a non-vanishing vacuum expectation value localized on the IR brane. Loop effects are responsible for the generation of supersymmetry breaking parameters for the rest of the quark, lepton and Higgs superfields. Although the right-handed stau becomes the lightest standard model superpartner, the lightest superparticle is given by the gravitino which becomes naturally light within this framework. Then, the collider phenomenology becomes similar to the one of gauge-mediated models with a light stau NLSP [26].

We have shown that, provided a relative phase exist between the two localized Majorana masses, the physical CP-violating phase necessary for the realization of soft leptogenesis is generated. An effective Majorana mass $M_4$ smaller than about $10^8$ GeV, as necessary for the realization of this scenario, is naturally generated by a proper localization of the right-handed neutrino zero modes. Moreover, this localization is also effective in avoiding the dilution of the baryon asymmetry by the entropy generated by the KK towers provided the Majorana mass is smaller than the local curvature term in the IR brane, $M_4 \lesssim k \exp(-k\pi R)$. Finally, the condition that the gaugino masses are at the TeV scale fixes the size of $F_T$. The resulting gravitino mass is of the order of a few eV, which satisfy the relic density and long range structure constraints dictated by cosmology.

A proper baryon asymmetry may be generated under the above conditions, provided the effective bilinear term $B_4$ is of the order of the sneutrino decay width. Due to the smallness of the Yukawa couplings, this implies a value of $B_4$ smaller than about 100 MeV. In our

<table>
<thead>
<tr>
<th>Input 3</th>
<th>Output 3</th>
</tr>
</thead>
</table>
| $c_{\nu R} = -0.105$  
$kR = 7.6$  
$M_1 = 3.3 \times 10^{14}$ GeV  
$M_2 = 3 \times 10^9$ GeV  
$k = 2 \times 10^{17}$ GeV  
$\lambda = 0.3/\sqrt{k}$  
$\eta = 1.5 \times 10^{-3}$ | $\lambda_4 = 2.138 \times 10^{-3}$  
$k e^{-k\pi R} = 8.546 \times 10^6$ GeV  
$M_{4,UV} = 1.27 \times 10^7$ GeV  
$M_{4,IR} = 0.77$ GeV  
$m_{\lambda_1} = 537$ GeV  
$A_4 = 74.4$ GeV  
$m_\nu = 1.086 \times 10^{-3}$ eV  
$B_4 = 0.0015$ GeV  
$\Gamma_4 = 0.00046$ GeV  
$\epsilon_L = 1.71 \times 10^{-6}$  
$m_{3/2} \approx 15$ eV  
$M_4/\lambda_2^2 = 2.786 \times 10^{16}$ GeV  
$N_{KK} = 1.38$  
$n_B/s \simeq 8.35 \times 10^{-11}$ |

Table 4: Results
model the value of $B_4$ is determined by the ratio of the localized Majorana mass term, and successful leptogenesis is achieved for values of the localized UV and IR mass terms $M_1 \simeq 10^{15}$ GeV and $M_2 \simeq 10^{10}$ GeV, respectively.

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References


