A brief survey of LISA sources and science

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Abstract. LISA is a planned space-based gravitational-wave (GW) detector that would be sensitive to waves from low-frequency sources, in the band of roughly $(0.03 - 0.1) \text{ mHz} \lesssim f \lesssim 0.1 \text{ Hz}$. This is expected to be an extremely rich chunk of the GW spectrum — observing these waves will provide a unique view of dynamical processes in astrophysics. Here we give a quick survey of some key LISA sources and what GWs can uniquely teach us about these sources. Particularly noteworthy science which is highlighted here is the potential for LISA to track the moderate to high redshift evolution of black hole masses and spins through the measurement of GWs generated from massive black hole binaries (which in turn form by the merger of galaxies and protogalaxies). Measurement of these binary black hole waves has the potential to determine the masses and spins of the constituent black holes with percent-level accuracy or better, providing a unique high-precision probe of an aspect of early structure growth. This article is based on the “Astrophysics Tutorial” talk given by the author at the Sixth International LISA Symposium.

Our view of the universe today is built almost entirely by exploiting the electromagnetic interaction\textsuperscript{1}. The leading order radiation comes, as is very well known, from time variations of the charge dipole moment $d_i$ of a source: writing

$$d_i = \int \rho_c(x') x'_i d^3 x' , \quad (1)$$

(where $\rho_c$ is the charge density and the integral is taken over an extended source), the radiative electromagnetic 4-potential is given by

$$A_i \simeq \frac{d_i}{r} \sim \frac{v q_{dip}}{r} . \quad (2)$$

The final order of magnitude formula tells us that the amplitude of this radiation is set by the amount of charge participating in dipole motions, and the speed of that charge’s motion. Since many astrophysical environments are rich in plasma and free charges, electromagnetic radiation is readily created; it is likewise detected with relative ease.

Gravity also generates radiation. Since “gravitational charge” (i.e., mass) only comes with one sign, the leading order radiation is quadrupolar rather than dipolar (see [1] for discussion): defining the mass quadrupole moment

$$Q_{ij} = \int \rho_m(x') \left( x'_i x'_j - \frac{1}{3} (r')^2 \delta_{ij} \right) d^3 x' , \quad (3)$$

\textsuperscript{1} With a relatively small, but extremely important, complementary exploitation of the weak interaction via neutrino astronomy.
the field which (at leading order) describes gravitational radiation is given by

\[ h_{ij} = \frac{G}{c^4} \frac{\ddot{Q}_{ij}}{r} \sim \frac{G m^{\text{quad}}}{r c^2} \frac{v^2}{c^2}. \]  

In the final order of magnitude formula, the mass \( m^{\text{quad}} \) is the portion of the source’s mass that participates in quadrupolar motions. Notice that the dimensionful constant coupling the radiation \( h_{ij} \) to the source \( Q_{ij} \) is incredibly small:

\[ \frac{G}{c^4} = \frac{6.673 \times 10^{-8} \text{ cm}^3 \text{ sec}^{-2} \text{ gm}^{-1}}{(2.998 \times 10^{10} \text{ cm/sec})^4} = 8.260 \times 10^{-50} \frac{\text{sec}^2}{\text{gm cm}}. \]  

Overcoming this tiny coupling requires enormous masses and speeds close to the speed of light. GW sources thus tend to be compact objects — objects that are both massive and small enough that they can whip around at relativistic speeds. It also means that, once generated, the waves barely interact with intervening matter. This unfortunately includes our detectors, so that detecting GWs is quite an experimental challenge; but it also means that sources cannot be obscured, as is often the case with electromagnetic sources. Thus, GWs present an opportunity to open a window directly onto the most violent and dynamic processes in the universe.

In this article, we quickly survey some of the most important LISA GW sources. A handy way of organizing these sources is by the spectral character of their waves (which in turn drives how we plan to analyze LISA data): Stochastic sources, with a broadband, flat or widely peaked spectrum; monochromatic sources, which radiate at nearly a pure tone in their rest frame; chirping sources, like a pure tone which quickly sweeps through the LISA band; and really complicated chirping sources — similar to chirping sources, but with a particularly ornate character that merits special consideration.

**STOCHASTIC SOURCES FROM COSMOLOGICAL BACKGROUNDS**

Stochastic GWs are “random” waves, arising from the superposition of many discrete, uncorrelated sources. They typically combine to produce a broadband spectrum that is nearly flat, or weakly peaked near some fiducial frequency.

Some of the most anticipated stochastic sources are cosmological in origin. By far the most eagerly sought waves are GWs arising from primordial fluctuations in the universe’s spacetime metric which have been parametrically amplified by inflation. In energy density units (commonly used to parameterize these waves; see [2] for a very readable discussion), these waves are flat across an extremely wide band, ranging from frequencies \( f \sim 10^{-16} \text{ Hz} \) to roughly \( 10^{10} \text{ Hz} \). The lower end of this scale corresponds to the inverse Hubble scale when the universe became matter dominated; below this frequency, the spectrum grows as \( f^{-2} \). At the high end, the spectrum is cut off by the (short but finite) time scale over which inflation ends and the universe enters a hot, radiation-dominated phase; see Fig. 12 of [2] and associated discussion. What’s particularly exciting about these waves is that the level of this spectrum is set by — and
thus encodes — the potential which drives inflation. Thus, measuring these waves would *directly probe inflationary physics*.

Unfortunately, these waves have such small amplitudes that direct detection with LISA (or any other near term GW experiment) is completely out of reach. LISA should be able to detect a background at a level \( \sim 10^{-11} \) of the universe’s closure density; current estimates suggest that this is at least 4 or 5 orders of magnitude from the sensitivity needed to measure these waves. Direct detection at the relatively high frequencies of man-made facilities requires finding a relatively quiet band (without too many “foreground” sources) and a much more sensitive instrument than LISA. At the lowest frequencies, detection is plausible in the next decade or so through cosmic microwave background (CMB) studies. The tensor nature of GWs has a unique impact on the polarization of CMB photons [3, 4]. The signal is weak compared to other signals in the CMB, and potentially contaminated by foreground effects. Searching for it is nonetheless one of the most important directions in CMB physics today.

Although inflationary waves are out of LISA’s reach, other cosmological backgrounds may not be. Backgrounds are associated with phase transitions that occur as the universe expanded and cooled. For example, when the mean temperature was roughly 1 TeV, the electroweak interaction separated into electromagnetic and weak interactions. This separation was not spatially homogeneous — some regions underwent this transition before others. Different regions were at different densities; boundaries between regions collided and coalesced, producing a GW background [2, 5]. More recently, there has been a flurry of work in superstring theory suggesting that cosmic strings may have been produced in the early universe and then expanded to cosmic sizes (see, for example [6] for an overview). A network of such objects could produce a background accessible to LISA; see Hogan’s contribution to these proceedings [7] for detailed discussion.

Though a relatively speculative source, cosmological stochastic backgrounds are easily searched for, and there is a tremendous amount of discovery space. The probability of payoff may be low, but the potential reward for discovery is extraordinarily high.

**PERIODIC SOURCES: BINARY STARS IN THE GALAXY**

The Milky Way contains trillions of stars in binary systems; each is a GW generator. A small fraction — tens of millions — are compact (mostly white dwarf binaries) and generate GWs in the LISA band. GW emission causes the binaries’ stars to gradually spiral towards one another, driving the frequency to slowly “chirp” upwards. At the masses and frequencies we’re discussing here, the chirp is extremely slow:

\[
\dot{f} = \frac{48}{5\pi} \mu M^{2/3} (2\pi f)^{11/3}
\]

\[
= 9.2 \times 10^{-18} \text{Hz/sec} \times \left( \frac{M}{1 M_\odot} \right)^{5/3} \left( \frac{f}{1 \text{mHz}} \right)^{11/3}. \tag{7}
\]

We’ve specialized to equal masses in the last line, putting the reduced mass \( \mu = M/4 \); we are also using units with \( G = 1 = c \), so that \( M_\odot = 4.92 \times 10^{-6} \text{sec} \). The frequency of these sources barely changes over a mission lifetime: \( \dot{f} \times T_{\text{mission}} \) is much less than the
rough binwidth, $\delta f \sim (a \text{ few})/T_{\text{mission}}$, except on the high end of the band, $f \gtrsim 0.01 \text{ Hz}$. These sources are thus mostly monochromatic, with a scattering of slowly chirping ones at the high end of the mass and frequency spectrum.

Such objects are extremely important for LISA science because they are guaranteed sources. Quite a few target binaries have already been catalogued by x-ray and optical studies (cf. contribution to this meeting by Gijs Nelemans [8], and Neleman’s webpage [9]). Indeed, population synthesis indicates there will be so many binaries at low frequencies that they will form a confused background — so many binaries radiate in a single frequency bin that they cannot be distinguished. For studying certain sources (e.g., massive black hole binaries, discussed in the next section), this background must actually be regarded as noise. It is of course signal to those interested in stellar populations! For example, Matt Benacquista and colleagues have shown that a galactic population can be clearly distinguished from a population based in globular clusters [10], and Benacquista and Holley-Bockelman have shown that the galactic scale height has important consequences for the mean level and detailed characteristics of this background [11]. A wealth of data about the distribution of stars in our galaxy is encoded in this “noise”.

**CHIRPING SOURCES: MASSIVE BLACK HOLE COALESCENCE**

Chirping sources can be thought of as a pure tone (like monochromatic periodic sources), but with that tone rapidly sweeping through the LISA band. Consulting Eq. (6), we see that making a binary sweep through the band requires large masses. The most important chirping LISA sources will be binaries in which both members are massive black holes: With a total system mass of $10^4 M_\odot - 10^7 M_\odot$ and a mass ratio of $1/20-1$ or so, a binary generates waves right in LISA’s main band of sensitivity, and sweeps across the band in a time ranging from a few months to a few years.

Binaries at these masses are created by the mergers of large structures — galaxies and protogalaxies, and the dark matter halos which host them. It has long been appreciated that the formation of massive binary black holes as a “side effect” of hierarchical structure formation could lead to healthy event rates. Quoting from Haehnelt’s contribution to the Proceedings of the 2nd LISA Symposium [12], “even a pessimist who assumes a rather long QSO lifetime and only one binary coalescence per newly-formed halo should expect a couple of SMBH binary coalescences during the lifetime of LISA while an optimistic might expect to see up to several hundred of these exciting events.”

Recent work suggests the rate may be near the high end of this range, though with most events at moderately high redshift. One input to this story is that it is now appreciated that almost all galaxies host a massive black hole at their core. It had long been argued that accretion onto black holes must power the emission of quasars and active galactic nuclei [13]; a corollary is that the cores of many galaxies should host massive black holes as “quasar fossils”. This expectation has been confirmed by observations which demonstrate kinematically that the cores of galaxies with large central bulges host a massive ($10^6 - 10^9 M_\odot$) black hole. Further, it has been established that the properties of these holes are strongly correlated with the properties of the galaxies that host them [14, 15], indicating that the growth of black holes and their host galaxies is tightly
FIGURE 1. Distribution of measured mass errors from a Monte-Carlo simulation of $10^4$ binaries, randomly distributed on the sky, randomly oriented, and with random spins and spin orientations. All binaries are placed at redshift $z = 1$, and have masses $m_1 = 10^6 M\odot$, $m_2 = 3 \times 10^5 M\odot$. The solid line gives the distribution of errors in $m_1$, the dotted line gives the distribution in $m_2$. Both distributions peak at a relative error of about 0.1%, and are almost entirely confined to errors smaller than 1%.

Second, there is a consensus that galaxies grow hierarchically, through repeated mergers of smaller galaxies as their host dark matter halos repeatedly merge; see for example Hopkins et al. [16]. Combining the apparent ubiquity of black holes in galaxies with the tendency of galaxies to merge suggests that the formation of massive binary black holes is fairly frequent, especially at high redshift when mergers were common. For example, Sesana et al. [17] calculate a total rate for LISA of about 60 – 70 events per year, with most ($\sim 50$) mergers occurring for a total system mass smaller than $10^5 M\odot$ and for redshifts $z > 10$. (Note that this mass is smaller than has been observed for any galactic core black hole, but is a reasonable “seed” mass for a hole that evolves to the masses observed in the local universe.) About 10 mergers per year are expected to occur in the mass range $10^5 M\odot \lesssim M \lesssim 10^6 M\odot$ and in the redshift range $2 \lesssim z \lesssim 6$. See Marta Volonteri’s contribution to this meeting for further details [18].

In the months to a year or so that the binary radiates in LISA’s most sensitive band, many thousands to tens of thousands of wave cycles accumulate. Even for high redshift sources, these signals can be detected with very high signal-to-noise ratio (of order hundreds after convolving with a model template). By tracking these many wave cycle at such high signal-to-noise, it will be possible to determine binary parameters with exquisite accuracy. Figure 1 (taken from Ref. [19]) shows how well masses can be determined — for this example, both black hole masses are determined with relative errors that are typically much less than 1%. This is because the masses strongly influence the rate at which the orbital phase evolves. In addition, the spins of the binary’s member holes impact the waveform through precessional effects; by modeling those precessions, the black hole spins can be determined. Figure 2 (also taken from Ref. [19]) shows that, for the same distribution of binaries as in Fig. 1, the Kerr spin parameter $\chi = |S|/M^2$ can...
FIGURE 2. Same as Fig. 1, but now showing errors in of the Kerr spin parameter $\chi = |S|/M^2$. For most events in the distribution, the spins are determined to within $\delta \chi = 0.01$. The spin of the larger black hole (solid line) is typically better measured than that of the smaller hole (dashed line), since the magnitude of its spin vector $|S|$ is larger and has a more important impact on the waveform.

typically be determined to within 0.01. LISA will be a tool for the precision measurement of the cosmic growth of black hole masses and spins. This will be a beautiful probe of the coevolution of black holes and galaxies.

“Extrinsic” source parameters — those having to do with its location and orientation relative to the detector — are also determined by measuring its GWs. What is particularly interesting is that GWs directly encode the luminosity distance to the source. In essence, inspiralling binaries are a standard candle in which the standardization is provided by general relativity. As shown in Ref. [19], source distances can be measured to within a few percent, even for fairly high redshift.

The position of a binary on the sky is not determined terribly well by astronomy standards, but is determined well enough that planned large scale surveys may be able to search for “electromagnetic counterparts” to merging binary black holes. At low redshift, sources are typically localized to an ellipse whose major axis is roughly $10 - a few 10s$ of arcminutes, and whose minor axis is typically a factor of $2 - 5$ smaller. At higher redshift, this “pixel” bloats by a factor of a few in each direction, so that a field of a few square degrees may need to be searched [19]. Hopefully, some kind of unique signature will be associated with the merger event, such as the onset of AGN or quasar activity. If this is the case, then the chance of associating a merger event with an electromagnetic counterpart is quite high [20].

Massive binary black hole inspirals provide perhaps the most broad-reaching, exciting LISA science. These sources can be measured to large redshifts, and can be measured well enough to determine the parameters of the black holes with high precision. Measuring these waves will make LISA an instrument for untangling the growth of black holes and tracking cosmic structure evolution.
**REALLY COMPLICATED CHIRPING SOURCES: EXTREME MASS RATIO INSPIRALS**

Extreme mass ratio inspirals (EMRIs) are binary systems in which one member is a massive, galactic core black hole, and the other is a stellar mass compact object. Such binaries are created by scattering processes in the cores of galaxies — the smaller member of the binary is scattered by multibody interactions onto a highly eccentric, strong-field orbit of the massive black hole. Current estimates suggest that hundreds of these sources may produce GWs accessible to LISA each year. Further details can be found in Clovis Hopman’s contribution to these proceedings [21].

Once scattered onto this orbit, the EMRI becomes a strong GW radiator. The small body executes $10^4 - 10^5$ orbits as it spirals in. These orbits are quite ornate. A strong-field black hole orbit has three orbital periods: $T_\phi$, describing motion in the axial direction; $T_\theta$, describing poloidal oscillations; and $T_r$, describing radial oscillations. These periods coincide in the Newtonian limit (mean orbit radius $r \gg M$), but differ quite strongly in the strong field, with $T_r > T_\theta > T_\phi$. The mismatch between for example $T_r$ and $T_\phi$ means that an orbiting body can appear to “whirl” around the black hole many times as it “zooms” in from apoapsis to periapsis and back. This “zoom-whirl” character leaves a distinctive stamp on the binary’s GWs [22].

The character of these modulations and the manner in which they evolve as the small body spirals in encodes a great deal of information about the binary’s spacetime. If it is possible to coherently track the inspiral over those $10^4 - 10^5$ orbits, it should be possible to determine the character of the spacetime with very high precision. It should be emphasized that building models capable of tracking the inspiral is quite a difficult task; see the author’s other contribution to these proceedings for further discussion [23].

A relatively prosaic application of these measurements will be to determine the mass and spins of quiescent black holes. Using a simplified model for EMRI waves, Barack and Cutler have shown that LISA will be able to determine massive black hole masses and spins with an accuracy of about a part in $10^4$ [24]. Such measurements will make it possible to precisely survey the distribution of black hole spins and masses in the nearby universe. More fundamentally, one can analyze these sources without assuming a priori that the massive object is described by the Kerr spacetime. The measurement then tests how well the Kerr hypothesis fits the data. More discussion of how this test can be done is given in the author’s other contribution to these proceedings [23].

**SUMMARY**

LISA is an astrophysics mission that will make possible unprecedented precision measurements of dynamic, strong-gravity phenomena. Perhaps most exciting will be the ability to track the mergers of massive black holes, precisely weighing their masses and spins. Such measurements will directly probe the early growth of black holes, opening a new

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\[ \text{In contrast to massive black hole coalescences, most EMRI events will be relatively close by} \quad z \lesssim 1. \]
window onto structure growth.

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