Critical Mass Transfer in Double-Degenerate Type Ia Supernovae

Rebecca G. Martin\textsuperscript{1⋆}, Christopher A. Tout\textsuperscript{1,2} and Pierre Lesaffre\textsuperscript{1}

\textsuperscript{1}University of Cambridge, Institute of Astronomy, The Observatories, Madingley Road, Cambridge CB3 0HA
\textsuperscript{2}Centre for Planetary and Stellar Astrophysics, School of Mathematics, Monash University, Clayton, Victoria 3800, Australia

ABSTRACT

Doubly-degenerate binary systems consisting of two white dwarfs both composed of carbon and oxygen and close enough that mass is transferred from the less massive to the more massive are possible progenitors of type Ia supernovae. If the mass transfer rate is slow enough that the accreting white dwarf can reach a mass of 1.38 M\textsubscript{⊙} then it can ignite carbon degenerately at its centre. This can lead to a thermonuclear runaway and thence a supernova explosion. However if the accretion rate is too high the outer layers of the white dwarf heat up too much and carbon ignites there non-degenerately. A series of mild carbon flashes can then propagate inwards and convert the carbon to neon relatively gently. There is no thermonuclear runaway and no supernova. We examine the critical accretion rate at which ignition switches from the centre to the surface for a variety of white dwarfs and find it to be about two fifths of the Eddington rate. In a real binary star the mass transfer rate falls off as mass transfer proceeds and the system widens. Even if the initial transfer rate is high enough for carbon to ignite at the outer edge, if such rapid accretion were to persist, we find that it can extinguish if the rate drops sufficiently quickly. The interior of the white dwarf remains carbon rich and, if sufficient mass can still be transferred from the companion, it can eventually ignite degenerately at the centre. The primary white dwarf must be about 1.1 M\textsubscript{⊙} or above and the companion about 0.3 M\textsubscript{⊙}. Though white dwarfs of such low mass are expected to be pure helium we note that a star of initial mass 2.5 M\textsubscript{⊙} has a CO core of about 0.3 M\textsubscript{⊙} when it begins to ascend the asymptotic giant branch. Alternatively if the accretion rate can be limited to a maximum of 0.46 of the Eddington rate then a 1.1 M\textsubscript{⊙} white dwarf accretes sufficiently slowly to explode from a companion white dwarf of any large enough mass.

Key words: white dwarfs, supernovae: general, stars: evolution, binaries: close.

1 INTRODUCTION

Type I supernovae (SNe Ia) are the brightest objects in normal galaxies. They appear to be standard candles and hence are useful cosmological measures of the Universe (Baade 1935). They are also a major source of iron (Tout et al. 2001) which tends to be trapped in the neutron star remnants of other supernovae. However, their progenitors remain uncertain (see for example Tout 2005) and this leaves their standard nature questionable particularly in the light of the chemical evolution of the Universe.

Type I supernovae have no hydrogen lines and type Ia are further distinguished by their prominent silicon lines. They are almost certainly exploding carbon and oxygen (CO) white dwarfs. Their available nuclear energy exceeds their binding energy so that the whole star can be destroyed in a thermonuclear runaway. Most of the material approaches nuclear statistical equilibrium and about 0.6 M\textsubscript{⊙} of 56Ni is expelled (Röpke & Hillebrandt 2005). The radioactive decay of 56Ni to 56Fe via 56Co powers the supernova and the decay signature has been clearly observed (Branch 1995). It is fairly certain that SNe Ia are detonated by mass accretion on to the CO white dwarf. As the mass of a cold white dwarf increases towards the Chandrasekhar mass, the gravitational collapse heats the material and fusion ignites in the degenerate core. Typically CO white dwarfs explode at about 1.38 M\textsubscript{⊙}.

Accreting white dwarfs have been known for some time as the engines of cataclysmic variables, the source of novae and dwarf novae (Warner 1995), and so were the first candidates to be considered. However if the accreting material is hydrogen-rich, accumulation of a layer of only $10^{-5} - 10^{-3}$ M\textsubscript{⊙} of cold material leads to degenerate igni-
tion of hydrogen burning sufficiently violent to eject most, if not all of or more than, the accreted layer in the well known nova outbursts of cataclysmic variables. The white dwarf mass does not significantly increase and ignition of its interior is usually avoided. However if the accretion rate is high, $M > 10^{-7} M_\odot \text{yr}^{-1}$, compressional heating of the surface layers raises the degeneracy and hydrogen can burn relatively gently as it is accreted, bypassing nova explosions \cite{Paczynski_1978}, allowing the white dwarf mass to grow. Though, if it is not much larger than this, $M > 3 \times 10^{-7} M_\odot \text{yr}^{-1}$, hydrogen cannot burn fast enough so that accreted material builds up a giant-like envelope around the core and burning shell which eventually leads to more drastic interaction with the companion and probably the end of the mass transfer episode. Rates in the narrow range for steady burning are found only when the companion is in the short-lived phase of thermal-timescale expansion as it evolves from the end of the main sequence to the base of the giant branch. Super-soft X-ray sources \cite{kahabka_1997} are probably in such a state but cannot be expected to remain in it for very long (a SNe Ia rate of only about one millionth of the observed rate form by this channel in the population synthesis of Hurley, Tout & Pols \cite{Hurley_2002}) and white dwarf masses almost never increase sufficiently to explode as SNe Ia. If He-rich material is accreted instead about 0.1 $M_\odot$ of degenerate material can accumulate before ignition \cite{Nomoto_1982} and an accretion rate above $3 \times 10^{-8} M_\odot \text{yr}^{-1}$ can lead to steady burning \cite{kawai_1987}.

The currently popular model overcomes these problems by postulating that, when the mass-transfer rate exceeds that allowed for steady burning, only just the right fraction of the mass transferred is actually accreted by the white dwarf. A viable mechanism for this is a strong wind from the accretion disc that expels material from the system before it reaches the white dwarf \cite{Hachisu_Kato_Nomoto_1996}. Alternatively the white dwarf might indeed swell up to giant dimensions but the resulting common-envelope evolution could be sufficiently efficient that the small amount of excess material can be ejected without the cores spiralling in. This is quite consistent with the findings of Nelemans & Tout \cite{Nelemans_2002} and Nelemans et al. \cite{Nelemans_2001} that such efficiency necessary for at least one phase of common-envelope evolution in the formation of close double white dwarf systems. Helium-accreting CO white dwarfs were once popular when it was thought that a thermonuclear runaway in the accreted helium layer could set off the CO core in an edge-lit detonation \cite{Woosley_1994}. However 2-D models indicate that central ignition is unlikely and light curves and spectra do not fit well so they are no longer considered viable progenitors \cite{Branch_1995}.

In this work we consider the alternative that many complications can be avoided altogether if the white dwarf can accrete material of similar composition to itself, carbon and oxygen. This can be achieved in a double degenerate model in which two CO white dwarfs merge \cite{webbink_1984}. Both form as the cores of asymptotic giant branch stars and, after one or two phases of mass transfer, one or both of which involves substantial orbital shrinkage through common-envelope evolution, they are brought close enough together that angular momentum losses owing to gravitational radiation lead to Roche lobe overflow. A major difficulty with this model lies in the fact that high accretion rates also heat the white dwarf. This heating is due mainly to the gravitational compression of the white-dwarf material as the mass increases. The actual accretion luminosity liberated by the material falling down the potential well on to the white-dwarf surface has a negligible heating effect in comparison and is in any case mostly liberated in an accretion disc or boundary layer and radiated away. A white dwarf can lose heat by conduction to the surface and radiation or by interior neutrino loss processes such as electron-positron pair production and annihilation. The material at the surface undergoes the most rapid compression as it becomes degenerate. At slow rates energy released in these surface layers can escape fast enough but at high rates the surface layers reach temperatures required for fusion to begin. True ignition occurs when energy production by carbon fusion exceeds that lost in neutrinos, a situation that is rapidly followed by the onset of convection but at velocities that are not sufficient to carry away igniting packets before thermonuclear runaway ensues. Nomoto & Iben \cite{Nomoto_1983} calculated that carbon ignites near the surface at constant accretion rates in excess of about one fifth of the Eddington rate (see section 4). Ignition at the surface raises the degeneracy so that burning of carbon to neon and sodium proceeds relatively gently. Surface burning triggers the gentle ignition of a deeper shell and carbon is thus burnt successively throughout the white dwarf. Once the central carbon has been exhausted the core can’t ignite until hot enough for neon burning by photo-disintegration or oxygen fusion but by this point electron capture by magnesium has taken the collapse beyond the point at which the available nuclear energy can explode the white dwarf. It simply collapses quietly to a neutron star releasing energy in neutrinos. Thus for a SN Ia a thermonuclear runaway must begin in the core before carbon ignites sufficiently at the surface.

Because white dwarfs expand as they lose mass, unless the Roche lobe of the mass donating companion expands even faster, the very process of mass transfer causes the mass-losing (lower mass) white dwarf to overfill its Roche lobe yet more and mass transfer accelerates to dynamical rates. Only when the mass ratio $q = M_\text{loser}/M_\text{accretor} < 0.628$ can this positive feedback be avoided. It is not known what the final outcome of dynamical mass transfer is but it is likely that much of the loser is lost to the interstellar medium and that the remainder is accreted at very high rates. This already excludes the majority (about four-fifths in the population synthesis models of Hurley, Tout & Pols \cite{Hurley_2002}) of merging CO white dwarf pairs which tend to have component masses between 0.5 and 1.1 $M_\odot$. In the remaining cases, where dynamical mass transfer is avoided, the initial rate is still in excess of Nomoto & Iben \cite{Nomoto_1983}’s limit and so merging CO white dwarfs are not currently favoured SNe Ia progenitors.

Various mechanisms might force merging white dwarfs to spin rapidly. Piersanti et al. \cite{Piersanti_2003} modelled the effects of rapid spin on carbon ignition in CO white dwarfs. They found that compressional heating was not much affected but that the rate of diffusion of heat to the centre was slowed. This generally reduces the maximum rate of accretion that allows central carbon ignition. On the other hand spinning near breakout prevents accretion but also distorts the white dwarf so that it spins down by its own gravitational wave emission, while accreting just enough mass to
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2 STELLAR EVOLUTION MODELS

We use the Cambridge STARS code to make, evolve and accrete on to white dwarfs. The STARS code is the most recent version of the Eggleton evolution program (Eggleton 1971, 1972, 1973). The equation of state, which includes molecular hydrogen, pressure ionization and Coulomb interactions is discussed by Pols et al. (1995). We take the initial composition of the star to be uniform with a hydrogen abundance \( X = 0.7 \), helium \( Y = 0.28 \) and metals \( Z = 0.02 \). The metal mixture is according to the meteoritic abundances determined by Anders & Grevesse (1989). Only nuclear burning that affects the structure is included, hydrogen burning by the p-p chain and the CNO cycles, helium burning by the triple-a reaction and reactions with\(^{12}\)C,\(^{14}\)N and\(^{16}\)O and carbon burning via\(^{12}\)C +\(^{12}\)C only. Other isotopes and reactions are not explicitly modelled. The reaction rates are taken from Caughlan & Fowler (1985) and opacity tables are from Iglesias, Rogers & Wilson (1992) and Alexander & Ferguson (1994). An Eddington approximation (Woolley & Stibbs 1953) is used for the surface boundary conditions at an optical depth of \( \tau = 2/3 \).

We initially construct white dwarf models from those of asymptotic giant branch stars with appropriate core masses by stripping them of their envelopes at very high mass-loss rates to mimic the binary formation process. This leaves CO cores which can cool, without further mass loss, to become white dwarfs. We obtain white dwarfs of different temperatures by varying the cooling time and we can form white dwarfs with different compositions by evolving to the AGB from main-sequence stars of different initial mass (see tables 1 and 2).

Once we have a white dwarf we can accrete mass back on to it. However the mass we accrete now is of the same composition as the surface of the white dwarf. There are then no problems with numerical diffusive mixing. Because our evolution code is hydrostatic, to ensure numerical convergence, we must prime our white dwarfs for rapid accretion by increasing the rate from zero to the desired amount in several short steps. These steps are short enough that this is effectively instantaneously compared with the thermal timescale in the regions of interest.

When carbon ignites in a thermonuclear runaway it manifests itself by a sudden rise in carbon burning luminosity followed by breakdown of the evolution code. Figure 4 shows how the internal run of temperature and density from the cool surface to hot relatively isothermal interior changes during accretion at rates just above and just below the critical rate at which ignition moves from the centre to the surface. To be sure of what has happened, we determine a carbon ignition curve as a function of temperature and density by equating the carbon burning energy production with that lost in neutrinos – see Figures 2 and 3. Above and to the right of this line we have the right conditions for carbon burning by the triple-\( \alpha \) reaction and reactions with\(^{12}\)C,\(^{14}\)N and\(^{16}\)O and carbon burning via\(^{12}\)C +\(^{12}\)C only. Other isotopes and reactions are not explicitly modelled. The reaction rates are taken from Caughlan & Fowler (1985) and opacity tables are from Iglesias, Rogers & Wilson (1992) and Alexander & Ferguson (1994). An Eddington approximation (Woolley & Stibbs 1953) is used for the surface boundary conditions at an optical depth of \( \tau = 2/3 \).

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Figure 2. The line crossing both axes in the carbon ignition curve on which energy generation by carbon burning equals the energy loss in neutrinos. The other lines are the run of temperature and density through the white dwarf, at late times, increasingly later from bottom to top, beginning with the dotted line of figure 1. In this case these cross the ignition curve at the centre and a thermonuclear runaway can begin there under degenerate conditions. The accretion rate is \(2 \times 10^{-6} \, M_{\odot} \, \text{yr}^{-1}\).

Figure 3. As figure 2 but beginning with the dashed line of figure 1. Ignition occurs close to the surface and carbon ignites under much less degenerate conditions. The accretion rate is \(3 \times 10^{-6} \, M_{\odot} \, \text{yr}^{-1}\). Grey filled circles indicate the path of the mass shell where carbon first ignites at mass \(1.354 \, M_{\odot}\) just \(0.008 \, M_{\odot}\) below the surface.

3 CONSTANT ACCRETION RATES

Nomoto & Iben (1985) considered accretion at constant rates. For a direct comparison we first examine a range of initial conditions and constant accretion rates (as in figures 2 and 3) to estimate the critical rate at which ignition switches from centre to surface and how it depends on initial temperature and central carbon to oxygen ratio. Table 1 shows the critical accretion rates at which the ignition switches from the centre to the outside of the star for different white dwarf models. We see that, in general, the hotter the initial central temperature, the higher the critical accretion rate. The hotter central temperature makes it easier for carbon to ignite at the centre before the outside. For white dwarfs of similar temperature, the higher the initial mass, the lower the critical accretion rate. Also, the higher the carbon to oxygen ratio, the lower the critical accretion rate.

4 EDDINGTON ACCRETION RATES

Because it has some physical meaning and could place a real limit on the accretion rate, it is also interesting to investigate the response of a white dwarf to accretion that varies at a fixed fraction of the Eddington rate \(\dot{M}_{\text{EDD}}\). Nomoto & Iben (1985) found the critical rate to be about one fifth of \(\dot{M}_{\text{EDD}}\).

In the case of one dimensional spherical accretion the kinetic energy of the accreting material is liberated in a shock at the surface of the white dwarf and, if radiative transfer is the only mechanism for the energy to escape, the consequent radiation pressure slows the infalling material. A maximum accretion rate, \(\dot{M}_{\text{EDD}}\), is reached when the radiation force on particles balances gravity at the surface of the white dwarf so

\[
\dot{M}_{\text{EDD}} = \frac{4\pi c}{\kappa} R_1,
\]

where \(R_1\) is the radius of the accreting white dwarf, \(c\) is the speed of light and \(\kappa\) is the opacity of the accreting material which is dominated by electron scattering. This is found by equating the Eddington luminosity and the accretion luminosity, that released by material falling from infinity to the ignition to run away. Below it, any carbon burning ceases if the accretion is turned off. On the same axes in figures 2 and 3 we plot the internal temperature against density for the models leading up to ignition. If a line from our internal model crosses the ignition curve then we know that the carbon in the star has ignited at that particular point in the model. There is then a clear distinction between those models that ignite at the centre (figure 2) and those which ignite in the outer layers just below the surface (figure 3). There is not a smooth transition of the ignition point from the centre to the surface because, in both cases, the intervening parts of the white dwarf lie well away from the ignition curve. The transition is sudden and so the critical rate at which it occurs is well determined for each of the cases we consider. We note further that this ignition in the outer layers differs from the concept of an off-centre but still relatively central ignition in the degenerate interior which could still lead to thermonuclear runaway and supernovae.
Table 1. Critical constant accretion rates. Column 1 is the original zero-age main-sequence mass of the star from which the white dwarf was made. Column 2 is the white dwarf mass. Its central carbon and oxygen abundances are given in columns 3 and 4. The initial central temperature \( T_c \) is in column 5. Column 6 is \( \dot{M}_{\text{crit}} \) the critical accretion rate at which the ignition switches from the centre to the outside of the white dwarf.

<table>
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<tr>
<th>ZAMS Mass/( M_\odot )</th>
<th>WD Mass/( M_\odot )</th>
<th>Carbon</th>
<th>Oxygen</th>
<th>( \log_{10}(T_c/K) )</th>
<th>( \dot{M}<em>{\text{crit}}/10^{-6}M</em>\odot \text{yr}^{-1} )</th>
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</table>

In all cases accretion at \( \dot{M}_{\text{EDD}} \) is sufficiently fast to cause ignition in the outer layers and so we also consider accretion at a fraction of \( \dot{M}_{\text{EDD}} \). In spherically symmetric accretion \( \dot{M}_{\text{EDD}} \) represents the absolute limit to the rate at which mass can be accreted. Han and Webbink (1999) make a more careful analysis, taking into account the potential difference between the inner Lagrangian point and the white dwarf surface and so permit higher accretion rates. If accretion is not spherically symmetric so that material can accrete along some directions and radiation escape in others even higher accretion rates could be possible. On the other hand, if the opacity \( \kappa \) increases rapidly as material is driven off, accretion may be limited to a lower rate in a similar manner to the limit on hydrogen accretion in the model of Hachisu, Kato & Nomoto (1996).

The radius \( R_1 \) depends on the mass \( M_1 \) of the accreting white dwarf according to the formula of Nauenberg (1972) as

\[
R_1 = 0.0115 \sqrt{\left( \frac{M_{\text{ch}}}{M_1} \right)^{2/3} - \left( \frac{M_1}{M_{\text{ch}}} \right)^{2/3}} R_\odot
\]  

where \( M_{\text{ch}} = 1.44 M_\odot \) is the Chandrasekhar mass. For a selection of white dwarfs table 2 lists the critical fraction of \( \dot{M}_{\text{EDD}} \) at which we move from central to outside ignition and Figure 3 shows how this critical rate varies with time for each case. We see that we can accrete up to about two fifths of \( \dot{M}_{\text{EDD}} \) and still find central ignition. This is double the rate estimated by Nomoto & Iben (1985). The higher the initial mass of the white dwarf the higher the critical fraction of the Eddington rate. We do not profess to have identified an actual physical mechanism that would limit the accretion in this way but rather wish to indicate what would be necessary.

5 ROCHE LOBE OVERFLOW IN WHITE DWARF SYSTEMS

The actual rate of mass transfer between double white dwarfs often exceeds \( \dot{M}_{\text{EDD}} \) but it is not constant. Rather it falls off as mass is transferred and the system widens in response to the expansion of the radius of the mass-losing (donor) white dwarf. The separation is controlled by two competing processes. Gravitational radiation acts to shrink the orbit while mass transfer from the less to more massive star acts to widen it. For stable mass transfer the Roche lobe must expand faster than the white dwarf, which grows in response to mass loss. The Roche lobe itself grows because the system expands as mass in transferred to the more massive companion. The combination of these changes determines...
Figure 4. The variation with time of accretion rate at the critical fractions of the Eddington rate for the four cases listed in Table 2. The rate is proportional to the radius of the white dwarf and so falls off as mass increases. The top line is for accretion on to the white dwarf of initial mass 1 \( M_\odot \) and the bottom of 1.3 \( M_\odot \).

The mass-transfer rate necessary to maintain \( R_2 = R_{L2} \) and the so the system widens. As it widens the system experiences weaker gravitational-radiation braking and the mass-transfer rate falls.

We set up a simple model that allows us to determine this mass-transfer rate and limit it to a maximum fraction \( f \) of \( M_{\text{edd}} \). In this way we can find systems in which the primary white dwarf ignites at the centre even though the initial mass transfer rate might be considerably higher than the critical rates calculated earlier.

In a binary system the Roche lobe radius \( R_{Li} \) is the radius of a sphere which encloses the same volume as that enclosed by the last stable equipotential surface around star \( i \) (\( i = 1,2 \)) as given by (Eggleton 1983),

\[
\frac{R_{Li}}{a} = \frac{0.49 q_i^{2/3}}{0.6 q_i^{2/3} + \log(1 + q_i^{1/3})},
\]

where \( q_i \) is the mass ratio \( M_i/M_{3-i} \) and \( a \) is the separation of the two stars. This formula is accurate to 1 percent for all \( q \).

In a binary system of two white dwarfs, when the less massive, of mass \( M_2 \) and radius \( R_2 \), overfills its Roche lobe it loses mass, some of which is accreted by its companion, of mass \( M_1 \) and radius \( R_1 \). For stable mass transfer the rate \( (\dot{M}_1 - \dot{M}_2) \) is determined by the angular momentum loss from the orbit because \( M_2 \) adjusts to keep \( R_2 \approx R_{L2} \) and \( R_2 \approx R_{L1} \). We model the mass-loss rate from the donor white dwarf by

\[
\dot{M}_2 = -M_0 e^{-f R},
\]

where \( \Delta R \) is the amount by which the white dwarf overfills its Roche lobe,

\[
\Delta R = R_2 - R_{L2}
\]

and \( H \) is the pressure scale height (Ritter 1982, Martin & Tout 2002). This formula takes account of the fact that stars have a thin atmosphere above their photosphere and Roche lobe overflow begins while the photosphere itself is still below the inner Lagrangian point. It also allows us to make a smooth transition to stable mass transfer as the white dwarfs are driven together. Because it is a steep function of \( \Delta R \) we have \( R_2 \approx R_{L2} \) once mass transfer has begun. The constant \( M_0 \) is the mass-transfer rate when the donor star fills its Roche lobe exactly so that \( \Delta R = 0 \). We use \( M_0 = 3 \times 10^{-6} \). This is approximately the critical mass transfer rate where ignition moves from the centre to the outside of the WD. If this rate is exceeded then we slightly over estimate the mass transfer rate because the stars are slightly closer than they would be if \( \Delta R = 0 \) and \( |\dot{J}_1| \) (see equation 3 below) is slightly larger. If \( M < M_0 \) we slightly underestimate the rate. Figure 5 below shows that this rate is typical of the systems in which we are interested.

We take \( H \) to be a constant fraction of one thousandth of the radius of the losing white dwarf. Such a small fraction is typical of white dwarfs so that \( R_2 \) is indeed very close to \( R_{L2} \). Any variation in \( H \) is unimportant because we are only interested here in the equilibrium mass-transfer rate.

We limit the mass accretion rate to \( f M_{\text{edd}} \), where \( 0 < f < \infty \) is a constant. A factor \( f \gg 1 \) allows all the mass transferred to be accreted \( (\dot{M}_1 = -\dot{M}_2) \) while \( f = 1 \) applies the Eddington limit. If the mass-transfer rate is less than \( f M_{\text{edd}} \) all of the mass lost by the donor is accreted on to the companion. In general the accretion rate on to star 1 is given by

\[
\dot{M}_1 = \min(f M_{\text{edd}}, -\dot{M}_2).
\]

When the limit is active there is mass loss from the system because star 2 is losing mass more quickly than star 1 is gaining it and \( \dot{M} = \dot{M}_1 + \dot{M}_2 \leq 0 \). This mass is blown away from the binary in a wind which carries angular momentum with it. We assume it has the specific angular momentum of the accreting white dwarf whence it is blown off and so the rate of loss of angular momentum is

<table>
<thead>
<tr>
<th>ZAMS Mass/( M_\odot )</th>
<th>WD Mass/( M_\odot )</th>
<th>Carbon</th>
<th>Oxygen</th>
<th>log((T_c/K))</th>
<th>Critical fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1.000</td>
<td>0.21744</td>
<td>0.75822</td>
<td>7.1625</td>
<td>0.31</td>
</tr>
<tr>
<td>2</td>
<td>1.100</td>
<td>0.14196</td>
<td>0.83390</td>
<td>7.2220</td>
<td>0.36</td>
</tr>
<tr>
<td>2</td>
<td>1.200</td>
<td>0.14193</td>
<td>0.83393</td>
<td>7.3406</td>
<td>0.38</td>
</tr>
<tr>
<td>5</td>
<td>1.300</td>
<td>0.21743</td>
<td>0.75823</td>
<td>7.5907</td>
<td>0.39</td>
</tr>
</tbody>
</table>

Table 2. Headings as for table 1 except the last column which is the fraction of the Eddington accretion rate for which the carbon first ignites in the outside of the white dwarf.
The limit on accretion rate has a stabilizing effect on dwarfs would have begun dynamically unstable mass transfer. The limit on accretion rate, the binary of $1 M_\odot$ limited by $M_{\text{Edd}}$ and this limit, which depends only on the mass of the gainer, is effective over the initial straighter section. Both tracks end when the accretor would have reached 1.38$M_\odot$. Note that no account of carbon burning has yet been taken.

\[ J_{\text{wind}} = \dot{M} \left( \frac{M_2}{M} \right)^2 \Omega, \]  

where $\Omega$ is the orbital angular velocity. For these close systems loss of angular momentum owing to gravitational radiation is the mechanism driving the evolution. For two point masses in a circular orbit the rate of change of angular momentum $\dot{J}_\text{gr}$ is given by [Landau & Lifshitz 1951].

\[ \frac{\dot{J}_\text{gr}}{J} = - \frac{32G^3}{5\pi^5} \frac{M_1 M_2 (M_1 + M_2)}{a^4}. \]  

We assume that there are no other angular momentum losses from the system so that the total angular momentum loss is

\[ \dot{J}_\text{total} = \dot{J}_\text{wind} + \dot{J}_\text{gr}. \]  

We can solve the three differential equations and with a Runge-Kutta method and hence find the accretion rate. Figure 5 shows an example of accretion rates on to the same mass of accreting white dwarf from two different mass companions. The rates both begin at the Eddington rate and fall off in such a way that the accretion rates on to a $1 M_\odot$ white dwarf would have begun dynamically unstable mass transfer. The limit on accretion rate has a stabilizing effect on the mass transfer.

We now consider detailed evolution of the accretion at variable rates according to the binary model. We begin with a separation such that the lower-mass white dwarf is about to fill its Roche lobe. The initial white dwarfs are those described in section 3 and used in section 4. In order for accretion to switch on smoothly by equation 4 we must prime our white dwarf model by rapidly building up the accretion rate over several short time steps to about $10^{-6} M_\odot \text{yr}^{-1}$ as described in section 2. This means our surface layers are slightly hotter than they should be at the onset of accretion but the effect is soon swamped and could in any case only reduce the likelihood of central ignition.

\[ f = \frac{\dot{M}_1}{\dot{M}_2} - \frac{\dot{M}_2}{\dot{M}_1}, \]  

\[ M_1 + M_2 = \frac{9}{8} M_\odot, \]  

\[ M_1 = 1 M_\odot, M_2 = 2 M_\odot. \]  

The cross-hatched region is always ruled out. To the right of this the models in the single shaded regions show central carbon ignition while those in the unshaded regions ignite at the surface. For a white dwarf of mass $1 M_\odot$ we find central ignition for any mass companion with an accretion rate 46 per cent of the Eddington limit. Similarly for a $1.2 M_\odot$ accreting white dwarf we can succeed with any mass companion with accretion limited to 40 per cent of the Eddington rate. The curves rise steeply as $f$ approaches 1 and the mass transfer can be fully conservative for lower-mass companions and still lead to central ignition.

Figure 5. Accretion rates on to a $1.2 M_\odot$ white dwarf. The solid line is from a white dwarf companion of mass $0.3 M_\odot$ and the dotted line of mass $0.9 M_\odot$. Both are limited by $M_{\text{Edd}}$ and this limit, which depends only on the mass of the gainer, is effective over the initial straighter section. Both tracks end when the accretor would have reached 1.38$M_\odot$. Note that no account of carbon burning has yet been taken.

**Figure 6.** Outcome for a $1.1 M_\odot$ white dwarf accreting from a companion white dwarf of initial mass $M_2$ with an upper limit to the accretion rate of $fM_{\text{Edd}}$. In the single shaded area the white dwarf ignites at the centre. The blank area it accretes too fast and ignite at the outside. In the cross-hatched area the initial total mass $M = M_1 + M_2 \leq 1.38 M_\odot$ so that central ignition cannot be reached.

### 6 CARBON IGNITION IN BINARY STAR MODELS

We now consider detailed evolution of the accretion at variable rates according to the binary model. We begin with a separation such that the lower-mass white dwarf is about to fill its Roche lobe. The initial white dwarfs are those described in section 3 and used in section 4. In order for accretion to switch on smoothly by equation 4 we must prime our white dwarf model by rapidly building up the accretion rate over several short time steps to about $10^{-6} M_\odot \text{yr}^{-1}$ as described in section 2. This means our surface layers are slightly hotter than they should be at the onset of accretion but the effect is soon swamped and could in any case only reduce the likelihood of central ignition.

Figures 5 and 6 illustrate these, our most realistic models, for initial mass accretors of 1.1 and 1.2$M_\odot$ of the compositions given in table 2. If the total mass of the system is less than 1.38$M_\odot$ central ignition can’t occur so the cross-hatched region is always ruled out. To the right of this the models in the single shaded regions show central carbon ignition while those in the unshaded regions ignite at the surface. For a white dwarf of mass $1.1 M_\odot$ we find central ignition for any mass companion with an accretion rate 46 per cent of the Eddington limit. Similarly for a $1.2 M_\odot$ accreting white dwarf we can succeed with any mass companion with accretion limited to 40 per cent of the Eddington rate. The curves rise steeply as $f$ approaches 1 and the mass transfer can be fully conservative for lower-mass companions and still lead to central ignition.

Figure 5 shows the highest mass of the donating white dwarf for which we find central ignition for a given mass of...
the accreting white dwarf with fully conservative evolution ($f \gg 1$). Above this mass the mass-transfer rate is too high and we find carbon ignition at the outside and consequently no supernova.

7 CONCLUSIONS

Using detailed models of accreting CO white dwarfs we have critically investigated what rates allow central degenerate carbon ignition as opposed to ignition near the surface. To compare with earlier work we first considered accretion at constant rates and then at fractions of the Eddington-limited rate. Our results are qualitatively similar to those of Nomoto & Iben (1985) but we find central ignition at rates up to twice what they did or two-fifths of $\dot{M}_{\text{edd}}$. This can be attributed to their relatively course grid and updated stellar physics. We have also considered real double white dwarf binaries with mass transfer by Roche lobe overflow. Variation in the initial temperature and composition of the accreting white dwarf have only a small effect. We note that with fully conservative mass transfer a 1.1 $M_\odot$ CO white dwarf with a companion CO white dwarf of 0.3 $M_\odot$ can reach the conditions required for a SN Ia even though the accretion rate initially rises to a maximum of almost $1.7 \times 10^{-5} M_\odot \text{yr}^{-1}$ and carbon burning in the non-degenerate layers reaches a luminosity of nearly 21 $L_\odot$. The mass transfer rate and hence the luminosity fall off sufficiently quickly with the mass-transfer rate that the burning is extinguished before it runs away and burns the entire white dwarf.

Though standard single star evolution with reasonable mass loss only produces CO white dwarfs from about 0.55 $M_\odot$ to 1.2 $M_\odot$ we note that a star of initial mass 2.5 $M_\odot$ has a CO core of 0.28 $M_\odot$ at the start of the asymptotic giant branch, immediately after convective core helium burning. We also recall that one or two phases of common envelope evolution are required to bring the two white dwarfs close enough for gravitational radiation to operate and that it would be easy to strip such a star’s hydrogen rich envelope to leave a naked helium star of about 0.55 $M_\odot$. Such a star is very small and would not undergo further interaction until gravitational radiation sets in but it is also very luminous (about 200 $L_\odot$) and maintains this luminosity for a long time (about $5 \times 10^7$ yr owing to helium burning. This could well drive off the helium rich material, in a similar way to Wolf-Rayet stars, and expose the required low-mass CO core. Hamann, Koesterke & Wessolowski (1992) suggest a mass-loss rate from naked helium stars of as much as $10^{-11.95} (L/L_\odot)^{1.5}$ which is considerably more than would be needed here.

Undoubtedly the parameter space leading to these conditions is small. In the population synthesis of Hurley, Tout & Pols (2002) only one in five hundred of the merging CO white dwarfs would actually make it to central carbon ignition under these conditions and thence a supernova rate of only about a thousandth of that observed. However much of the physics, including that of common envelope evolution and mass loss from naked helium stars is highly uncertain, as are the initial mass functions, mass ratio distributions and separation distributions of binary stars so any population synthesis model is still very uncertain. We note also that interactions between stars in dense stellar environments can significantly increase the number of merging white dwarfs. Shara & Hurley (2002) find a factor of fifteen increase in open clusters.

On the other hand if the mass accretion rate can be limited to 0.46 of $\dot{M}_{\text{edd}}$ then a 1.1 $M_\odot$ white dwarf accreting from a companion of any mass above 0.28 $M_\odot$ can make...
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...it to a supernova. This would indeed be the case if a disc wind similar to that postulated by [Hachisu, Kato & Nomoto (1996)] operated in the double-degenerate case in a similar way to the single-degenerate model.

We conclude that double CO white dwarfs remain as viable progenitors of SNe Ia as any others currently proposed but are unlikely to be responsible for the majority of SNe Ia. In particular, if the accretion luminosity were somehow limited to less than 46 per cent of the Eddington rate in the early stages, then stable mass transfer could account for an important fraction of observed SNe Ia.

ACKNOWLEDGEMENTS

We are grateful to John Lattanzio and Monash University for their hospitality during which a substantial amount of this work was carried out. RGM thanks Churchill College for travel grants to do this work. CAT thanks Churchill College for a fellowship. We thank the referee for carefully reading the manuscript and pointing many opportunities for improvement.

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