Hibernation Revived by Weak Magnetic Braking

Rebecca G. Martin and Christopher A. Tout *
University of Cambridge, Institute of Astronomy, The Observatories, Madingley Road, Cambridge CB3 0HA

ABSTRACT

Cataclysmic variables undergo periodic nova explosions during which a finite mass of material is expelled on a short timescale. The system widens and, as a result, the mass-transfer rate drops. This state of hibernation may account for the variety of cataclysmic variable types observed in systems of similar mass and period. In the light of recent changes to the theory of nova ignition and magnetic braking we investigate whether hibernation remains a viable mechanism for creating cataclysmic variable diversity. We model the ratio of time spent as dwarf novae (DNes) to nova-like systems (NLs). Above a critical mass-transfer rate the system is NL and below it a DN. The dominant loss of angular momentum is by magnetic braking but the rate is uncertain. It is also uncertain what fraction of the mass accreted is expelled during the novae. We compare the models of the ratios against the period of the system for different magnetic braking rates and different ejected masses with the ratio of the number of observed NLs to DNe. We deduce that a rate of angular momentum loss a factor of ten smaller than that traditionally assumed is necessary if hibernation is to account for the observed ratios.

Key words: stars: dwarf novae, novae, cataclysmic variables

1 INTRODUCTION

Cataclysmic variables (Warner 1995) are interacting close binary stars in which a white dwarf is accreting from a companion. In the vast majority of cases the companion is a low-mass main-sequence star or red dwarf and the periods are measured in hours. The red dwarf is transferring hydrogen-rich material to the white dwarf by Roche lobe overflow at a rate, $10^{-11}$ to $10^{-8} M_\odot \, \text{yr}^{-1}$. As the hydrogen settles on the surface, the base of the layer becomes degenerate and heats. When enough, $10^{-5}$ to $10^{-3} M_\odot$, has accumulated it ignites. The degenerate conditions lead to a thermonuclear runaway and a nova explosion. The system brightens by ten or so magnitudes and most of the layer if not all of it together with some of the underlying white dwarf material is blown off.

Though some of this material may fall back on to the white dwarf and some of it may be accreted back by the red dwarf, a significant fraction escapes carrying with it only the intrinsic angular momentum of the white dwarf. The system expands such that $a \propto M = \text{const}$, where $a$ is the separation and $M$ the total mass which has just decreased. If the red dwarf had no atmosphere, that is a distinct boundary just filling its Roche lobe, it would no longer do so and mass transfer would cease. This is the idea of hibernation first documented by Shara et al. (1986). At that time they were motivated by the apparent lack of cataclysmic variables in the solar neighbourhood when compared with theoretical estimates and the frequency of classical novae in M31. The complete detachment during hibernation would render the majority of cataclysmic variables unobservable at any given time.

Because the red dwarf has a slightly extended atmosphere it actually must overfill its Roche lobe in order to maintain an equilibrium mass-transfer rate. The ratio of the atmosphere scale height to the radius of the star is comparable to the ratio of the mass lost from the system to its total mass, so the change in separation is not sufficient to shut off mass transfer altogether but does reduce the rate significantly. This change in rate could be responsible for the diversity of cataclysmic variable types at a given period: dwarf novae that undergo disc instability outbursts require a lower mean mass-transfer rate than nova-like systems that do not suffer the disc instability (Faulkner, Lin & Papaloizou 1983). However the idea lost favour when it was realised that the nova ejecta would temporarily form a common envelope around the system and consequential angular momentum loss would actually cause lower-period systems to shrink (Livio, Gorarie, & Ritter 1991). Recent work by Nelemans et al. (2000) indicated that the common-envelope phase is very poorly understood and that in some cases systems with tenuous envelopes can actually expand. Since we
wish to compare the simplest case we neglect any common-envelope phase in this work.

A detailed analysis of the period distribution was made by Shafter (1992). He emphasized that the lack of dwarf novae between 3 and 4 hr is difficult to reproduce theoretically with any reasonable magnetic braking law. He speculated that the solution might lie in a better understanding of any of additional angular momentum loss alongside mass transfer, weak but unnoticed magnetic fields that disrupt the inner parts of the disc, correlation between white dwarf mass and orbital period or hibernation. Among these he deduced that hibernation might prove the most promising and it is this that we explore here in more detail.

We investigate the hibernation scenario for cataclysmic variable diversity in the light of two recent changes to the accepted model of cataclysmic variables. These are

(i) the relation between the mass of hydrogen that must be accreted to ignite $M_{\text{ign}}$, and the secular accretion rate


(ii) the possibility that magnetic braking, that drives the evolution of cataclysmic variables, could be much weaker than previously thought (Andronov, Pinsonneault & Sills 2003).

We consider hibernation in its simplest form to deduce what combinations of $J$, $M_{\text{ej}}$ and $M_{\text{d}}$, the mass actually ejected in the nova, could be consistent with the observed variety of cataclysmic variable types.

2 SIMPLE ANALYTIC MODEL

We first consider a simple analytical model to get some idea of what fraction of the time is spent in hibernation and then consider the simplest numerical model that includes the pertinent effects of the atmosphere and variations of $M_{\text{d}}$ and $J$.

2.1 Binary Systems

The cataclysmic binaries we consider consist of a white dwarf, of mass $M_1$, and a low-mass main-sequence star, of mass $M_2$ and radius $R_2$.

The hydrostatic and thermal equilibrium radius of the main-sequence star can be approximated by,

$$\frac{R_2}{L_2} = \frac{M_2}{M_0}.$$  \hfill (1)

The Roche lobe radius $R_{\text{L}}$, is the radius of a sphere which encloses the same volume as that enclosed by the last stable equipotential surface around star $i$. The Roche lobe radius of star 2, for a restricted range of mass ratio $q = M_2/M_1$, is

$$\frac{R_{\text{L}2}}{a} = \frac{2}{3^{1/3}} \frac{M_2}{M_1}^{1/3}, \quad 0 < q < 0.8,$$  \hfill (2)

where $M = M_1 + M_2$ is the total mass of the system (Paczynski 1971). The simplicity of this formula makes it useful for analytic work within the restricted but useful range of $q$. When $R_2 > R_{\text{L}2}$ mass is transferred from the red dwarf to the white dwarf.

Kepler’s third law, that relates the angular velocity $\Omega$ of each star and the binary system to the total mass $M$ and the separation of the two stars $a$, is

$$\Omega^2 = \frac{GM}{a^3}.$$  \hfill (3)

When the red dwarf is filling its Roche lobe, $R_2 = R_{\text{L}2}$, we combine equations (1), (2) and (3) to find

$$P = \frac{9\pi}{(2\pi)^{1/2}} \left( \frac{R_\odot}{M_\odot} \right)^{3/2} M_2,$$  \hfill (4)

where $P = 2\pi/\Omega$ is the orbital period. So in this model the orbital period depends only on the mass of the main-sequence star when $R_2 = R_{\text{L}2}$.

We assume the spin angular momentum of the stars is negligible compared to the orbital angular momentum. Hence the total angular momentum of the system is

$$J = M_1 a_1^2 \Omega + M_2 a_2^2 \Omega,$$  \hfill (5)

where $a_1$ and $a_2$ are the distances from star 1 and star 2 respectively to the centre of mass of the system. We can rearrange

$$a_1 M_1 = a_2 M_2 \quad \text{and} \quad a = a_1 + a_2$$  \hfill (6)

to find

$$a_1 = \frac{M_2}{M} a,$$  \hfill (7)

and similarly for $a_2$. Substituting into equation (5) we have

$$J = \frac{M_1 M_2}{M} a^2 \Omega.$$  \hfill (8)

Using Kepler’s law, equation (3), and equation (8) we can rearrange to get the separation

$$a = \frac{J^2 M}{GM_1^2 M_2^2}$$

and the angular velocity

$$\Omega = \frac{GM_1 M_2}{J^3 M}$$  \hfill (10)

in terms of the three independent variables, $J$, $M_1$ and $M_2$.

2.2 Nova Eruption

When a critical mass $\Delta m$, which we examine later, has accumulated on the surface of the white dwarf star, thermonuclear reactions ignite in the degenerate material. Suppose the entire layer of $\Delta m$ is expelled from the system in a nova explosion. Since $\Delta M_1 = \Delta M = -\Delta m$, the angular momentum of the system changes by

$$\Delta J = \Delta M_1 a_1^2 \Omega = -\Delta m \left( \frac{a M_2}{M} \right)^2 \Omega.$$  \hfill (11)

We want to find an expression for the change in separation during the nova eruption. Equations (8) and (11) give

$$\frac{\Delta J}{J} = -\frac{\Delta m M_2}{MM_1}.$$  \hfill (12)

Differentiating equation (8) gives

$$\frac{\Delta J}{J} = \frac{\Delta M_1}{M_1} + \frac{\Delta M_2}{M_2} - \frac{\Delta M}{M} + \frac{2 \Delta a}{a} + \frac{\Delta \Omega}{\Omega},$$  \hfill (13)

but, because $M_2$ is constant during the eruption, $\Delta M_2 = 0$. Differentiating equation (3) we get,
After the eruption the cycle is repeated. We have many eruptions. If the mass of the white dwarf decreases, mass transfer doesn’t end at \( q \) and steady mass transfer occurs. For a red dwarf to be in a state of hibernation.

\[ \Delta \Omega = \frac{2 \Delta M}{M} - 3 \Delta a \]  \hspace{1cm} (14)

We can combine equations (12), (13) and (14) to get the familiar result

\[ \frac{\Delta a}{a} = \frac{\Delta m}{M}. \]  \hspace{1cm} (15)

Because of the mass loss from the system, the separation of the stars increases during the explosion. Equations (12) and (15) ensure that \( a(M_1 + M_2) = \text{const} \) during the eruption. The Roche lobe radius must also increase as the separation increases. Differentiating equation (2) and substituting (15) we find

\[ \frac{\Delta R_{L,2}}{R_{L,2}} = \frac{4 \Delta m}{3M}. \]  \hspace{1cm} (16)

After the eruption \( R_2 < R_{L,2} \) and so the mass-transfer rate decreases sharply, switches off in this model. In this low mass-transfer rate the cataclysmic binary is said to be in a state of hibernation.

2.3 Conservative Case

We now consider a fully conservative case for which \( M = \text{const} \) and \( J = 0 \) so

\[ \frac{\dot{J}}{J} = \frac{\dot{M}_1}{M_1} + \frac{\dot{M}_2}{M_2} + \frac{\dot{a}}{2a} = 0. \]  \hspace{1cm} (17)

Since \( \dot{M}_1 = -\dot{M}_2 \) we can substitute to find

\[ \frac{\dot{a}}{a} = 2(q - 1) \frac{\dot{M}_2}{M_2}. \]  \hspace{1cm} (18)

and

\[ \frac{\dot{P}}{P} = 3(q - 1) \frac{\dot{M}_2}{M_2}. \]  \hspace{1cm} (19)

where \( q = M_2/M_1 \). These two equations show that, as the mass of the red dwarf decreases, \( a \) and \( P \) only decrease if \( q > 1 \). But because the Roche lobe continues to shrink as \( q \) decreases, mass transfer doesn’t end at \( q = 1 \). Differentiating equation (3) we get

\[ \frac{R_{L,2}}{R_{L,2}} = \frac{\dot{a}}{a} + \frac{\dot{M}_2}{M_2}, \]  \hspace{1cm} (20)

and combining with equation (18) we find

\[ \frac{R_{L,2}}{R_{L,2}} = \frac{\dot{M}_2}{M_2} \left( 2q - \frac{5}{3} \right). \]  \hspace{1cm} (21)

This shows that the minimum Roche lobe radius is when \( q = 5/6 \). But for most observed cataclysmic binaries \( q < 5/6 \) and steady mass transfer occurs. For a red dwarf to be in steady contact with its Roche lobe there must be something causing loss of orbital angular momentum.

So after an eruption the stars gradually get closer together because of the loss of angular momentum which brings the main-sequence star back into contact with its Roche lobe. The mass-transfer rate increases once more and the cycle is repeated. We have many eruptions.

We want to find the range of mass ratios \( q \) for which mass transfer is stable. Since we have, by differentiating equation (1)

\[ \frac{R_2}{R_{L,2}} = \frac{\dot{M}_2}{M_2} \]  \hspace{1cm} (22)

and in order to be stable we need

\[ \frac{R_{L,2} - a}{R_{L,2}} < \frac{R_2 - a}{R_2}. \]  \hspace{1cm} (23)

by comparing equations (21), (22) and (23) we therefore must have

\[ q < 4/3. \]  \hspace{1cm} (24)

2.4 Analytic Ratio of Times

After a nova eruption the main-sequence star no longer fills its Roche lobe because the orbit has expanded. Here we assume that the rate of angular momentum loss \( |\dot{J}| \) remains constant until the next nova explosion.

When the system is detached (when there is no mass transfer) \( M_1 = M_2 = M = 0 \). So from one nova explosion, the time until mass transfer begins again is the time it takes until \( R_{L,2} \) shrinks back to \( R_2 \). From equations (13), (14) and (16),

\[ \frac{\Delta J}{J} = \frac{|\dot{J}| t_d}{J} = \frac{\Delta a}{2a} = \frac{\Delta R_{L,2}}{2R_{L,2}} = \frac{2 \Delta m}{3M}. \]  \hspace{1cm} (25)

So the time the system spends detached is,

\[ t_d = \frac{J}{2 \Delta m} \frac{2 \Delta m}{3M}. \]  \hspace{1cm} (26)

When the system is semi-detached mass is transferred from the red dwarf to the white dwarf, \( M_1 = -M_2 \) and \( M = 0 \). The Roche lobe of the red dwarf is filled so that \( R_2 = R_{L,2} \) and \( R_1 = R_{L,1} \) so

\[ \frac{\Delta R_{L,2}}{R_{L,2}} = \frac{\Delta a}{a} + \frac{\Delta M_2}{3M_2} = \frac{\Delta R_1}{R_1} = \frac{\Delta M_2}{M_2}. \]  \hspace{1cm} (27)

Rearranging this we find

\[ \frac{\Delta a}{a} = \frac{2 \Delta M_2}{3M_2}. \]  \hspace{1cm} (28)

Substituting (28) into (13) we get

\[ \frac{\Delta J}{J} = \frac{|\dot{J}| t_s}{J} = -\frac{\Delta m}{M} + \frac{\Delta m}{M_2} + \frac{\Delta m}{3M_2}. \]  \hspace{1cm} (29)

The time spent semi-detached is

\[ t_s = \frac{J}{|\dot{J}|} \frac{4 - 3q}{3M_2}. \]  \hspace{1cm} (30)

This model predicts the ratio of time spent detached to the time spent semi-detached to be,

\[ \frac{t_s}{t_d} = \frac{(1 + q)(4 - 3q)}{2q}. \]  \hspace{1cm} (31)

We plot it in Fig. 10 for later comparison with our numerical model and note that this is consistent with the result of Livio & Shara (1987) that hibernation should be deepest when \( q \) is close to one.

3 OBSERVED CATAclySMIC VARIABLES

Classical novae are a class of novae and cataclysmic variables that have had only a single observed eruption. Their brightness ranges from 6 to greater than 19 magnitudes. Dwarf novae (DNe) are a class of cataclysmic variables...
Figure 1. The solid line is $t_\text{e}/t_\text{d}$ against the mass ratio $q$ (equation 31). We compare with the dashed line which is the ratio of the time spent as a nova-like to dwarf nova against $q$ with a constant period $P = 3\, \text{hr}$. $J$ factor of 0.1 and $M_{\text{ign}} = M_{\text{ej}}$ from our numerical model (section 5).

that have had several observed outbursts. These outbursts however range in brightness from 2 to 5 magnitudes. They are associated with a disc instability. Nova-like systems (NLs) include all non-outbursting cataclysmic variables. Their spectra indicate that they could be novae in a pre- or post-eruption stage.

The nature of the cataclysmic variable, DN or NL, should depend only on the mass-transfer rate which itself should only depend on the system masses and period. However at any given period some of each type are seen.

4 NUMERICAL MODEL

We now estimate the radius and luminosity of the main-sequence star with the formulae of Tout et al. (1996) and use the more accurate formula for the Roche lobe radius for star i, Eggleton (1983),

$$ R_{\text{L}i} = \frac{0.49 q_i^{2/3}}{0.6 q_i^{2/3} + \log(1 + q_i^{1/3})}, $$

(32)

where $q_i$ is the mass ratio $M_i/M_{3-i}$. This formula is valid for all $q_i$.

4.1 Ignition Mass

By considering cold white dwarf models Fujimoto (1982) calculated the critical pressure at the base of the hydrogen rich layer for ignition. Based on this Truran & Livio (1986) derived

$$ M_{\text{ign}} = \frac{9.42 \times 10^{-4} (R_1/10^3 R_\odot)^4}{M_1/M_\odot}, $$

(33)

where $M_{\text{ign}}$ is the mass needed for ignition on the white dwarf. We make a linear fit to their figure 8 and find

$$ \frac{M_{\text{ign}}}{M_\odot} = \alpha \left( \frac{\langle M \rangle}{10^{-6} M_\odot \text{yr}^{-1}} \right)^{-\beta}, $$

(34)

where

$$ \alpha = 3.2 \times 10^{-4} \left( \frac{M_1}{M_\odot} \right)^{1.231} $$

(35)

and

$$ \beta = 0.534 \left( \frac{M_1}{M_\odot} \right)^{0.605}. $$

(36)

When $M_{\text{ign}}$ has accumulated on the white dwarf we have a nova explosion. We consider three different models, two in which the mass ejected is constant in all the explosions,

(i) $M_{\text{ej}} = \text{const} = 1 \times 10^{-3} M_\odot$

(ii) $M_{\text{ej}} = \text{const} = 5 \times 10^{-4} M_\odot$

and one in which the mass ejected in the nova equals the amount of mass needed for ignition,

(iii) $M_{\text{ej}} = M_{\text{ign}}$.

4.2 Rate of loss of Mass

When the main-sequence star is overfilling its Roche lobe mass is transferred on to the white dwarf so that

$$ M_1 = -M_2. $$

(37)

We approximate the atmosphere of the red dwarf by a simple isothermal model. The rate of mass loss from the red dwarf is then given by,

$$ M_2 = -M_2 \left( \frac{GM_2}{R_2^2} \right)^{1/2} e^{-\frac{\Delta R}{H}}, $$

(38)

where $\Delta R$ is the amount by which the star overfills its Roche lobe,

$$ \Delta R = R_2 - R_{L2}. $$

(39)

When the red dwarf underfills its Roche lobe, $R_2 < R_{L2}, \Delta R$ is negative and the mass-transfer rate is very small. When the star overfills its Roche lobe, $R_2 > R_{L2}, \Delta R > 0$ and the mass transfer rate increases.

The scale height $H$ of the unperturbed isothermal atmosphere of the main-sequence star is

$$ H = -\frac{\rho(r)}{d\rho/dr}, $$

(40)

so we need to find $\rho(r)$, the density at position $r$. The equations of an isothermal atmosphere are

$$ \frac{dP(r)}{dr} = -\rho(r) g \quad \text{and} \quad P(r) = \frac{\gamma T_e}{\mu} \rho(r), $$

(41)

where $P(r)$ is the pressure, $\gamma$ is the gas constant, $T_e$ is the effective temperature at the surface of the main-sequence star and $\mu$ is the mean molecular weight. The atmosphere is
a mixture of hydrogen, helium and metals in various states of ionization and in molecules. So we use \( \mu \) equal to 1. The gravitational field strength at the surface of the red dwarf is,
\[
g = \frac{G M_2}{R_2^2}. \tag{42}
\]
The surface temperature of the main-sequence star is found by Stefan’s law,
\[
T_e = \left( \frac{L_2}{4\pi\sigma R_2^2} \right)^{1/4}.
\tag{43}
\]
Integrating the equations in (11) for \( \rho \),
\[
\rho = \rho_0 e^{-\frac{\mu g}{\mu g}}, \tag{44}
\]
where \( \rho_0 \) is some constant. Hence we find the scale height,
\[
H = \frac{\sqrt{T_e}}{\mu g}. \tag{45}
\]
The factor in front of the exponential in equation (38) is an estimated maximum mass-transfer rate equal to the mass of the secondary star divided by its dynamical timescale. We have verified that varying it by large factors has almost no effect on our results because \( \Delta R \) simply adjusts to compensate when mass transfer is stable.

### 4.3 Rate of Loss of Angular Momentum

Angular momentum is continually lost from the system. For the closest systems gravitational radiation is the most important cause of mass transfer. The rate of loss of angular momentum in gravitational radiation from two point masses in a circular orbit is \( \text{(Landau & Lifshitz, 1951)} \)
\[
\frac{J_{GR}}{J} = \frac{32 G^3 M_1 M_2 (M_1 + M_2)}{5c^5 a^4}.
\tag{46}
\]
In wider systems however magnetic braking is the most important mechanism. The rate of change of angular momentum owing to magnetic braking is uncertain. Based on the work of \( \text{Skumanich (1972), Rappaport, Verbunt & Joss (1983)} \) postulated a rate of the form,
\[
\frac{J_{MB}}{J} = -5.83 \times 10^{-16} (R_2/R_\odot)^3 \Omega \, \frac{\sqrt{T_e}}{\mu g} M_\odot R_\odot^2 \, \text{yr}^{-2}. \tag{47}
\]
However recent work by \( \text{Andronov, Pinsoneault & Sills (2003)} \) based on the spins of stars in clusters suggests that this rate is orders of magnitude too high. But such a low rate would not be able to drive the interrupted magnetic braking model for the period gap between two and three hours in cataclysmic variables. In this model a system evolves to lower period as \( M_2 \) drops at such a high rate that the red dwarf is out of thermal equilibrium. At a period of three hours, a mass of about 0.3\( M_\odot \), magnetic braking abruptly falls off, perhaps because the red dwarf has become fully convective. The mass-transfer rate drops and the red dwarf shrinks back to its equilibrium radius and must wait for some much weaker angular momentum loss mechanism to shrink the orbit until \( R_{L2} \) is again reduced to \( R_2 \). With the high rate the period gap is very well reproduced. With the low rate the stars are not far enough from thermal equilibrium and, even if magnetic braking were interrupted, the gap would be much too narrow. If such a low rate proves to be correct an alternative reason for the period gap must be found.

### 4.4 Irradiation

The nova decay time is 50 to 100 yr \( \text{(Livio & Shara, 1987)} \). So although irradiation of the red dwarf from the white dwarf may increase the mass-transfer rate initially, it will only be for a small fraction of the inter nova period of several thousand years. Though we find increased mass transfer while the white dwarf is hot the time for which it is negligible and we ignore it. We note that the irradiation cycles discussed by \( \text{Binning & Ritter (2004)} \) operate on a much longer timescale and would have to survive several episodes of hibernation.

### 5 Ratio of Number of Observed NLS to DNE

We select the systems from Ritter’s Catalogue \( \text{(Ritter & Kolb, 2003)} \) which are classified as nova-likes or as dwarf novae and divide them into period bins with at least five systems of each kind in each bin. We then divide these numbers to find the observed ratio of NLS to DNe. Though there are obvious selection effects, such as the fact that dwarf novae, owing to their variability, are far more easily spotted, we can hope to reproduce trends in this ratio.

Fig. 2 shows that the ratio peaks at a period of just over 3 hr then decays as the period increases further. After a period of 5 hr there are very few observed binaries but the data do suggest that the ratio increases again slightly with increasing period. We use the period of the peak ratio 3.5 hr and the fact that it has dropped by a factor of twelve by 4.4 hr to test our models. We do not expect to reproduce the exact height of the peak because different selection effects apply to DNe and NLS. We exclude identified polars and intermediate polars but we cannot be certain that others, particularly IPs, do not contaminate the sample of NLS. However their contribution is small, certainly when compared with other probable selection effects, at the relatively high periods we consider here.

#### 5.1 Critical Mass-Transfer Rate

We calculate the times spent as a dwarf nova \( t_{DN} \) and as a nova-like \( t_{NL} \) by considering the mass-transfer rate. Fig. 3 shows an example of the mass-loss rate of the red dwarf for three nova explosions.

There is a critical mass-transfer rate above and below which the system appears to be in two different states. This critical rate of mass loss from the main-sequence star is
\[
\dot{M}_{2\text{crit}} = -8.08 \times 10^{15} \left( \frac{\alpha_H}{0.3} \right)^{0.3} (1 + q)^{7/8} \left( \frac{P}{\text{yr}} \right)^{7/4} \, \text{g s}^{-1}. \tag{48}
\]
\( \text{(Faulkner, Lin & Papaloizou, 1983)} \), where \( \alpha_H = 0.1 \). Dwarf novae lie below the critical value. They have \( \langle M_2 \rangle < M_{2\text{crit}} \) whereas nova-likes all lie above the critical \( M_{2\text{crit}} \) relationship. Just after a nova explosion, the mass-loss rate from the red dwarf is very small. The time from the explosion until the mass-loss rate has increased up to the critical rate is...
t_{DN}. The binary then appears to be a nova-like from this time until the next eruption, t_{NL}. We calculate the ratio of these times.

\[ \text{Ratio} = \frac{t_{NL}}{t_{DN}} \]  

(49)

If the critical mass-transfer rate is higher than the rate of the model for the whole cycle, then we are predicting that there are no nova-likes, only dwarf novae. So we have \( t_{NL} = 0 \) and \( t_{DN} = \) time between nova eruptions. Similarly, if the critical rate is lower than the mass-transfer rate, we have only nova-likes, \( t_{DN} = 0 \) and \( t_{NL} = \) time between nova eruptions.

In Fig. 1 we compare this ratio for \( P = 3 \) hr against \( q \) with the simple analytic calculation in section 2.4. At a period of 3 hr there are both NLs and DNe at approximately all mass ratios and we see that both models are indeed very similar.

5.2 Model Ratio Averaging

In order to compare our models with the observed data we need to plot the predicted ratio against the period but the ratio also depends on \( q \). For a given period, we average the ratio over varying \( q \). Averaging by adding the total times, \( t_{NL} \) and \( t_{DN} \), over a flat \( q \) distribution and then finding the ratio of these total times is our preferred method. It automatically takes account of system lifetimes. The longer the time, the more weight it is given in the averaging. In the observed data, the longer a system is in a state, the more likely it is to be observed. We do not try to model the birth rate of various systems with population synthesis because we consider this to be too uncertain. We therefore implicitly assume all cataclysmic variables equally likely to form.

6 RESULTS

To examine the effects of a much lower angular momentum loss rate we multiply the formula for magnetic braking (equation 47) by a constant factor. We try combinations of different \( \dot{J} \) factors and \( M_{ej} \) and compare the ratios in Table 1.

In Table 1 we list various models and properties which can be compared with the observations. As we have discussed, selection effects make any truly quantitative comparison difficult but we have identified key features in Fig. 2 that ought to be reproduced. We also concentrate on periods above the gap because we are interested in what can be said about magnetic braking irrespective of whether or not it is interrupted. Then we see a peak in the ratio of NLs to DNe just above 3 hr with a rapid decrease to higher periods.

Table 1. Models with different factor in \( \dot{J} \) and \( M_{ej} \)

<table>
<thead>
<tr>
<th>( \dot{J} ) Factor</th>
<th>( M_{ej}/M_{\odot} )</th>
<th>Peak</th>
<th>Peak Ratio</th>
<th>Ratio after peak at ( P = 4.4 ) hr</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>( M_{\odot} )</td>
<td>4.5-7.25</td>
<td>inf</td>
<td>-</td>
</tr>
<tr>
<td>1.0</td>
<td>( 5 \times 10^{-4} )</td>
<td>6.0</td>
<td>inf</td>
<td>-</td>
</tr>
<tr>
<td>1.0</td>
<td>( 1 \times 10^{-3} )</td>
<td>3.5</td>
<td>4.7</td>
<td>4.3</td>
</tr>
<tr>
<td>0.5</td>
<td>( M_{\odot} )</td>
<td>5.75</td>
<td>13.5</td>
<td>-</td>
</tr>
<tr>
<td>0.2</td>
<td>( M_{\odot} )</td>
<td>3.25</td>
<td>3.1</td>
<td>2.0</td>
</tr>
<tr>
<td>0.15</td>
<td>( M_{\odot} )</td>
<td>3.25</td>
<td>2.8</td>
<td>1.0</td>
</tr>
<tr>
<td>0.1</td>
<td>( M_{\odot} )</td>
<td>3.0</td>
<td>2.4</td>
<td>0.2</td>
</tr>
<tr>
<td>0.1</td>
<td>( 5 \times 10^{-4} )</td>
<td>&lt;2.0</td>
<td>&gt;4.0</td>
<td>0.2</td>
</tr>
<tr>
<td>0.1</td>
<td>( 1 \times 10^{-3} )</td>
<td>&lt;2.0</td>
<td>&gt;3.4</td>
<td>0.2</td>
</tr>
<tr>
<td>0.1</td>
<td>( M'_{\odot} )</td>
<td>&lt;3</td>
<td>&gt;1.2</td>
<td>0.6</td>
</tr>
</tbody>
</table>
By 4.4 hr it should have fallen by a factor of 12. A record of ‘inf’ for the peak ratio indicates no dwarf novae.

With the full [Rappaport, Verbunt & Joss 1983] braking rate (J factor = 1) we cannot reproduce these features. When just the accreted mass is blown off there are no DNe at all between about 4.5 and 7.25 hr. If a constant $5 \times 10^{-4} M_\odot$ mass is ejected then there is a peak in the ratio at 6 hr. Only when $M_{ej} = 10^{-3} M_\odot$ do we find a peak at about 3.5 hr but then the ratio has hardly fallen by 4.4 hr. In any case such a large ejected mass is unrealistic. [Kolb et al. 2001] investigated the effects of hibernation at these standard rates of magnetic braking. They deduced that $M_{ej} > 2 \times 10^{-4} M_\odot$ would be necessary to account for the observed spread in $M$ at a given period above 3 hr.

It is not until we have reduced the braking rate by a factor of ten that we can reproduce the observed features. There is little variation between the three different $M_{ej}$, all of which show a peak in the ratio at about 3 hr with a drop by a factor of five or more by 4.4 hr. We also made one model with the old cold white dwarf ignition mass $M_{ej} = M_{ign}$. Here, the ratio is only 1.2 at $P = 3$ hr predicting about equal numbers of NLs and DNe. Because DNe should be easier to observe than NLs this is inconsistent with the data too.

Fig. 4 shows the ratios as a function of period for $J$ factors of 0.1, 0.15 and 0.2. All have the correct shape but as the $J$ factor increases the curves flatten, no longer reproducing the drop of a factor of 12 by 4.4 hr.

For a $J$ factor of 0.01, which gives a rate close to that of [Andronov, Pinsonneault & Sills 2003], all of the mass-transfer rates are lower than the critical value and so we see only dwarf novae. The only way this could be consistent with observations would be if another mechanism such as irradiation or magnetic cycles can periodically raise the mass transfer by an order of magnitude. This would then be responsible for the variety of cataclysmic variable types.

7 CONCLUSIONS

For the full [Rappaport, Verbunt & Joss 1983] rate ($J$ factor of 1) only the unrealistically high $M_{ej} = 10^{-3} M_\odot$ follows the observed trend but then the variation of ratio with period is too small.

With $J$ reduced by a factor of ten all the models show the right trends with the ratio rising to a peak at a period of about 3 hr and then falling distinctly by a period of 4.4 hr. We cannot then distinguish between the different mass ejection models but do find a distinct improvement when $M_{ej} = M_{ign}$ when proper account is taken of white dwarf heating by accretion [Townsley & Bildsten 2004]. Hibernation is most effective at reproducing observed trends when our $J$ factor is about 0.1. This is ten times smaller than that proposed by [Rappaport, Verbunt & Joss 1983] and ten times larger than that proposed by [Andronov, Pinsonneault & Sills 2003].

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