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SUPERSYMMETRY, PARTICLE PHYSICS AND GRAVITATION

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ABSTRACT

Supersymmetry leads one to introduce bosonic partners for leptons and quarks, and fermionic partners for the photon and gluons. It also relates intermediate gauge bosons and Higgs bosons.

The couplings of the massless spin- $\frac{1}{2}$ Goldstone fermion, the goldstino, and those of its bosonic partner, are fixed by the mass spectrum. This implies the existence of mass relations. When gravitation is introduced, there is no longer a goldstino, but instead a massive spin- $\frac{3}{2}$ gravitino. Its interactions are practically negligible unless its mass is very small, in which case it interacts like a goldstino. We discuss how to search for the new particles in e^+e^- annihilation, high-energy hadronic reactions and Ψ decays.

Finally, we compare briefly the supersymmetry and grand unification approaches.

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1. INTRODUCTION

Supersymmetry is a beautiful symmetry between bosons and fermions, although there is no evidence of it in Nature¹. This does not mean that supersymmetry is not present, but that it must be well hidden. Supersymmetric theories of weak, electromagnetic and strong interactions involve new particles such as heavy bosonic leptons and quarks, and fermionic partners for the photon and gluons². They have a new, and very rich, phenomenology³⁻⁵. Supersymmetry also provides a natural framework for the introduction of gravity due to its close connection with the geometry of space-time⁶, and a possible link between interactions of gravitational and weak types due to a massive spin-3/2 particle, the gravitino^{4,7}. (For earlier review articles on supersymmetric theories of particles, see Refs. 8-10.)

Searching for supersymmetry leads one to postulate the existence of new particles, many of them being heavy. This is not unreasonable since, for example, the gauge symmetry of weak and electromagnetic interactions involves mass splittings as large as the mass of the intermediate boson W . A very useful guide in the construction of theories of weak and electromagnetic interactions was the similarity between weak and electromagnetic currents. This could be noticed due to the existence of multiplets of light particles such as (proton, neutron) and (neutrino, electron).

We do not have the same chance for supersymmetry, so that the construction of supersymmetric theories of particles cannot rely on similarities between known currents. Although such a construction could seem arbitrary, this is not the case due to the symmetry

itself: supersymmetry is so constraining that, initially, it did not even seem compatible with particle physics. Let us mention some of the problems one has to solve: the identification of multiplets of particles related by supersymmetry; the spontaneous breaking of gauge invariance (the problem is mainly to keep an unbroken $SU(3) \times U(1)$ subgroup when spin-0 fields carry colour and charge) and of supersymmetry; the roles of the fermionic partners of the photon (photino), and of the Goldstone fermion (goldstino). The hardest problem is, in fact, to get a realistic mass spectrum for bosons and fermions, despite the existence of linear relations between the squares of the masses.

In Section 2 we explain why we have to introduce new particles, and present the general features of supersymmetric gauge theories. In Section 3 we discuss spontaneous supersymmetry breaking; we study the couplings of the Goldstone fermion and of its bosonic partner, in connection with the mass spectrum. In Section 4 we examine the phenomenological consequences of these theories. In Section 5 we discuss the interactions of the spin-3/2 gauge particle of supersymmetry, the gravitino; if it is very light it would interact essentially like a massless spin- $\frac{1}{2}$ Goldstone fermion. Finally, we comment in Section 6 on the possible relations between supersymmetry and grand unified theories.

2. PARTICLE CONTENT OF THE MULTIPLETS OF SUPERSYMMETRY

2.1 The supersymmetry algebra

Supersymmetry transformations turn bosons into fermions and conversely¹. They are generated by a (self-conjugate) Majorana spinor Q_α satisfying the algebra

$$[Q_\alpha, P^\mu] = 0 \quad (1)$$

$$\{Q_\alpha, \bar{Q}_\beta\} = -2 \gamma_{\alpha\beta}^\mu P_\mu. \quad (2)$$

The first relation expresses that the result of a supersymmetry transformation does not depend on the space-time point where it is performed. Since they carry half a unit of spin, the supersymmetry generators satisfy anticommutation relations. Due to the antisymmetry and Lorentz covariance, the anticommutator must be a vector or an antisymmetric tensor. In addition it should commute with the generators of translations P^μ . This leads to formula (2). One can also consider extended supersymmetry algebras with several supersymmetry generators.

It follows from the supersymmetry algebra that all particles in a given representation have the same mass. Therefore supersymmetry has to be broken. A spontaneous breaking of global supersymmetry generates a massless spin- $\frac{1}{2}$ Goldstone particle, the goldstino¹¹. What happens to it when supersymmetry is realized locally will be discussed in Section 5.

2.2 Why do we need to introduce new fields?

The first task is to find what are the multiplets of supersymmetry. We shall present here what seems to be the simplest solution, although we do not know which one, if any, is chosen by Nature. In any case we have to introduce new fields.

Consider for example the quarks and gluons. Supersymmetry will transform them into boson and fermion fields, respectively. We could try to associate quarks with gluons. This would require the existence of several supersymmetry generators carrying colour and charge, at least six (a colour triplet and an antitriplet), and in fact more. When acting on ordinary fields these would generate many new fields.

Such a feature seems unavoidable if one wants to implement supersymmetry. The simplest and most economic situation corresponds to the smallest supersymmetry algebra with a single gauge-invariant spinorial generator. This one turns leptons and quarks into spin-0 partners, and the photon and gluons into spin- $\frac{1}{2}$ partners.

We could also consider the possibility that leptons and quarks have spin-1 partners. If we are in the framework of ordinary gauge theories this would lead to enlarge the gauge group, and also requires the existence of spin-0 fields to make the spin-1 fields massive.

In addition, the world may be invariant under an algebra with several supersymmetry generators. Some may be apparent at relatively low energies, and the others (including, possibly, those carrying charge and colour) only at ultra-high energies. In such a framework leptons and quarks would have both spin-1 and spin-0 partners, and the photon and gluons both spin- $\frac{3}{2}$ and spin- $\frac{1}{2}$ partners.

At the present stage we shall mostly consider the smallest algebra and the lower spin partners of the ordinary particles, although many results on symmetry breaking and phenomenological consequences apply to more general situations. This symmetry may become apparent at relatively low energies of the order of the W mass. It allows for the construction of supersymmetric theories of weak, electromagnetic and strong interactions², as we shall see now, leaving for Section 5 the introduction of gravity.

2.3 Spontaneously broken gauge theories

We now assume that spin- $\frac{1}{2}$ leptons and quarks have spin-0 partners. There are two of them for each Dirac fermion, but only one for a two-component fermion. We denote by s_e and t_e the complex spin-0 fields associated with the left-handed and right-handed parts of the electron field, respectively; we use similar notations for other matter fields².

The photon and the SU(3) colour octet of gluons have neutral spin- $\frac{1}{2}$ partners named the photino and the gluinos, respectively. Their Yukawa couplings between the spin- $\frac{1}{2}$ leptons and quarks and their spin-0 partners are fixed by the electrical charge for the photino, and the colour for the gluinos. These interactions are described by Abelian¹² or non-Abelian¹³ supersymmetric gauge theories, with U(1) or SU(3) as the gauge group.

In order to describe weak as well as electromagnetic and strong interactions one needs to consider also supersymmetric spontaneously broken gauge theories. In a supersymmetric theory the Higgs mechanism operates as usual. For each generator of the gauge group which is spontaneously broken, a gauge boson acquires a mass while the corresponding would-be Goldstone boson is eliminated. These bosons belong to two originally different representations of supersymmetry, which join to give a single massive representation. It is the massive gauge multiplet describing a vector, a Dirac spinor and a real scalar, which are degenerate in mass as long as supersymmetry is conserved (see, for example, Ref. 14).

The real scalar in this multiplet is the physical Higgs field, so that supersymmetry gives us a relation between intermediate gauge bosons and Higgs bosons. In particular, it leads to the existence of a charged Higgs boson w_- associated with the intermediate gauge boson W_- and a pair of Dirac fermions L_- and ℓ_- .

In order to write Lagrangian densities which are supersymmetric (up to a four-derivative), it is often convenient to use the superfield formalism. We will not need it here, so we only indicate that a superfield is a function of a space-time point and of an anticommuting Grassman co-ordinate θ ; the coefficients of its expansion with respect to θ are ordinary fields called components of the superfield¹⁵.

The original theory is formulated in terms of massless gauge superfields (with spins 1 and $\frac{1}{2}$) and chiral superfields (with spins $\frac{1}{2}$ and 0). After the spontaneous breaking of gauge invariance, some of the gauge superfields acquire masses while the corresponding chiral superfields are eliminated. The particle content of such a theory is summarized in Table 1.

Table 1

Particle content of a supersymmetric
spontaneously broken gauge theory

Superfields		Spin-1	Spin- $\frac{1}{2}$	Spin-0
Gauge superfields	Massless gauge superfields	Photon Gluons	Photino Gluinos	
	Massive gauge superfields	Intermediate gauge bosons	Heavy fermions	Higgs bosons
Chiral superfields	Matter superfields		Leptons Quarks	Spin-0 leptons Spin-0 quarks

2.4 R-parity and R-invariance

Before discussing the spontaneous breaking of the supersymmetry we present briefly a useful symmetry denoted by R . In order to distinguish ordinary particles from their partners under supersymmetry it is convenient to define an R -parity character: gauge and Higgs bosons, spin- $\frac{1}{2}$ leptons and quarks are R -even, while their superpartners (photino, gluinos, heavy fermions, spin-0 leptons and quarks) are R -odd. The R -even sector of a supersymmetric gauge theory is like an ordinary gauge theory, but with further constraints due to supersymmetry; for example, relations between gauge and Higgs bosons and constraints on the mass spectrum leading to restrictions on the gauge group, as we shall see later.

It follows from R -parity conservation that ordinary particles cannot exchange R -odd particles at lowest order. Spin- $\frac{1}{2}$ leptons and quarks only exchange gauge or Higgs bosons at the classical level. R -odd particles are produced in pairs^{2,8}.

Moreover, it is often possible to define R not only as a discrete symmetry but as a continuous one. R -transformations act as phase transformations on spin-0 fields, phase or γ_5 transformations on spin- $\frac{1}{2}$ fields^{16,17}. R -invariance can be useful to restrict a Lagrangian density beyond the constraints already imposed by

supersymmetry, gauge invariance, and baryon or lepton number conservation. These restrictions are sometimes necessary to get a spontaneous breaking of supersymmetry. In any case, a continuous R-invariance allows for the definition of the spin- $\frac{1}{2}$ partners of the gauge bosons as Dirac fermions with their own conserved quantum number. This fact has important consequences, since it may forbid the appearance of masses for the photino and gluinos, as we shall discuss Subsection 4.8.

3. SPONTANEOUS SUPERSYMMETRY BREAKING, THE GOLDSTINO, AND ITS BOSONIC PARTNER

3.1 The spontaneous breaking of the supersymmetry

The spontaneous breaking of the supersymmetry presents particular features due to the presence of the Hamiltonian in the algebra. Let us illustrate them, for global supersymmetry, by the following argument: the Hamiltonian can be expressed in terms of the components of the supersymmetry generator:

$$H = \frac{1}{4} \sum_{\alpha=1}^4 Q_{\alpha}^2 . \quad (3)$$

Any supersymmetric state has zero energy, certainly the minimum possible one, and therefore is stable.

Despite this fact, spontaneous supersymmetry breaking is possible. One can find models for which no state at all is annihilated by the supersymmetry generator. This is the case, in particular, for the state of minimal energy chosen as the vacuum, so that supersymmetry is spontaneously broken¹¹.

A massless Goldstone particle appears which has the same quantum numbers as the supersymmetry generator, i.e., a massless, neutral, spin- $\frac{1}{2}$, R-odd particle named the goldstino. The masses of bosons and fermions are no longer systematically degenerate, but they turn out to be still related.

3.2 Relations between the goldstino couplings and the boson-fermion mass²-splittings

The boson-fermion pairs which are not directly coupled to the goldstino remain mass-degenerate at lowest order, even if supersymmetry is spontaneously broken. For the others, the couplings of the goldstino are fixed by the mass spectrum, as one sees from the examination of the conserved vector-spinor current of supersymmetry⁷.

In a spontaneously broken globally supersymmetric theory this current reads

$$J_{\alpha}^{\mu} = d \gamma^{\mu} \gamma_5 \lambda_g + J_{2\alpha}^{\mu} + \dots \quad (4)$$

d is a parameter with the dimension of a mass²; λ_g is the Majorana spinor describing the goldstino, and $J_{2\alpha}^{\mu}$, which denotes the bilinear terms in the current, involves all boson-fermion pairs related by supersymmetry.

Let us now express current conservation; it reads:

$$\gamma_5 \not{\partial} \lambda_g = \frac{1}{d} \partial_{\mu} J_{2\alpha}^{\mu} + \dots \quad (5)$$

This gives the equation of motion for the goldstino.

If we are only interested in the trilinear couplings of the goldstino (i.e., to the bilinear terms in its equation of motion) we can use the free equations of motion to compute $\partial_{\mu} J_{2\alpha}^{\mu}$. This involves only the knowledge of the masses and mixing angles. We rewrite (5) as follows:

$$\gamma_5 \not{\partial} \lambda_g = \frac{1}{d} \left(\partial_{\mu} J_{2\alpha}^{\mu} \right)_{\text{free}} + \dots \quad (6)$$

The ... stand for terms which are, at least, trilinear in the fields. There are no such terms in ordinary supersymmetric gauge theories, in which the goldstino has only gauge and Yukawa couplings.

Mass-degenerate boson-fermion pairs do not contribute to (6) so that they have no direct coupling to the goldstino. The couplings of the goldstino are completely determined by the mass spectrum, up to the parameter d which fixes the magnitude of the spontaneous supersymmetry breaking. We find the following relation between the goldstino Yukawa coupling constant $e_g \sqrt{2}$ to a spin-0 - spin- $\frac{1}{2}$ pair, and the mass²-splitting of the pair⁷:

$$e_g = \mp \frac{\Delta m^2}{d} . \quad (7)$$

The sign depends on the chirality of the fermion considered. The same relation is also valid for the coupling of the goldstino to a spin-1 - spin- $\frac{1}{2}$ pair, as one finds out easily from the results of Ref. 16; the sign is fixed by the R-quantum number of the fermion. These formulae can be extended to include the effects of mixing angles.

3.3 Are the photino and the goldstino identical?

Which particle could be the goldstino? Low-energy theorems forbid identifying it with the electron neutrino¹⁸. Identifying the goldstino with one of the neutrinos, or a linear combination of them, could lead to difficulties with the conservation of quantum numbers and universality. But, mostly, the couplings of the goldstino, determined by the mass spectrum, are not like those of a neutrino.

Looking at Table 1 one might think of the photino as the natural candidate for being the goldstino. Identifying the goldstino, coupled to boson-fermion pairs proportionally to their mass²-splittings, with the photino, which is coupled proportionally to the electrical charges, would lead to a proportionality between the mass²-splittings Δm^2 and the electrical charges $Q e$:

$$\Delta m^2 = \mp Q e d . \quad (8)$$

The sign $\mp$ depends on the chirality, or R-quantum number, of the spin- $\frac{1}{2}$ field considered.

Formula (8), together with the conservation of the electrical charge, leads to relations between mass²-splittings. To examine whether these are acceptable or not, we shall consider the most simple situation in which spin- $\frac{1}{2}$ leptons and quarks are associated with spin-0 partners.

We denote by s_e and t_e the spin-0 fields associated with the left-handed and right-handed electron fields, respectively, and use similar notations for quarks. Examples of the mass relations we would get are

$$m^2(s_e) + m^2(t_e) = 2m^2(e) \quad (9)$$

$$[m^2(s_u) - m^2(u)] = -2 [m^2(s_d) - m^2(d)] . \quad (10)$$

Such relations are obviously excluded: any attempt to make one of those spin-0 fields heavy would lead to a negative mass² for another. Translating the latter would lead to a spontaneous breaking of QED or QCD gauge invariances. Therefore if leptons and quarks have spin-0 partners the photino and the goldstino must be different¹⁹.

It is, precisely, the existence of mass relations such as those written above which makes hard the construction of spontaneously broken supersymmetric theories having a realistic mass spectrum.

3.4 The bosonic partner of the goldstino

Formula (7) relates the boson-fermion mass²-splitting to the coupling of the goldstino to the pair. Owing to supersymmetry the latter is related to the coupling of the bosonic partner of the goldstino. Therefore the formula

$$\Delta m^2 = \mp e_g d \quad (11)$$

also expresses the proportionality of the boson-fermion mass²-splitting Δm^2 to the coupling e_g of the bosonic partner of the goldstino to the fermion.

Assuming that the electron has spin-0 partners we get the unacceptable mass relation (9) if the goldstino has the photon for superpartner; but, also, if it is any spin-1 particle vectorially coupled to the electron. (If the goldstino is a mixing of the fermionic partner of a spin-1 particle vectorially coupled to the electron with the one of a spin-0 particle, the only effect of the latter could be to mix the two spin-0 fields s_e and t_e and to raise one mass² while lowering the other, the average mass² remaining equal to the mass² of the electron.) The only way to raise the mass² of both spin-0 partners of the electron (leptons and quarks) is to make use of a spin-1 particle having an axial coupling with the electron (leptons and quarks).

This leads us to consider the weak neutral currents, which do have axial parts. Could the goldstino be a linear combination of the photon γ and the standard neutral gauge boson Z ? Then we would get the mass relations:

$$\Delta m^2 = \pm (x Q_\gamma + y Q_Z) . \quad (12)$$

But we know experimentally the electrical and weak charges for the u and d quarks of both chiralities²⁰, and relations (12) turn out to be unacceptable.

Therefore, if spin-0 leptons and quarks have spin-0 partners getting heavy masses through spontaneous supersymmetry breaking, we need, at least, one neutral spin-1 boson in addition to the bosons of the minimal gauge group $SU(3) \times SU(2) \times U(1)$ ¹⁹.

This is the reason we chose $SU(3) \times SU(2) \times U'(1) \times U''(1)$ as the gauge group for constructing an example of a realistic model². In the simplest case, $SU(2) \times U'(1)$ is the standard gauge group of weak and electromagnetic interactions, and there is in addition a new gauge boson axially coupled to leptons and quarks. The goldstino is a linear combination of the spin- $\frac{1}{2}$ partners of the new

gauge boson and of a spin-0 particle carrying two units of R. The latter does not couple directly to leptons and quarks.

Other gauge groups could be used as well, provided there is one neutral spin-1 boson having couplings of opposite signs with (light) leptons and quarks of opposite chiralities. Its fermionic partner will be the goldstino, up to a mixing angle. Note that the couplings do not need to be exactly axial, but may have vector parts if these are smaller in magnitude than the axial parts.

Note, also, that supersymmetry alone does not require the couplings of the new spin-1 particle to be fixed by a gauge principle. If this is the case the corresponding gauge invariance is spontaneously broken, allowing the new gauge boson, as well as leptons and quarks, to acquire masses.

If the new spin-1 boson is indeed an ordinary gauge particle it is desirable to have a cancellation of anomalies. This requires the introduction of new heavy spin- $\frac{1}{2}$ leptons and quarks, with axial couplings of the opposite sign. They would be heavier than their spin-0 partners while ordinary leptons and quarks are lighter. (Consideration of the $N = 2$ extended supersymmetry algebra would also lead to such a new class of leptons and quarks¹⁰.)

However, the interactions of the new boson are also, in a sense which will be specified in Section 5, gravitational interactions (with, possibly, a very small coupling constant), so that the cancellation of anomalies may not be required as in ordinary gauge theories.

3.5 Mass relations involving Higgs bosons and spin-0 leptons and quarks

Whenever we have a symmetry, or a scheme of symmetry breaking, we get mass relations.

The simplest possibility is that supersymmetry is conserved and bosons and fermions are mass-degenerate. This is obviously excluded. The next possibility is that the goldstino is the spin- $\frac{1}{2}$ partner of the photon, the boson-fermion mass²-splittings being proportional to the electrical charges at the classical level. This is also excluded if leptons and quarks have spin-0 partners.

Therefore we considered a more elaborate scheme of spontaneous supersymmetry breaking, in which the goldstino is partly the spin- $\frac{1}{2}$ partner of a new spin-1 boson.

If this new boson is axially coupled (and if the $R = 2$ spin-0 boson which is also, partly, the partner of the goldstino does not couple directly to leptons and quarks), the masses of the two spin-0 particles s_f and t_f associated with each Dirac lepton or quark f are equal:

$$m^2(s_f) = m^2(t_f) . \quad (13)$$

If the couplings of the new spin-1 boson are fixed by a gauge principle, this gauge invariance leads to relations between the various mass²-splittings. This is the case, in particular, if all the usual leptons and quarks get their masses from couplings with a single pair of Higgs doublets, as in Ref. 2. The simplest set of relations is obtained when the new gauge boson has a universal axial coupling with spin- $\frac{1}{2}$ leptons and quarks. Then we find a universal mass²-splitting Δm^2 between spin-0 and spin- $\frac{1}{2}$ leptons and quarks

$$m^2(s_f) - m^2(f) = m^2(t_f) - m^2(f) = \Delta m^2 . \quad (14)$$

The mass²-splittings in the lepton and quark multiplets may also be related to the mass²-splittings between the intermediate boson W_- and the Higgs boson w_- . In a rather general class of models we have:

$$m^2(W_-) - m^2(w_-) = 2 \left[m^2(s_f) + m^2(t_f) - 2m^2(f) \right] = 4\Delta m^2 . \quad (15)$$

We see, again, that increasing the mass² of a spin-0 boson makes another one decrease. This is now acceptable since the latter can be the one of the charged Higgs boson w_- , which is degenerate with the W_- when supersymmetry is conserved. With this scheme of spontaneous supersymmetry breaking the masses of spin-0 leptons and quarks cannot be too large, since they verify

$$\left(\frac{m^2(s_f) + m^2(t_f)}{2} \right)^{\frac{1}{2}} \lesssim \frac{1}{2} m(W_-) \sim 40 \text{ GeV}/c^2 . \quad (16)$$

The existence of the mass relation (16) which holds the masses of spin-0 leptons and quarks at reasonable values is encouraging in view of the experimental searches for these particles. However, the absence of spin-0 leptons and quarks below $40 \text{ GeV}/c^2$ would not be a signal against supersymmetry, but only against the general association of spin- $\frac{1}{2}$ leptons and quarks with spin-0 partners with the supersymmetry breaking mechanism considered above: (i.e., essentially, the spin-1 partner of the goldstino has its couplings constrained by a gauge principle leading to the mass relation (15)).

4. PHENOMENOLOGICAL CONSEQUENCES OF SUPERSYMMETRIC THEORIES OF PARTICLES

4.1 Foreword

In the supersymmetric theories of particles which have been constructed up to now and appear as conventional gauge theories, leptons and quarks have spin-0 partners while the photon and gluons have spin- $\frac{1}{2}$ partners. We shall discuss the experimental consequences of supersymmetry mostly in that framework, but many of the features described can be extended to partners with different spins.

Whether such phenomena can be observed or not depends on how much the supersymmetry is broken, i.e. on quantities which are not yet fixed by the theory:

i) the order of magnitude for the mass²-splittings Δm^2 inside the multiplets of supersymmetry; is it $\lesssim (m_W/2)^2$, as suggested in Subsection 3.5, or on the contrary much larger, and possibly related to the Planck mass $\sim 10^{19} \text{ GeV}/c^2$? In the latter case the chances of finding direct experimental evidence for supersymmetry would be very slim;

ii) the strength e_g of the coupling of the goldstino to boson-fermion pairs; is it of the order of the electromagnetic or weak gauge coupling constants, or on the contrary much smaller, maybe some extremely small number related to the strength of gravitational interactions? (See also Section 5.)

iii) the masses that the fermionic partners of the gluons (and photon) might acquire at the quantum level (see Subsection 4.8).

4.2 The R-even sector

The R-even sector of a supersymmetric gauge theory is similar to any ordinary gauge theory, but with extra constraints due to supersymmetry.

One is the association between intermediate gauge bosons and physical Higgs bosons, together with heavy fermions, in massive gauge multiplets. This implies, in particular, the existence of a charged Higgs boson w_- associated with the intermediate boson W_- . They would be mass-degenerate if supersymmetry were conserved. If the mass relation (15) holds, the Higgs boson w_- is lighter than the W_- . It gets lighter when spin-0 leptons and quarks get heavier; this leads to the upper limit $\sim m_W/2 \sim 40 \text{ GeV}/c^2$ for the masses of spin-0 leptons and quarks^{2,19}.

The bosonic partner of the goldstino is an R-even particle which determines the mass²-splittings between bosons and fermions in the multiplets of supersymmetry. It must be, in part, a new neutral spin-1 boson with, dominantly, axial couplings to leptons and quarks. This particle, related to gravitation, plays a fundamental role in the structure of spontaneously broken supersymmetric theories. It could lead to modifications of the neutral current phenomenology and to many interesting effects which will be discussed elsewhere.

4.3 Bosonic leptons and quarks: indirect constraints

Lower limits on the masses of bosonic leptons and quarks only come from experiments. Let us consider the extra contributions to the muon anomalous magnetic moment due to spin-0 muons s_μ and t_μ .

Fig. 1 Contributions of spin-0 muons to the muon anomalous magnetic moment.

The contributions of the diagrams of Fig. 1 to the muon anomaly $a_\mu = (g_\mu - 2)/2$ are:

$$\Delta a_\mu \simeq -\frac{\alpha}{12\pi} \left[\frac{m^2(\mu)}{m^2(s_\mu)} + \frac{m^2(\mu)}{m^2(t_\mu)} \right] \quad (17)$$

For $m(s_\mu) = m(t_\mu) = 15 \text{ GeV}/c^2$, we find

$$\Delta a_\mu \simeq -2 \times 10^{-8}. \quad (18)$$

The sum of all extra contributions to the muon anomaly should not be larger than 2×10^{-8} in magnitude²¹. Unless there is some cancellation mechanism between various contributions, this can be taken as an indirect indication that spin-0 muons cannot be too light:

$$m(s_\mu), m(t_\mu) \gtrsim 15 \text{ GeV}/c^2. \quad (19)$$

If, in addition, the leptons and quarks (f) acquire their masses through couplings with a single pair of Higgs doublets, as in Ref. 2, the relations between mass²-splittings imply that

$$\left[m^2(s_f) - m^2(f) \right]^{\frac{1}{2}}, \left[m^2(t_f) - m^2(f) \right]^{\frac{1}{2}} \gtrsim 15 \text{ GeV}/c^2, \quad (20)$$

i.e., spin-0 leptons and quarks should be heavier than $\sim 15 \text{ GeV}/c^2$ (see Ref. 19). However, one should look directly for such particles, irrespectively of this indication on their mass, due to the hypotheses made in the derivation. Similar arguments could be applied if the muons had spin-1 particles for superpartners. Those should not be too light.

4.4 Direct experimental search for bosonic leptons and quarks

Bosonic leptons and quarks are unstable R-odd particles. They decay towards the corresponding spin- $\frac{1}{2}$ lepton or quark by emission of a photino or a goldstino. The decay rate for the emission of a photino is $\sim \alpha$ times the mass of the particle (if it is charged). The lifetime is extremely short, at least for charged particles (neutral ones may have longer lifetimes if the interactions of the goldstino are very weak; see Section 5). As a result one should look for their decay products^{2,5,8}.

R-odd particles can be pair produced, for example in e^+e^- annihilation, or by the Drell-Yan mechanism. (It is also possible to produce a single bosonic lepton or quark in association with its spin- $\frac{1}{2}$ partner and a photino or a goldstino.) Figure 2 illustrates the pair production of bosonic leptons or quarks in e^+e^- annihilation.

Fig. 2 Pair production of bosonic leptons or quarks in e^+e^- annihilation.

Once produced, the pair decays towards the corresponding pair of spin- $\frac{1}{2}$ leptons or quarks, by emitting photinos or goldstinos carrying away half of the energy, in average:

$$\begin{aligned}
 e^+e^- &\rightarrow \text{Pair of bosonic leptons or quarks} \\
 &\rightarrow \text{Non coplanar pair } (e^+e^-, \mu^+\mu^-, \tau^+\tau^-, \text{ or } q\bar{q}) \\
 &\quad + \text{Pair of photinos or goldstinos} .
 \end{aligned}
 \tag{21}$$

The pair production of bosonic electrons and muons resemble the pair production of heavy leptons like the τ . But they differ since (at least in the simple case considered here) their decays lead to e^+e^- or $\mu^+\mu^-$ pairs without μ^+e^- or μ^-e^+ pairs. From SPEAR data taken at $\sim 7.2 \text{ GeV}/c^2$ ²² one can put a lower limit $\sim 3\frac{1}{2} \text{ GeV}/c^2$ on the masses of bosonic electrons and muons, but better limits (~ 10 to $15 \text{ GeV}/c^2$) are probably already available from PETRA experiments. Bosonic quarks would lead to non-coplanar jets with missing energy.

If the new bosons are spin-0 particles, one expects a β^3 -dependence of the production cross-section, and an isotropic distribution of the events just above the threshold. Asymptotically each spin-0 quark would give a contribution to the ratio

$$R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

equal to $\frac{1}{4} Q_i^2$, where Q_i is its electrical charge. Above the threshold for pair production of the spin- $\frac{1}{2}$ quarks u, d, c, s , and b and their spin-0 partners, we would get

$$R = \frac{3}{2} \sum_i Q_i^2 = \frac{11}{2} . \quad (22)$$

4.5 Interactions of the photino and the goldstino

Both the photino and the goldstino couple leptons and quarks to their bosonic partners so that they can scatter on matter^{2,23}. Their couplings are fixed by the electrical charges and the boson-fermion mass²-splittings, respectively. If the lepton and quark fields, denoted by ψ , have spin-0 partners s and t (for which we assume equal masses for simplicity), the relevant Yukawa couplings are fixed by the following terms in the Lagrangian density:

$$\begin{aligned} \mathcal{L} = & i Q_e \sqrt{2} \left[(\bar{\psi}_L s + \bar{\psi}_R t) \lambda_\gamma + \bar{\lambda}_\gamma (s^* \psi_L + t^* \psi_R) \right] \\ & + i e_g \sqrt{2} \left[(-\bar{\psi}_L s + \bar{\psi}_R t) \lambda_g + \bar{\lambda}_g (-s^* \psi_L + t^* \psi_R) \right] \end{aligned} \quad (23)$$

λ_γ and λ_g are Majorana spinors representing the photino and the goldstino, Q_e is the electrical charge, and

$$e_g = \frac{\Delta m^2}{d} \quad (24)$$

fixes the goldstino coupling constant in terms of the boson-fermion mass²-splitting Δm^2 . In this simple case the spin-1 particle which is, in part, the bosonic partner of the goldstino is axially coupled to the lepton or quark field ψ , and the photino and the goldstino have charge conjugation $C = -$ and $C = +$, respectively.

We shall study three types of scatterings on matter: photino $\rightarrow$ photino, photino $\leftrightarrow$ goldstino and goldstino $\rightarrow$ goldstino. They can occur through the exchanges of spin-0 or spin-1 particles as illustrated in Figs 3 and 4, respectively.

Fig. 3 Scattering of photinos or goldstinos on matter due to the exchange of spin-0 leptons or quarks

Fig. 4 Scattering of goldstinos on matter due to the exchange of the new spin-1 boson

In the local limit these processes can be described by the effective Lagrangian density

$$\begin{aligned}
 \mathcal{L}_{\text{eff}} = & \frac{Q^2 e^2}{\Delta m^2} (\bar{\lambda}_{\gamma L} \gamma^\mu \lambda_{\gamma L}) (\bar{\psi} i \gamma_\mu \gamma_5 \psi) \\
 & + \frac{Q e e_g}{\Delta m^2} (\bar{\lambda}_{\gamma L} \gamma^\mu \lambda_{g L} + \bar{\lambda}_{g L} \gamma^\mu \lambda_{\gamma L}) (\bar{\psi} \gamma_\mu \psi)
 \end{aligned}
 \tag{25}$$

The first term describes the scattering photino $\rightarrow$ photino. It depends on the electrical charge of the scattered lepton or quark and on the mass of its spin-0 partners.

The second term describes the scatterings photino $\leftrightarrow$ goldstino. It involves the electromagnetic current $Q e \bar{\psi} \gamma_\mu \psi$. The proportionality coefficient $(e_g/\Delta m^2) = 1/d$ is fixed by the magnitude of the spontaneous supersymmetry breaking (i.e., ultimately, by the mass of the spin-3/2 gauge particle of supersymmetry, the gravitino: see Section 5).

Finally there is no term responsible for the scattering goldstino $\rightarrow$ goldstino since the diagrams of Figs 3 and 4 cancel exactly at low energy due to the relations between masses and coupling constants resulting from supersymmetry. The fact that the new spin-1 boson is axially coupled to the lepton or quark (if $m_s = m_t$) is crucial in this cancellation.

These results are not surprising since the amplitudes involving a goldstino satisfy low-energy theorems (cf. Ref.18). In the present situation the photino and the goldstino are orthogonal states. The goldstino $\rightarrow$ goldstino amplitude must vanish at low energy, since there is no pole term resulting from a boson-fermion mass degeneracy. The goldstino $\leftrightarrow$ photino amplitudes are proportional to the matrix elements of the electromagnetic current, due to the photon-photino mass degeneracy.

These cross-sections have been evaluated numerically in Ref. 23 in terms of the masses of spin-0 leptons and quarks, and of the (unknown) goldstino coupling constant e_g . Here we only indicate the most significant result:

$$\begin{aligned} \sigma(\text{photino} + \text{nucleon} \rightarrow \text{photino} + \text{hadrons}) &\simeq \\ &\simeq \begin{cases} 30 & 10^{-38} \text{ cm}^2 \text{ E(GeV)} & \text{for } m_s = m_t = 15 \text{ GeV/c}^2 \\ 0.7 & 10^{-38} \text{ cm}^2 \text{ E(GeV)} & \text{for } m_s = m_t = 40 \text{ GeV/c}^2 \end{cases} \quad (26) \end{aligned}$$

The last number is still about three times larger than the usual neutral current cross-section $\sigma_{\nu\mu N}^{\text{NC}}$.

For the goldstino cross-sections we can only give upper limits since the goldstino coupling constant may be very small (in fact these cross-sections are fixed by the ratio

$$\frac{G_{\text{Newton}}}{m_{3/2}^2},$$

where $m_{3/2}$ is the gravitino mass: see Section 5).

We expect the photino to have the same effect as a weakly-interacting two-component neutrino on the helium production in the early Universe. This is not necessarily the case for the goldstino: its interaction cross-sections might be very small so that it would drop out of the equilibrium much earlier, and would not affect significantly the evolution of the Universe at the time of helium production²⁴.

We also mention that both the photino and the goldstino, when scattering on quarks, may produce the fermionic partners of the gluons (gluinos): new hadronic states called R-hadrons can be excited if they are not too heavy^{3,23} (see Subsection 4.7).

4.6 The decay $\Psi \rightarrow \text{goldstino} + \text{antiphotino}$

Lepton-antilepton or quark-antiquark pairs can annihilate into pairs photino-antiphotino, photino-antigoldstino or goldstino-antiphotino. In particular, this can be the case for the Ψ resonance, interpreted as a cc bound state⁴. In the simple situation considered earlier, for which the two spin-0 charmed quarks have equal masses, the decay mode photino + antiphotino is forbidden by parity or charge conjugation invariances (see Eq. (25) and Ref. 23).

We only have to consider the decay modes goldstino-antiphotino and photino-antigoldstino. The corresponding diagrams can be obtained easily from those of Fig. 3, and formula (25) gives the effective Lagrangian density responsible for the decay, in the local limit. Comparing with the decay mode $\Psi \rightarrow e^+e^-$, we find:

$$r = \frac{\Gamma(\Psi \rightarrow \text{goldstino} + \text{antiphotino}) + \Gamma(\Psi \rightarrow \text{photino} + \text{antigoldstino})}{\Gamma(\Psi \rightarrow e^+e^-)}$$

$$= \left(\frac{m_\Psi^2}{ed} \right)^2 = \left(\frac{e_g m_\Psi^2}{e \Delta m^2} \right)^2 \quad (27)$$

As an example with spin-0 quarks of $15 \text{ GeV}/c^2$ and $e_g = e/2$ we get

$$\Gamma(\text{goldstino} + \text{antiphotino}) + \Gamma(\text{photino} + \text{antigoldstino}) \simeq$$

$$\simeq 3 \times 10^{-5} \quad (28)$$

This number is to be understood as an illustration and not as an estimate. The branching ratio would be completely negligible if the goldstino coupling constant e_g turned out to be very small. It should be compared to the branching ratio expected for a $\nu\bar{\nu}$ decay mode ($\sim 10^{-8}$), and to the present experimental limit: preliminary results from the SLAC-LBL collaboration, obtained from the search for the decay

$$\Psi' \rightarrow \pi^+ \pi^- \Psi \quad \downarrow \text{unobserved neutrals} \quad (29)$$

indicate that ²⁵

$$\Gamma(\Psi \rightarrow \text{unobserved neutrals}) < 7 \times 10^{-3} . \quad (30)$$

As a result we get the lower limit on the parameter d :

$$d = \frac{\Delta m^2}{\frac{e}{g}} > 90 \text{ (GeV/c}^2\text{)}^2 . \quad (31)$$

This is not a stringent bound since spin-0 charmed quarks are likely to be heavier than 15 GeV/c^2 .

However, this type of experiment is particularly interesting since it gives a direct limit on a fundamental parameter of the theory, d . It measures the magnitude of the spontaneous supersymmetry breaking, and will be related in Section 5 to the mass and effective interaction strength of the spin-3/2 gravitino. It would be useful to be able to improve the limit (30), even if the chances of detecting these decay modes are very slim. The situation would probably be more favourable for the decays of heavier resonances, such as the T or the toponium if it is found.

4.7 New hadronic states

Supersymmetry associates to the colour octet of gluons a colour octet of fermions which we call gluinos. They are neutral and R-odd. In the simplest situations they are massless, but they may also acquire a mass by quantum effects, as discussed in Subsection 4.8. The contribution to the β -function of an octet of self-conjugate spin- $\frac{1}{2}$ gluinos is the same as for three quark flavours, so that gluinos do not destroy the property of asymptotic freedom of the strong interaction theory.

Gluinos may combine with quarks, antiquarks and gluons to give new R-odd colour-singlet hadronic states called R-hadrons. R-mesons, formed of a $q\bar{q}$ pair and a gluino, would be fermions, while R-baryons, formed of three quarks and a gluino, would be bosons. If R-hadrons are relatively light, one can search for their effects in present experiments^{3,8,9}.

Fig. 5 A diagram responsible for the decay
 $R\text{-pion} \rightarrow \pi\pi \text{ photino}$.

R-hadrons would be unstable and decay towards ordinary hadrons by emitting a photino or a goldstino. Figure 5 shows one of the diagrams responsible for the decay $R\text{-pion} \rightarrow \pi\pi \text{ photino}$.

The lifetime is comparable to those of non-leptonic weak decays but shorter due to the presence of a strong rather than a weak or electromagnetic coupling constant at the gluino vertex. For an R-pion of mass $1.2 \text{ GeV}/c^2$ the lifetime is estimated to be $\sim 10^{-15}$ to 10^{-12} s.

R-hadrons could be pair-produced, in particular in hadronic reactions, and would decay into ordinary hadrons by emission of a pair of photinos or goldstinos carrying away part of the energy-momentum. Experiments looking for the pair production and semi-leptonic decays of charmed particles can be used to search for R-hadrons. One can look for short tracks, reinteraction of the emitted neutrals, or missing energy in the final state.

From the BEBC beam dump experiment²⁶ we get the result

$$2 \sigma_{pN \rightarrow RR+X} \cdot \sigma_i < 2 \times 10^{-66} \text{ cm}^4 \quad (32)$$

σ_i is the reinteraction cross-section of the photino or goldstino emitted. If spin-0 quarks are lighter than $m_W/2 \sim 40 \text{ GeV}/c^2$, the reinteraction cross-section of the photino is larger than $0.7 \times 10^{-38} \text{ cm}^2 E \text{ (GeV)}$ (see Eqs (16) and (26)). Unless the decay modes towards the photino are inhibited we get

$$\sigma_{pN \rightarrow R\bar{R}+X} \lesssim 6 \text{ } \mu\text{b} \quad \text{at } \sqrt{s} = 27 \text{ GeV} . \quad (33)$$

The limit would be smaller if the photinos re-excite R-hadrons in the final state³.

One can also look, in calorimeter experiments, for events without charged leptons and with missing energy in the final state. Such events could be attributed to the production and decay of a pair of R-hadrons. From the results of the Caltech-Stanford experiment at Fermilab²⁷ one can give upper limits on the R-hadron pair production cross-section. This requires various assumptions on the A-dependence of the cross-section, the masses of R-hadrons and the spectrum of the emitted neutrals. As an example for an R-hadron mass of $2 \text{ GeV}/c^2$ we get³

$$\sigma_{pN \rightarrow R\bar{R}+X} \lesssim 40 \text{ } \mu\text{b} \quad \text{at } \sqrt{s} = 27 \text{ GeV} . \quad (34)$$

This limit is less constraining than the one given by formula (33), but it does not depend on estimates of the photino and goldstino reinteraction cross-sections.

From there one can conclude that if R-hadrons have a normal hadronic production they cannot be very light, and their masses should be at least of the order of $1\frac{1}{2}$ or $2 \text{ GeV}/c^2$.

4.8 The question of the mass of gluinos

We do not know whether massless gluinos can lead to R-hadrons somewhat heavier than ordinary hadrons, as experimental results seem to indicate. Therefore it is natural to ask whether gluinos can be massive.

We assume, again, that we are in the usual framework of supersymmetric gauge theories. The Lagrangian density does not contain any direct mass term for gluons, nor for gluinos. None can be generated by the Higgs mechanism since the Yukawa couplings of gluinos involve coloured spin-0 fields which cannot be translated if colour gauge invariance is to be conserved. Therefore, gluinos are massless at the classical level.

Could they get a mass by some other effect? If R-invariance is realized as a continuous symmetry, acting on gluino fields as a γ_5 transformation, it would forbid the appearance of a gluino mass, unless the theory contains a second octet with opposite R-transformation properties. But there is also the possibility that R-invariance is not realized as a continuous symmetry, whether it is broken explicitly or spontaneously (maybe by dynamical effects).

An example shows that if the symmetries of the theory do not prevent the appearance of a gluino mass, this one is, indeed, generated by quantum effects²⁸. The value of the mass depends on the detail of the model so that, although it is calculable, it appears in practice as an arbitrary parameter to be determined experimentally. Similar remarks could be applied to the generation of a mass for the photino by quantum effects.

5. SUPERSYMMETRY AND GRAVITATION

5.1 Supergravity: the graviton and the gravitino

There is a close connection between supersymmetry and the geometrical structure of space-time due to the presence of the generators of translations in the anticommutation relations. If the supersymmetry algebra is realized locally, the theory must be invariant under local co-ordinate transformations, so that it includes general relativity. This is the theory of supergravity⁶. The spin-2 graviton is coupled to the conserved energy momentum tensor $T^{\mu\nu}$ with strength $\sim \kappa \sim (G_{\text{Newton}})^{1/2}$. Its spin-3/2 partner, the gravitino, is the gauge particle of supersymmetry. It is coupled, also with strength $\sim \kappa$, to the conserved vector-spinor current of supersymmetry, J^μ_α .

5.2 Spontaneous breaking of local supersymmetry

Spontaneous breaking of global supersymmetry generates a massless Goldstone spin- $\frac{1}{2}$ particle, the goldstino. The magnitude of the spontaneous breaking of the supersymmetry, and therefore the effective strength of the interactions of the goldstino, are characterized by a parameter d with the dimension of a mass². It plays the same role as the vacuum expectation value of a spin-0 field when a global internal symmetry is spontaneously broken, generating a massless Goldstone boson.

When supersymmetry is realized locally the spin- $\frac{1}{2}$ goldstino is eliminated while the corresponding gauge particle, the spin-3/2 gravitino, acquires a mass, the total number of degrees of freedom being conserved²⁹. The value of this mass, proportional to d and to the gauge coupling constant κ of gravitation, is

$$m_{3/2} = \frac{\kappa d}{\sqrt{6}} \quad (35)$$

with

$$\kappa = (8\pi G_{\text{Newton}})^{1/2} \approx 4 \times 10^{-19} \text{ (GeV/c}^2\text{)}^{-1} . \quad (36)$$

Formula (35) is quite similar to the formula $m_W \sim gv$ giving the intermediate boson mass in a gauge theory.

We have seen earlier that the boson-fermion mass²-splitting Δm^2 is proportional to the goldstino coupling constant to the pair, e_g (in the globally supersymmetric theory), which is also the coupling constant of the bosonic partner of the goldstino to the fermion of the pair (formulas (7) and (11)). Disregarding the sign, we can rewrite formula (35) as follows⁷:

$$m_{3/2} = \frac{\kappa \Delta m^2}{e_g \sqrt{6}} . \quad (37)$$

5.3 What is the mass of the gravitino?

Even if we knew the mass²-splitting Δm^2 we could not deduce the gravitino mass unless we also knew the value of the coupling constant e_g . As discussed previously, there are experimental indications that $(\Delta m^2)^{1/2} \gtrsim 15 \text{ GeV}/c^2$, while it should be $\lesssim 40 \text{ GeV}/c^2$ if the mass relation (16) holds. But we do not know the value of e_g . It could be of the order of ordinary gauge coupling constants. It could also be much smaller, since it can be viewed as a gravitational effect.

In the first case the gravitino mass may be extremely small. For $(\Delta m^2)^{1/2} = 15 \text{ GeV}/c^2$ and $e_g = e/2$ we get

$$m_{3/2} \simeq 2 \times 10^{-7} \text{ eV}/c^2 . \quad (38)$$

This number is not an estimate of the gravitino mass but an illustration of how small it might be. If Δm^2 is larger and/or e_g is smaller, the gravitino mass would not be so tiny. Conceivably it might be as large as the Planck mass $\sim 10^{19} \text{ GeV}/c^2$.

Therefore, we should consider the mass of the gravitino as an unknown parameter which, in principle, should be determined from the experiment (!), as is the mass of the W in the theory of weak interactions. In fact, we shall see that in both cases the masses of the gauge particles (W or gravitino) determine, together with the gauge coupling constants (g or κ), the effective strength of the corresponding (weak or gravitational) interaction: the exchange of a W boson is fixed by $G_{\text{Fermi}} \sim (g^2/m_W^2) \sim (1/v^2)$, while the production of a gravitino turns out to be fixed by

$$\frac{G_{\text{Newton}}}{m_{3/2}^2} \sim \frac{\kappa^2}{m_{3/2}^2} \sim \frac{1}{d^2} . \quad (39)$$

5.4 A very light gravitino interacts essentially like a goldstino

Since the interactions of the gravitino are, like those of the graviton, fixed by the extremely small coupling constant κ , it could seem, naïvely, that its effects in particle physics would be negligible. However, whether this is indeed true or not depends on the value of the gravitino mass, as we shall see.

It is somewhat surprising that in a spontaneously broken supersymmetric gauge theory, realizing locally the supersymmetry algebra with an extremely small coupling constant κ leads to the disappearance of the goldstino which would, otherwise, have weak-type interactions (with the coupling constant e_g). In fact, the goldstino does disappear while the gravitino acquires new degrees of freedom by getting a mass. But if this mass is very small the four states of the ultra-relativistic massive spin-3/2 gravitino behave as those of massless particles with helicities $\pm 3/2$ and $\pm 1/2$. The former have practically negligible gravitational interactions, while the latter do interact as the goldstino of the globally supersymmetric theory.

Let us consider the amplitude for emitting, by gravitational interaction, a very light gravitino of mass $m_{3/2}$, momentum $k^\mu \gg m_{3/2}$ and helicity $\pm 1/2$. We first have a factor

$$\frac{\kappa}{2}$$

at the gravitino vertex. Since we select the $\pm 1/2$ helicity states of the massive gravitino we then have an extra factor

$$\frac{k^\mu}{m_{3/2}} \sqrt{\frac{2}{3}} \quad (40)$$

coming from the gravitino wave function. Since $m_{3/2}$ is also proportional to κ , this factor cancels out in the expression

$$\frac{\kappa}{2} \frac{k^\mu}{m_{3/2}} \sqrt{\frac{2}{3}} = \frac{k^\mu}{d} \quad (41)$$

k^μ is to be contracted with the relevant term in the vector-spinor current of supersymmetry.

For example, if a gravitino is produced in the decay of a bosonic partner of a lepton or a quark, the amplitude reads, schematically

$$A \sim \frac{\kappa}{2} \frac{k^\mu}{m_{3/2}} \sqrt{\frac{2}{3}} J_2^\mu \sim \frac{\kappa}{m_{3/2}} \frac{\Delta m^2}{\sqrt{6}} \sim e_g \quad (42)$$

and we recover the coupling constant of the goldstino to the pair (see Ref. 7 for more details).

The amplitude for emitting a gravitino does not vanish in the limit of small κ and $m_{3/2}$ but has a finite limit which is the amplitude for emitting a massless spin- $\frac{1}{2}$ goldstino in a globally supersymmetric theory. The same argument applies to the exchange of these particles:

$$\begin{array}{ccc} \text{Gravitational interactions} & \xrightarrow[m_{3/2} \rightarrow 0]{\kappa \rightarrow 0} & \text{"Weak" interactions} \\ \text{of the gravitino} & & \text{of the goldstino} \end{array} \quad (43)$$

This illustrates the continuity of the theory for $\kappa \rightarrow 0$, giving an independent determination of the gravitino mass^{4,7}. It offers a possibility to look for the effects of the gravitino in particle physics. However, one would not detect its spin-3/2 character, since the only states which could be produced and interact in a non-negligible way would have essentially $\pm \frac{1}{2}$ polarizations.

Consider now a theory which has, in the absence of gravity, a continuous R-invariance, one unit of R being carried by the supersymmetry generator. The introduction of gravity and the generation of a mass for the gravitino breaks R-invariance and reduces it to a discrete symmetry, R-parity. The existence of a mass for the gravitino would lead to $\Delta R = \pm 2$ processes but they

would be extremely small (at the classical level): for example, the $\Delta R = \pm 2$ amplitude due to the exchange of a massive gravitino would be

$$\sim \frac{G_{\text{Newton}}}{m_{3/2}^2} m_{3/2} \sim \frac{\kappa e g}{\Delta m^2}.$$

5.5 Low energy production of gravitinos

It is not necessary to produce R-odd heavy particles to get gravitinos among their decay products. Gravitinos, if not too heavy, can be produced at low energies since they couple to the photon-photino and gluon-gluino pairs.

As said earlier, a very light gravitino interacts essentially like a goldstino. From formula (41), or equivalently from the low-energy theorems satisfied by a goldstino, we expect the amplitudes to vanish in the low-energy limit, unless there is a pole term contribution associated with a mass-degenerate boson-fermion pair.

Therefore we consider the gravitino-photon-photino vertex³⁰. Let q^ρ and k^μ be the momenta of the photon and the gravitino, respectively. The vertex factor is

$$\frac{\kappa}{4} [\gamma^\rho, \gamma^\sigma] \gamma^\mu q_\rho. \quad (44)$$

For a light gravitino of helicity $\pm \frac{1}{2}$ we have the extra factor (40) coming from the wave function. For external gravitinos and photinos we get the effective vertex factor

$$\frac{\kappa}{m_{3/2} \sqrt{6}} \gamma^\sigma q^2. \quad (45)$$

The factor q^2 cancels the $1/q^2$ coming from the photon propagator. The one-photon exchange contribution to the production of gravitino-antiphotino or photino-antigravitino pairs is exactly given (in the limit of small κ and $m_{3/2}$) by the local four-fermion interaction⁴

$$\mathcal{L}_{\text{eff}} = \frac{\kappa}{m_{3/2} \sqrt{6}} (\bar{\lambda}_{gL} \gamma_\mu \lambda_{\gamma L} + \bar{\lambda}_{\gamma L} \gamma_\mu \lambda_{gL}) J_{\text{electromagnetic}}^\mu \quad (46)$$

This expression for gravitino production is identical to the second term in expression (25) found for goldstino production²³. The relation

$$\frac{\kappa}{m_{3/2} \sqrt{6}} = \frac{e_g}{\Delta m^2} \quad (47)$$

expresses the continuity of the theory when gravity is neglected. This is illustrated in Fig. 6:

Fig. 6 Equivalence between the production of a gravitino antiphotino and a goldstino-antiphotino pairs.

6. SUPERSYMMETRY AND GRAND UNIFICATION

It is believed that weak, electromagnetic and strong interactions are due to the exchanges of $SU(2) \times U(1) \times SU(3)$ gauge bosons between fermions, leptons and quarks. Among the possible ways to go further are supersymmetry and grand unification³¹. The former relates bosons and fermions, the latter leptons and quarks. We shall discuss briefly the similarities and differences between these two approaches, and whether they can be combined.

6.1 Effective strengths and distinctive features of the interactions

In grand unification schemes $SU(2)$, $U(1)$ and $SU(3)$ are subgroups of a simple group such as $SU(5)$, $O(10)$ or larger groups. The intermediate bosons $W_{\pm}$ and Z , with masses $\sim 10^2 \text{ GeV}/c^2$, belong to the same multiplet as the massless photon and gluons together with very heavy gauge bosons with masses estimated to be $\sim 10^{15} \text{ GeV}/c^2$. The scale of the breaking of the grand unification group for lepton and quark multiplets is considerably smaller. The existence of these different scales is the so-called hierarchy problem. The effective strength of weak interactions is fixed by the Fermi coupling constant

$$G_F \sim \frac{g^2}{m_W^2} \sim \frac{1}{v^2}.$$

The effective strength of the exchanges of the very heavy bosons, collectively denoted by X , is fixed by

$$\frac{g_{GUT}^2}{m_X^2} \sim \frac{1}{v_{GUT}^2}.$$

The observability of their effects, for example in B - or L -violating processes such as proton decay, will depend essentially on the very heavy masses m_X .

The situation for grand unification, where the effects of the very heavy bosons are fixed by α_{GUT}/m_X^2 , presents some formal similarity with the situation for supersymmetry and supergravity, where the effects of the gravitino are fixed by $G_{\text{Newton}}/m_{3/2}^2$. One may be able to detect B - or L -violating effects of the X -bosons, e.g., proton decay, even if these gauge bosons are very heavy. On the contrary, because of the smallness of the Newton coupling constant, direct effects of gravitinos may only be observed for those of them, if any, which would be very light. Those would be neutral.

More precisely a complete study involving other graphs shows that the equivalence between the gravitino and the goldstino formulations holds at any values of the momenta, and not only in the local limit approximation.

5.6 Looking for gravitinos in Ψ decays

It is now easy to re-interpret the results of Subsection 4.6 in terms of gravitinos instead of goldstinos (see Fig. 7).

The experiment gives an upper limit on the effective strength of the gravitino interaction (proportional to $\kappa/m_{3/2} \sim 1/d$), so that we get a lower limit on the gravitino mass⁴:

$$m_{3/2} > 1.5 \times 10^{-8} \text{ eV}/c^2. \quad (48)$$

This is not yet a constraining limit on the theory (although it does exclude the too naïve estimate $m_{3/2} \sim \kappa m_{\text{proton}}^2$), but it has the merit of being model-independent.

It is the mass of the gravitino which will determine its production rate and its interaction cross-section. Both are proportional to

$$\frac{4\pi}{3} \frac{G_{\text{Newton}}}{m_{3/2}^2} = \frac{\kappa^2}{6 m_{3/2}^2} = \frac{1}{d^2} = \frac{e^2 g}{(\Delta m^2)^2}. \quad (49)$$

Only if the gravitino mass is very small do we have a chance to detect directly its effects, i.e., the effects of gravitational interactions, in particle physics. But then it would, also, behave essentially like a spin- $\frac{1}{2}$ goldstino.

Fig. 7 The main diagram responsible for the decay $\Psi \rightarrow \text{gravitino} + \text{antiphotino}$.

Three facts contribute to make the search for gravitinos harder than the search for proton decay:

i) the Newton coupling constant (with the proton mass as unit) is very small compared to the grand unification fine-structure constant α_{GUT} (or α).

ii) While for grand unification we think we know complete multiplets of leptons and quarks, for supersymmetry we do not have this chance with bosons and fermions.

iii) While the interactions of the very heavy X bosons may violate B- or L-quantum numbers and induce processes such as proton decay, the interactions of a very light gravitino would not have such a striking property. However, a gravitino mass term would violate R-invariance by two units so that it would lead to $\Delta R = \pm 2$ processes, unfortunately both very rare and less remarkable than proton-decay.

6.2 Compatibility of supersymmetry and grand unification

The most naïve approach would be some direct product of these invariances. Then we have of course the consequences of each invariance separately, but we could also look for both of them at the same time by searching for gravitino production in proton decay (see Fig. 8)!

The decay rate is

$$\Gamma \sim \alpha \left[\frac{\alpha_{\text{GUT}}}{m_X^2} \right]^2 \frac{G_{\text{Newton}}}{m_{3/2}^2} m_p^9 \quad (50)$$

Fig. 8 One of the graphs responsible for the decay $\text{proton} \rightarrow \pi^0 e^+ \text{gravitino antiphotino}$.

If the gravitino mass lies somewhere between $10^{-(6 \text{ or } 7)} \text{ eV}/c^2$ and the proton mass, a very crude estimate of the partial lifetime is:

$$10^{40} \text{ years} \lesssim \tau(\text{proton} \rightarrow \text{gravitino} + \dots) \lesssim 10^{70} \text{ years} . \quad (51)$$

We shall not comment on the feasibility of this experiment, since there are more important consequences. The existence of the superpartners of the ordinary particles (gluinos and photino, bosonic leptons and quarks, heavy fermions) would modify the estimates which have been made for the $SU(2) \times U(1)$ mixing angle θ and the proton lifetime in the minimal $SU(5)$ model. A precise evaluation of these effects would require the knowledge of the particle spectrum.

Moreover, the possibility of realizing a spontaneous breaking of supersymmetry implies very strong constraints on a grand unified theory. If we assume that spin- $\frac{1}{2}$ leptons and quarks have spin-0 partners, the bosonic partner of the goldstino should be, in part, a new spin-1 boson with axial couplings to leptons and quarks. In the gauge theory framework, the gauge group must be bigger than $SU(5)$. For example, it could be $SU(5) \times U(1)$; $SU(5)$ is the grand unified gauge group and the $U(1)$ factor corresponds to a new gauge interaction responsible for the spontaneous supersymmetry breaking and related to gravitation (see Subsection 3.4 for a necessary condition on the gauge group).

If the supersymmetry is such that spin- $\frac{1}{2}$ leptons and quarks can have spin-1 partners, the constraints on the gauge group are different. We need either a larger gauge group or the consideration of spin-1 particles which would not have ordinary gauge interactions. In this case, and in particular in the framework of extended supersymmetry which involves large multiplets with many new fields, it seems hard to find out whether supersymmetry is indeed broken or not, and what are the masses and interactions of the boson and fermion states.

7. CONCLUDING REMARKS

Whatever the fundamental theory of Nature is, the framework we have presented may be used as soon as there is a trace of a boson-fermion supersymmetry at energies where field theory is still valid.

The actual observability of this supersymmetry depends on the mass scale at which the boson-fermion multiplet structure becomes apparent. In addition, the direct observability of the effects of the corresponding gauge particle, the gravitino, depends on its mass which fixes the effective strength of its interaction.

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There is no incompatibility between the supersymmetry and grand unification approaches, but complementarity. Grand unification aims at a unified description of weak, electromagnetic and strong interactions while supersymmetry seems the natural framework for the introduction of gravity.

We know how to construct spontaneously broken supersymmetric theories of weak, electromagnetic and strong interactions, and how to search experimentally for the effects of supersymmetry. Introducing gravity does not lead to any presently observable consequence, but it is conceptually very important for the connection between gravity and gauge interactions.

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