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SYMMETRY BREAKING IN SUPERSYMMETRIC GUTS *)

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ABSTRACT

In the context of supersymmetric grand unified theories, we give a group-theoretical analysis of possible patterns of gauge symmetry breakings which are compatible with unbroken supersymmetry.

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INTRODUCTION

Supersymmetry [1] is one of the most attractive and puzzling symmetries of relativistic quantum field theory. It is attractive because of several reasons : among them we mention the enlargement of the space-time symmetry algebra giving rise to supermultiplets of fields with different spins and the softening of quantum divergences [2]. Ultraviolet improvement of radiative corrections is especially dramatic in the supersymmetric extensions of Einstein theory, the so-called supergravities [3], which provide us with theories of interacting particles with spin ranging from $J = 0$ up to $J = 2$ and possessing amazing convergence properties [4]. This is related to the previously mentioned softening of quantum divergences in supersymmetric field theories. More specifically in a globally supersymmetric renormalizable theory with or without gauge interactions there is a set of non-renormalization theorems [5] which ensure to all orders of perturbation theory the absence of radiative corrections to masses and coupling constants; this would certainly not be true in an ordinary gauge theory. As an important consequence of these theorems, it becomes technically natural to impose numerical relations between the bare parameters

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of the superpotential which describes the self-interactions of the $J = 0, 1/2$ chiral multiplets : such relations will survive renormalization. Moreover, there will be no quadratically divergent corrections to masses of scalar particles.

On the other hand, supersymmetry is puzzling since it has to be broken in order to have any relevance to Physics. And there is no indication at what scale should this breaking occur. In some attempts at superunification [6], it was assumed that supersymmetry is broken at the Planck scale. In this case the low energy implications of supersymmetry are very remote and one can only hope to get indirect predictions like constraints on the gauge group unifying the non gravitational interactions and some limitations on the fermion representations. Recently many authors [7-14] have pointed out that a supersymmetric world could emerge if one supersymmetry remains unbroken all the way down to the weak interaction scale. The possible relevance of a supersymmetry broken at low energy, $M_W \ll M_{\text{Planck}}$, is suggested by the so-called hierarchy problem [15] one encounters in attempts to unify electroweak and strong interactions. In conventional unified theories (GUT's), the scales of symmetry breaking are strongly related to the masses of the Higgs scalars which acquire v.e.v.'s and are responsible for the breaking of the gauge group. However there is no symmetry to ensure masslessness of scalar fields, as chiral symmetry does for fermions. Thus there is no natural mechanism to provide us with the huge ratio between the scale of baryon number violating processes ($\gtrsim 10^{15}\text{GeV}$) and the scale of weak interactions ($\sim 100\text{GeV}$). Supersymmetry however potentially possesses the desirable features which allow to solve the hierarchy problem : if the bare parameters of the Lagrangian allow for two different scales, the higher order corrections will not be able to mix these scales as long as supersymmetry is unbroken (or softly broken at the lower scale). Then a Weinberg-Salam Higgs doublet with bare mass of order 250GeV will not receive mass "corrections" of order 10^{15}GeV as it is usually the case in unified theories.

In the present work, we analyze the first step of symmetry breaking in $N = 1$ supersymmetric unified theories. As usual, these theories have (at least) two scales of breaking : at super-heavy energies ($\sim 10^{15}\text{GeV}$) the gauge group G is broken down into $G' \supseteq SU(3) \times SU(2) \times U(1)$ and baryon number nonconserving gauge interactions decouple. Supersymmetry however remains unbroken : its breaking will occur only at the weak interaction scale, the necessary hierarchy being technically natural due to supersymmetry. Our aim is to characterize the possible patterns of gauge symmetry breaking consistent with supersymmetry.

Let us first review some well-known properties of the scalar potential in supersymmetric gauge theories [16-18]. The scalar fields are complex : $z = A + iB$, transforming according to some representation $\underline{R}$ of the gauge group G , which is in general reducible. We assume G to be simple, but our results apply as well to semisimple gauge groups. The scalar potential is then given by the following non-negative G -invariant function of z and z^* :

$$V(z, z^*) = \sum_a |F_a(z)|^2 + \sum_\alpha |D^\alpha(z)|^2 \geq 0 \quad (1)$$

where the "gauge term", $D^\alpha(z)$, ($\alpha = 1, 2, \dots, \dim G$) is

$$D^\alpha(z) = g \sum_{a,b} z_a^* (T^\alpha)_b^a z^b \quad (2)$$

the matrices T^α being the generators of G in the representation $\underline{R}$, in the basis where z has components z_a ($a = 1, 2, \dots, \dim R$). We refer to the second term in $V(z, z^*)$ as the D^2 -term. The first term, or F^2 -term, is defined in terms of the gradient :

$$F_a = \frac{\partial F(z)}{\partial z^a} \quad (3)$$

where $F(z)$ is a G -invariant polynomial in the z -fields (but not in the z^* -fields) which is at most cubic in a renormalizable theory.

Supersymmetry survives the spontaneous symmetry breaking of G if $V(\langle z \rangle, \langle z^* \rangle) = 0$, where $\langle z \rangle$ is the vacuum expectation value (v.e.v.) of z ; the residual gauge group is the little group of $\langle z \rangle$. Since $V(z, z^*)$ is non-negative, if a supersymmetry preserving solution exists, all the solutions are supersymmetric. In this case the v.e.v.'s have to satisfy the following conditions

$$F_a(\langle z \rangle) = 0, \quad \forall a \quad (4)$$

$$D^\alpha(\langle z \rangle) = 0, \quad \forall \alpha \quad (5)$$

If $F(z)$ has no linear term in some component of z (which has to be a singlet under G) then all possible sets of v.e.v.'s preserve supersymmetry since the trivial solution, without spontaneous gauge symmetry breaking, $\langle z \rangle = 0$ gives the minimum value $V(0, 0) = 0$. In this case all the v.e.v.'s are solutions of Eqs (4) and (5). Since we are interested in supersymmetric solutions we will first assume that there are no G -invariant components in the (reducible) multiplet of the scalar fields z , and we will discuss later the possible effects when there are G -singlets among the scalar fields.

In this paper, we introduce some simple but rather powerful methods to discover which v.e.v.'s of a given multiplet of scalar fields are consistent with Eqs (4) and (5). Since we assume supersymmetry to be preserved, we say that all v.e.v.'s which are excluded by these conditions, Eqs (4) and (5), are protected. In general, besides the trivial solution, $\langle z \rangle = 0$, there are several non-trivial ones which break the gauge group G . In practice, we would like to have $SU(3) \times SU(2) \times U(1)$ (perhaps, a larger group, consistent with a breaking at low energies) as the residual little group and protect all other solutions (but $\langle z \rangle = 0$). This, in general, is not possible because there are degenerate sets of supersymmetric v.e.v.'s. In these cases we restrict ourselves to ensure that a solution exists with $SU(3) \times SU(2) \times U(1)$ as little group and that the v.e.v.'s which subsequently break $SU(3) \times SU(2) \times U(1)$ are protected. In this sense, we will say that the supersymmetric gauge theory has a natural hierarchy. The breaking of both supersymmetry and the Weinberg-Salam gauge group at much lower energies should need including new terms in the Lagrangian with parameters of the order of 1TeV. In the models proposed so far these new terms are also used to resolve the degeneracy of the supersymmetric solutions in favour of the $SU(3) \times SU(2) \times U(1)$ residual group at high energies.

It must be stressed, however, that in this paper we do not address ourselves to the construction of realistic models. Even if we illustrate our discussion with examples which mimic some well-known GUTS, the emphasis is on the protection of v.e.v.'s and on the methods to select the allowed supersymmetric gauge breaking patterns.

The paper is organized as follows. The two contributions to the scalar potential are analyzed separately. In Section 2, we discuss the vanishing of the D^2 -term, and we derive simple but powerful lemmas; we illustrate their application with several examples, leading to a fairly simple selection of the unprotected v.e.v.'s. Section 3 is devoted to the study of the vanishing of the F^2 -term. We find a sort of complementarity between the D^2 -term and the F^2 -term in the sense that v.e.v.'s which are allowed by Eq. (5) are in general forbidden by Eq. (4) and vice-versa. Simple group theoretical arguments can be used to observe this property. In Section 4, we apply the results of the preceding sections to some possible candidates to a supersymmetric gauge theory, based on the gauge groups $SU(5)$, $O(10)$ and E_6 . Even if the models involve several irreducible representations (irreps) of chiral supermultiplets, by exploiting the properties of the potential in Eq. (1), one is able to find possible patterns of symmetry break-

ing consistently with supersymmetry. Section 5 presents a short discussion of the next step in the construction of supersymmetric unified theories, namely the mechanism of supersymmetry breaking.

VANISHING OF GAUGE POTENTIAL TERM

Let us start by discussing the condition $D^\alpha(\langle z \rangle) = 0$. We introduce the notation

$$D^\alpha(z) = g\langle z | T^\alpha | z \rangle = \sum_i g\langle z_i | T^\alpha | z_i \rangle,$$

where the sum is over the irreps appearing in the representation of the chiral multiplets, the last identity following from the characteristic property of the generators T^α .

We characterize the possible v.e.v.'s, z , by their little group, g_z . For the generators of g_z , by definition, $T^\alpha | z \rangle = 0$, so that the corresponding $D^\alpha(z)$ trivially vanish. The remaining generators of G , associated to the coset G/g_z , are such that $T^\alpha | z \rangle \neq 0$. However, for these T^α of G/g_z which do not transform as singlets under g_z , one has $\langle z | T^\alpha | z \rangle = 0$, by the Wigner-Eckart theorem as z is a g_z -singlet. Therefore, one has the sufficient condition :

Lemma 1 : If G/g_z does not contain any generator transforming as a singlet under g_z , then all $D^\alpha(z) = 0$ (i.e., the v.e.v. z is allowed by the D^2 -term).

In general this condition is not necessary, since if a T^α of G/g_z is a g_z -singlet, in principle, $T^\alpha | z \rangle$ can be orthogonal to $| z \rangle$. If z is on a critical orbit (which means that there is no element in the representation with a little group containing g_z), then $T^\alpha | z \rangle \propto | z \rangle$, so that the above condition is also necessary :

Lemma 2 : For z on a critical orbit, if G/g_z has a generator T^α transforming as a singlet under g_z , then the corresponding $D^\alpha(z) \neq 0$ (the critical v.e.v. z is then protected by the D^2 -term).

For $z = A + iB$ transforming as a real representation of G , the vanishing of the D^2 -term imposes particularly simple conditions. Indeed, the generators T^α can be represented by antisymmetric matrices $T_{ab}^\alpha = -T_{ba}^\alpha$, so that the only contribution to D^α comes from $iT_{ab}^\alpha (A_a B_b - B_a A_b)$. For instance

$A \propto B$ is sufficient to have all $D^\alpha(z) = 0$. But, in general, the $D^\alpha(z)$ vanish in more general circumstances : if, e.g., z transforms as the adjoint representation, A and B can be written as Hermitean matrices and the condition $[A, B] = 0$ ensures the vanishing of the D^2 -term. This is an example of Lemma 1 since, if $[A, B] = 0$, z can be rotated into a diagonal form and has a little group which contains the Cartan subgroup of G , so that G/g_z does not contain any g_z -singlet.

These results apply as well to the contribution $D^\alpha(z_i)$ of each irrep appearing in the decomposition of z . We first analyze according to Lemmas 1 and 2, some complex irreps of various (simple) groups that could be relevant for grand unification.

The spinorial representation of $O(10)$, 16, has two critical orbits, with little groups $SU(5)$ and $O(7)$, respectively. But both of them correspond to a non-vanishing D^2 -term, since $O(10)/SU(5)$ has a $SU(5)$ -singlet generator, and $O(10)/O(7)$ has an $O(7)$ -singlet generator. Also the 126 of $O(10)$ has an $SU(5)$ -invariant critical orbit which gives a non-vanishing D^2 -term. However, there are directions of the 126 for which the D^2 -term vanishes, like that with $O(5) \times O(5)$ invariance, for instance. In $O(14)$, the next group in the $O(4n+2)$ series, the spinorial representation, 64, has a $(G_2 \times G_2)$ -invariant direction for which the D^2 -term vanishes.

In the case of the E_6 group, for the 27 fundamental representation, the direction with F_4 -invariance gives a vanishing D^2 -term, while the direction with the little group $O(10)$ contributes to the D^2 -term, since $E_6 \supset O(10) \times U(1)$. For the 351_S representation, contained in the symmetric product $(27 \times 27)_S$, one finds that among all $[SU(3) \times SU(2) \times U(1)]$ -invariant directions, the D^2 -term vanishes only on one (non-critical) orbit with little group $O(6) \times O(4)$. Indeed, $E_6/O(6) \times O(4)$ contains just one $[O(6) \times O(4)]$ -singlet generator, T , and this orbit is precisely characterized by the fact that $T|z\rangle$ is orthogonal to $|z\rangle$.

We now study some complex irreps of $SU(N)$: i) any element of the fundamental representation has little group $SU(N-1)$ and so contributes to the D^2 -term; ii) the D^2 -term vanishes for an antisymmetric tensor, $z^{[ab]}$ of rank two, only for even N , on the critical orbit with little group $Sp(N)$; iii) the symmetric tensor of rank two, $z^{\{ab\}}$ has the same property (for any N) in the critical direction with little group $O(N)$; for the N -fold symmetric representation, $z^{\{a_1 a_2 \dots a_N\}}$, the critical orbit with the little group $[U(1)]^{N-1}$ (Cartan subgroup) does not contribute to the D^2 -term.

The vanishing of the D^2 -term for a given direction of an irrep means that a v.e.v. in this direction is not protected by the D^2 -term. The converse is not true, in general. Indeed, in physical applications one always needs a reducible representation of chiral supermultiplets. The non-vanishing of the D^2 -term on a given orbit of an irrep does not ensure the protection of the associated v.e.v.'s, since a cancellation may occur between different contributions. In this case, however, the scales and the directions of the different v.e.v.'s are strictly related. This implies a much more restricted pattern of possible symmetry breakings that in the case of non-supersymmetric theories with the same set of irreps for the scalars. As a trivial example, take a real reducible representation, $\underline{R} + \overline{\underline{R}}$: the D^2 -term obviously vanishes if the v.e.v.'s in $\underline{R}$ and $\overline{\underline{R}}$ have the same little group and the same strength.

Let us present some examples where two complex irreps may conspire to give a vanishing overall contribution to the D^2 -term :

a) In $SU(N)$, the fundamental representation, z^a , and the symmetric tensor of rank r , $Z_{\{a_1 \dots a_r\}}$: if their v.e.v.'s are related by :

$$Z_{\{a_1 a_2 \dots a_r\}} = \frac{1}{\sqrt{r}} \sum_b \delta^{a_1 b} \dots \delta^{a_r b} z^b,$$

they both have $SU(N-1)$ as little group and the $U(1)$ in the coset $SU(N)/SU(N-1)$ has a generator with opposite signs for the two irreps.

b) In $SU(2n+1)$, the fundamental representation, z^a , and the two-fold antisymmetric representation, $Z^{[ab]}$: if $Z^{[ab]}$ has the little group $Sp(2n)$ which is contained in the little group $SU(2n)$ of z^a , the D^2 -term will vanish for $\text{Tr} Z Z^* = n |z|^2$.

c) In $O(10)$, a $\underline{126}$ and a $\underline{16}$: if the v.e.v.'s are $SU(5)$ -invariant and if the strengths of the $\underline{16}$ and the $\underline{126}$ are in the ratio $\sqrt{2}$, the D^2 -term vanishes. The same occurs in E_6 , with a $\underline{351}_s$ and a $\underline{27}$ if the v.e.v.'s have $O(10)$ as little group.

d) In $O(10)$, the complex v.e.v. of a $\underline{10}$ with little group $O(8)$ and the v.e.v. of a $\underline{16}$ with little group $O(7)$ can cancel to give a vanishing D^2 -term.

In all these examples, the cosets G/g_z of the v.e.v.'s in each irrep have the same g_z -invariant generator which takes opposite sign for the two v.e.v.'s. We now construct simple examples where a cancellation occurs between contributions of v.e.v.'s in several irreps :

e) In $SU(N)$, take the r -fold antisymmetric and r fundamental representations, $Z_{[a_1 \dots a_r]}$ and $z^a_{(i)}$, ($i = 1, \dots, r$) : if their

v.e.v.'s are such that $z_{(i)}^a = \mu \delta^{ai}$ and that $Z_{[a_1 \dots a_r]}$ has only one non-vanishing component, $Z_{[12 \dots r]} = \mu$, then the D^2 -term vanishes and the little group of the whole set of v.e.v.'s is $SU(N-r)$.

f) In $SU(N)$, with N fundamental representations $z_{(i)}^a$, ($i = 1, \dots, N$) the D^2 -term vanishes when the v.e.v.'s are all orthogonal, e.g., $z_{(i)}^a = \mu \delta^{ai}$; the $SU(N)$ symmetry is completely broken.

From the examples above it should be clear how to build sets of irreps such that the D^2 -term vanishes when the v.e.v.'s are properly chosen. Notice that in the cases above the strengths of all v.e.v.'s are given in terms of only one arbitrary scale. Obviously, more scales may be introduced if one uses a larger set of v.e.v.'s to obtain a vanishing D^2 -term.

A remarkable property in these examples is that a G -invariant exists for the set of irreps. This is shown in Table I. If one substitutes in the G -invariant the v.e.v.'s which cancel in the D^2 -term, the result does not vanish. Furthermore, the strength of each v.e.v. is proportional to $\sqrt{n}$ where n is the number of times the corresponding irrep appears in the G -invariant.

VANISHING OF THE SUPERPOTENTIAL TERM

We now turn to the discussion of the F^2 -term in the potential. It is determined by the superpotential, which may have terms mixing different irreps with several arbitrary parameters. Therefore, for a given (in general, reducible) representation of chiral multiplets many cases are possible depending on the structure of the superpotential. In practice, the analysis simplifies a lot as only directions which are unprotected by the D^2 -term are to be considered.

The F^2 -term vanishes at the stationary points of the superpotential. We have noticed that a direction with vanishing D^2 -term corresponds to a non-vanishing contribution to some invariant under the gauge group which may be present in the superpotential. Examples are the quadratic invariant for self-conjugate representations and the invariants in Table I for complex representations. The gradient of these invariants in the directions selected by the D^2 -term will not vanish so that these v.e.v.'s could be protected by the F^2 -term. There is a sort of complementarity between the protection by the D^2 -term and the F^2 -term. The v.e.v.'s which are not protected by the D^2 -term will be protected by the F^2 -term unless one of the following conditions is satisfied :

I) The gradients of two (or more) invariants are such to exactly

Table I Examples of solution of Eq. (5). The little groups of the corresponding orbits are given. In the last column, the non-vanishing invariants associated to these solutions are indicated (see text).

Group Representations	Little group	Invariant
$SU(N)$ $z^{[ij]}$ (N even)	$Sp(N)$	$\epsilon_{i_1 \dots i_N} z^{[i_1 i_2]} \dots z^{[i_{N-1} i_N]}$
$z^{\{ij\}}$	$O(N)$	$\epsilon_{i_1 \dots i_N} \epsilon_{j_1 \dots j_N} z^{\{i_1 j_1\}} \dots z^{\{i_N j_N\}}$
$z^{\{i_1 \dots i_N\}}$	$[U(1)]^{N-1}$	$\epsilon_{i_1 \dots i_N} \epsilon_{j_1 \dots j_N} \dots \epsilon_{r_1 \dots r_N} z^{\{i_1 j_1 \dots r_1\}} \dots z^{\{i_N j_N \dots r_N\}}$
z^i $z^{\{i_1 \dots i_r\}}$	$SU(N-1)$	$z^{i_1} \dots z^{i_r} z^{\{i_1 \dots i_r\}}$
z^i $z^{[ij]}$ (N odd)	$SU(N-1)$ U $Sp(N-1)$	$\epsilon_{i_1 \dots i_N} z^{i_1} z^{[i_2 i_3]} \dots z^{[i_{N-1} i_N]}$
$z^i_{(1)} \dots z^i_{(r)}$ $z^{[i_1 \dots i_r]}$	$SU(N-r)$ $SU(r) \times SU(N-r)$	$z^i_{(1)} \dots z^i_{(r)} z^{[i_1 \dots i_r]}$
$z^i_{(1)} \dots z^i_{(N)}$	Identity	$\epsilon_{i_1 \dots i_N} z^i_{(1)} \dots z^i_{(N)}$
$O(10)$ $\underline{126}$	e.g. $O(5) \times O(5)$	$(\underline{126} \times \underline{126} \times \underline{126} \times \underline{126})_1$
$\frac{\underline{126}}{\underline{16}}$	e.g. $SU(5)$ $SU(5)$	$[\underline{126} \times (\underline{16} \times \underline{16})_s]_1$
$\frac{\underline{16}}{\underline{10}}$ (complex)	e.g. $O(7)$ $O(8)$	$[(\underline{16} \times \underline{16})_s \times \underline{10}]_1$
E_6 $\underline{27}$	F_4	$(\underline{27} \times \underline{27} \times \underline{27})_1$
$\underline{351}_s$	e.g. $O(6) \times O(4)$	$(\underline{315}_s \times \underline{351}_s \times \underline{351}_s)_1$
$\frac{\underline{351}_s}{\underline{27}}$	$O(10)$ $O(10)$	$[(\underline{27} \times \underline{27})_s \times \underline{351}_s]_1$

compensate each other, and the relations which are needed between v.e.v.'s are consistent with the vanishing of the D^2 -term.

II) The G-invariants one is able to construct for the irreps in correspondence with $|D^2| = 0$ are all of degree ≥ 3 (several examples are available in Table I).

III) Invariants that imply the protection of some particular set of v.e.v.'s can be excluded from the superpotential. As already noticed, the procedure of fixing the parameters in the superpotential, although unaesthetical, is (technically) natural in supersymmetric theories. However, the pattern of fermionic masses is also related to the superpotential and some G-invariants are needed for this purpose in the superpotential. Also, the elimination of some invariants may introduce additional global symmetries in the superpotential, whose breaking might lead to the existence of Goldstone bosons.

We shall illustrate these properties with some simple examples. If there is only one irrep of chiral multiplets, a quadratic invariant may appear in the superpotential if the irrep is real. If there is no cubic invariant, the v.e.v.'s of these fields are protected by the F^2 -term. Examples are the adjoint representations of $O(N)$, ($N \neq 6$) and of the exceptional groups. (This remains true, in general, with two adjoint representations.)

Therefore, it is interesting to consider self-conjugate irreps which admit a cubic invariant. Take for instance, the adjoint of $SU(N)$, ($N \geq 3$) and the rank-two symmetric traceless tensor of $O(N)$, ($N \geq 5$), which can be written as a matrix ϕ . If both the invariants, $\text{Tr}\phi^2$ and $\text{Tr}\phi^3$ are present in the superpotential, the vanishing of the F^2 -term requires ϕ to be in a critical orbit, with little groups $SU(M) \times SU(N-M) \times U(1)$ and $O(M) \times O(N-M)$, respectively ($2M \neq N$, in both cases). With the cubic invariant $\text{Tr}\phi^3$ alone, only the directions with little groups $SU(N/2) \times SU(N/2) \times U(1)$ and $O(N/2) \times O(N/2)$ are allowed, provided that N be even.

More generally, two self-conjugate irreps can conspire to give a vanishing F^2 -term (if they share some bilinear invariant), in which case the little groups of their v.e.v.'s are closely related. Consider, for instance, the antisymmetric tensor (adjoint representation) and the symmetric tensor representations of $O(2N)$, represented by the traceless matrices: $A^{ab} = -A^{ba}$, $S^{ab} = S^{ba}$, ($a, b = 1, \dots, 2N$). The possible $O(2N)$ invariants are: $\text{Tr}A^2$, $\text{Tr}S^2$, $\text{Tr}S^3$, $\text{Tr}A^2S$. The F^2 -term vanishes when the v.e.v. of S has little group $O(2p) \times O(2k-2p) \times O(2N-2k)$, ($p < k/2$, $k \neq N/2$) while the v.e.v. of A has little group $O(2k) \times U(N-k)$, the residual gauge symmetry being $O(2p) \times O(2k-2p) \times U(N-k)$. This

property is easy to understand by noticing, for instance, that at a stationary point of the superpotential the v.e.v.'s of A and of the adjoint part of (AS) have the same invariances.

Also in the case of two conjugate representations, R and $\bar{R}$, of chiral multiplets, the invariant (RR) in the superpotential would tend to protect their v.e.v.'s. Again, if there are bilinear couplings in the superpotential which involve R and (or) $\bar{R}$ a conspiracy may occur allowing for v.e.v.'s of $\bar{R}$ and (or) R . As an example, take an $O(10)$ gauge model with chiral multiplets in the $(\underline{16} + \underline{16} + \underline{45})$ representation. Take the v.e.v.'s of the $\underline{16}$ and $\underline{16}$ to be parallel and of equal strength to get a vanishing D^2 -term. These v.e.v.'s have a little group $\supset SU(4)$ [19]. The possible invariants in the superpotential are : $(\underline{16} \times \underline{16})$, $(\underline{45})^2$, $(\underline{16} \times \underline{16} \times \underline{45})$. Hence, the v.e.v. of the $\underline{45}$ has to have little group $\supset SU(4) \times U(1) \times U(1)$ in order to compensate for the v.e.v. of the $\underline{45}$ components of $(\underline{16} \times \underline{16})$. But since the v.e.v. of the $\underline{16}$ -part of $(\underline{16} \times \underline{45})$ has to be proportional to the v.e.v. of the $\underline{16}$, the only possible solution must have an overall $SU(5)$ invariance.

The property that the little groups of the v.e.v.'s for the various irreps have to be strongly related is general. It explains why supersymmetry allows for, and often requires, a richer pattern of Higgs representations than in non-supersymmetric GUTS, in order to achieve a certain pattern of symmetry breaking.

It is interesting to consider complex irreps with a cubic invariant. Take for instance the $\underline{27}$ of E_6 . The F_4 -invariant v.e.v. is allowed by the D^2 -term and protected by the F^2 -term, while the $O(10)$ -invariant v.e.v. is allowed by the F^2 -term and protected by the D^2 -term. The complementarity between these two terms in the potential can be proved here in a very illustrative way [20]. The D^2 -term has the form $(\underline{27} \times \underline{27})_{\underline{28}} \times (\underline{27} \times \underline{27})_{\underline{28}}$, while the F^2 -term is of the form $(\underline{27} \times \underline{27})_{\underline{27}} \times (\underline{27} \times \underline{27})_{\underline{27}}$. Since $(\underline{27} \times \underline{27})_S = \underline{351} + \underline{27}$, there are only two independent quartic invariants built with $\underline{27} \times \underline{27}$, so that the F^2 -term can be written as a linear combination of the D^2 -term and a term proportional to $|(\underline{27} \times \underline{27})_{\underline{1}}|^2$. When the D^2 -term vanishes the potential is proportional to the square of the norm of the v.e.v. so that this v.e.v. is protected. Another example is the antisymmetric tensor of rank two $(\underline{15})$ of $SU(6)$, for which v.e.v.'s in the $Sp(6)$ invariant critical orbit are allowed by the D^2 -term, while the F^2 -term vanishes for v.e.v.'s with the little group $SU(4) \times SU(2)$.

Up to now, gauge singlet superfields were omitted, since

supersymmetry was left unbroken. However singlet superfields may have interesting implications, if they are used with some real multiplets, even without any breaking of supersymmetry. It is clear that the analysis of the D^2 -term remains unchanged in presence of singlet fields : they do not contribute to it. In general, the introduction of a singlet X will modify the superpotential by a polynomial $\mu^2 X + m \underline{X}^2 + \ell X^3$ and by couplings to conjugate representations R_i and $\bar{R}_j$ of the form $\lambda_{ij} R_i \bar{R}_j X$. The superpotential in this case contains already terms like $M_{ij} R_i \bar{R}_j$. Thus in general the study of the F^2 -term remains qualitatively unchanged, with M_{ij} replaced by $M_{ij} + \lambda_{ij} \langle X \rangle$. The v.e.v. of X has to satisfy the constraint

$$0 = \mu^2 + 2m\langle X \rangle + 3\ell\langle X \rangle^2 + \sum_{i,j} \lambda_{ij} \langle R_i \rangle \langle \bar{R}_j \rangle$$

The main difference appears when some v.e.v.'s $\langle R_i \rangle$, $\langle \bar{R}_j \rangle$ are protected by the $M_{ij} R_i \bar{R}_j$ term. It is then possible to choose parameters such that $M_{ij} + \lambda_{ij} \langle X \rangle = 0$, and thus to avoid protection of $\langle R_i \rangle$ and $\langle \bar{R}_j \rangle$. However in this case the directions of the v.e.v.'s are not determined.

APPLICATIONS TO GUTS

We have constructed a method to determine if a given pattern of gauge symmetry breaking is consistent with supersymmetry. From the little groups of the v.e.v.'s in the various irreps one can quickly select the generators T^α of the gauge group for which the corresponding $D^\alpha(z)$ might receive contributions from the v.e.v.'s and so check if $D^\alpha(z) = 0$. If this is the case, one has to consider the superpotential, take the v.e.v. of its gradient, $F^a(z)$, and see if a cancellation may occur between terms arising from quadratic and cubic invariants in the superpotential. Since this obviously requires these terms to have the same invariances, most of the analysis of the F^2 -term is performed in terms of the little groups of the v.e.v.'s.

In this section we study the possibility of obtaining the phenomenological $SU(3) \times SU(2) \times U(1)$ gauge group as the residual symmetry in some supersymmetric models, with spontaneously broken gauge groups : $SU(5)$, $O(10)$ and E_6 .

a) $SU(5)$. - A minimal supersymmetric model with $SU(5)$ gauge group has been constructed by Dimopoulos and Georgi [9] and independently by Sakai [10]. One needs at least one adjoint representation (24) and one $(\underline{5} + \bar{\underline{5}})$ of "Higgs" chiral supermultiplets, and $n(\underline{5} + \bar{\underline{10}})$ of "matter" chiral supermultiplets (n is the

number of fermion families). The v.e.v.'s of the $\underline{5}, \bar{\underline{5}}$'s and $\underline{10}$'s have to be protected in order to preserve $SU(3) \times SU(2) \times U(1)$ down to low energies (where SUSY is broken). According to the discussion in Section 2, there are four basic ways in which these complex representations can conspire to give a vanishing D^2 -term, and the associated v.e.v.'s must be protected by the F^2 -term. To these four possibilities there correspond the four invariants : $(\underline{5} \times \underline{5})$, $(\underline{5} \times \underline{10} \times \underline{10})$, $(\underline{5} \times \bar{\underline{5}} \times \underline{10})$ and $(\underline{5} \times \bar{\underline{5}} \times \bar{\underline{5}} \times \bar{\underline{5}} \times \bar{\underline{5}})_A$. The last invariant, of degree five, cannot appear in the superpotential and the corresponding v.e.v.'s are not protected. Since it requires at least five $\underline{5}$'s, the hierarchy is not naturally ensured by supersymmetry if there are more than four families of light fermions.

The superpotential of Dimopoulos, Georgi and Sakai includes all the possible invariants but those which are trilinear in the "matter" fields and so are forbidden by the stability of the proton. One can verify that with this superpotential the simultaneous vanishing of the D^2 -term and the F^2 -term implies the protection of the v.e.v.'s of all these representations. The presence in the superpotential of both the quadratic and the cubic invariants of the $\underline{24}$ allows for these fields to take v.e.v.'s on the critical orbits with little group $SU(3) \times SU(2) \times U(1)$ or $SU(4) \times U(1)$ as discussed in Section 3.

b) $\underline{0}(10)$. - As we have advocated on more general grounds at the end of Section 3, a minimal supersymmetric model based on $\underline{0}(10)$ also requires more irreps of Higgs scalars than the non-supersymmetric versions [21-25]. We will discuss the case with the (Higgs) chiral multiplets in the reducible representation : $\underline{54} + \underline{45} + \underline{16} + \underline{16} + \underline{10}$. The D^2 -term vanishes if one takes the solutions discussed in Section 2 for the real irreps and v.e.v.'s for the $\underline{16}$ and $\underline{16}$ with the same little group and equal strengths. The superpotential has the following types of invariants : $(\underline{54})^2$, $(\underline{54})^3$, $(\underline{45})^2$, $(\underline{54} \times \underline{45} \times \underline{45})$, $(\underline{16} \times \underline{16})$, $(\underline{16} \times \underline{16} \times \underline{45})$, $(\underline{10})^2$, $(\underline{10} \times \underline{16} \times \underline{16})$, $(\underline{10} \times \underline{16} \times \underline{16})$. There is a supersymmetric solution where the v.e.v. of the $\underline{10}$ is indeed protected, the $\underline{16}$ and $\underline{16}$ have $SU(5)$ -invariant v.e.v.'s, the $\underline{54}$ takes v.e.v.'s with little group $\underline{0}(2k) \times \underline{0}(10-2k)$ and the v.e.v.'s of the $\underline{45}$ have little group $U(k) \times U(5-k)$, where $k = 1, 2$. For $k = 2$, the residual symmetry is then $SU(3) \times SU(2) \times U(1)$, as a consequence of the conspiracy between all the irreps. The $\underline{54}$ is needed, otherwise the $\underline{45}$ would choose the $SU(5)$ -invariant direction. The $\underline{16}$ and $\underline{16}$ are included to avoid a larger residual symmetry, $\underline{0}(2k) \times U(5-k)$, $k = 1, \dots, 4$. The $\underline{45}$ is also needed in order to link the little groups of the $\underline{54}$ and the $\underline{16} + \underline{16}$. The $SU(5)$ invariance of the $\underline{16} + \underline{16}$ helps in protecting the $\underline{10}$, which contains the Higgs doublets that break the Weinberg-Salam

$SU(2) \times U(1)$ and give masses to the light fermions once supersymmetry is broken at low energy (the matter supermultiplets are in additional $\underline{16}$'s and appear in the superpotential through invariants of the type $(\underline{10} \times \underline{16} \times \underline{16})$).

c) E_6 . - As an illustration, let us consider the supersymmetric version of an E_6 GUT [24,26,27] with the matter supermultiplets in $\underline{27}$'s and the Higgs chiral multiplets only in the representations that couple to matter in the superpotential, i.e., the $\underline{27}$ and the $\underline{351}_s$. The most general Higgs superpotential with these irreps contains the following invariants : $(\underline{27})^3$, $(\underline{351}_s)^3$, $(\underline{351}_s \times \underline{27} \times \underline{27})$. We have already shown that there is no non-trivial solution if only the $\underline{27}$ is present. We find that there is no solution with little group containing $SU(3) \times SU(2) \times U(1)$ if all the invariants appear in the superpotential. On the other hand, if the invariant $(\underline{351}_s)^3$ is absent, the $\underline{27}$ is still protected, but the $\underline{351}_s$ can take the v.e.v. allowed by the D^2 -term, with little group $O(6) \times O(4)$, as discussed in Section 2.

It seems that a feature of these superGUTS is that more irreps of chiral supermultiplets are needed than the irreps of Higgs scalars in usual GUTS without supersymmetry, and also that complex irreps require the addition of their conjugates, in order to allow for physically interesting v.e.v.'s. In this case all the particles in the Higgs supermultiplets tend to acquire a large mass.

THE PROBLEM OF SUSY BREAKING

In the previous sections, we have considered the first step of symmetry breaking at superheavy energies. In order to protect the hierarchy, supersymmetry was left unbroken. The second step of symmetry breaking will occur at the weak interaction scale; at this stage, supersymmetry and $SU(2) \times U(1)$ must break.

Spontaneous breaking of supersymmetry has well-known features which render its use in realistic models difficult. It will happen when there is no solution to the vanishing of the scalar potential, Eqs (4) and (5). However D^0 is quadratic in the scalar fields, except if the gauge group has a $U(1)$ factor which allows a Fayet-Iliopoulos ξ term [28]. In particular if there is no $U(1)$ factor, the D^2 -term will vanish for zero value of the scalar fields. Correspondingly the F term will be zero for vanishing scalar fields if there is no linear term in the gauge invariant superpotential. Spontaneous supersymmetry breaking might thus occur only if one has either a $U(1)$ factor in the gauge group [28] or gauge singlet chiral multiplets in the theory [29]. We recall that if a semi-simple gauge group is broken at superheavy energies

($\sim 10^{15}\text{GeV}$), the effective supersymmetric theory at intermediate scales (far from 1TeV or 10^{15}GeV) will not contain a Fayet-Iliopoulos ξ term: the D^α 's in the effective theory correspond to generators of unbroken symmetries and these generators annihilate all superheavy v.e.v.'s [8].

Additional $U(1)$ factors must be chosen anomaly-free and traceless in order to avoid quadratic divergences spoiling naturalness of hierarchy. Gauge singlet superfields which break supersymmetry are also useful in some other cases, as it was mentioned in Section 3, to obtain a correct pattern of superheavy gauge symmetry breaking.

The main problem with spontaneous supersymmetry breaking is to reconcile the mass spectrum with the known light states [30]. Unwanted light colored scalars or massless fermions are in general difficult to avoid.

Breaking of supersymmetry might also be introduced explicitly, with non supersymmetric gauge invariant terms in the Lagrangian [9,10]. Restrictions on the possible breaking terms follow from the necessity that the non renormalizability properties of the theory are not spoiled [31]. In particular, these "soft" terms have to be at most of dimension three: they all carry a mass scale which should be of the TeV order. Apart from supersymmetry breaking it is possible in this way to lift the masses of some unwanted light scalars or fermions up to the TeV scale.

Since the breaking of supersymmetry occurs at the TeV scale, the energy of the non supersymmetric ground state will be of the same order. In our analysis of the gauge symmetry breaking in the supersymmetric limit the unprotected v.e.v.'s were of the order of 10^{15}GeV . The value of the potential in the protected directions was of the same order. The breaking of supersymmetry will then not be able to alter the pattern of superheavy gauge breaking; however, if there are degenerate supersymmetric ground states, the true vacuum might be selected with supersymmetry breaking. For instance, we have shown that the adjoint representation of $SU(N)$ and the second rank symmetric traceless tensor of $O(N)$ give a vanishing potential for v.e.v.'s with little group $SU(k) \times SU(N-k) \times U(1)$ and $O(k) \times O(N-k)$ respectively and with $k \neq N/2$. Adding to the Lagrangian a non supersymmetric negative term quadratic in these representations will select unambiguously $k = [\frac{N}{2}] + 1$. Then for instance $SU(5)$ will break into $SU(3) \times SU(2) \times U(1)$ with the 24, $SU(8)$ into $SU(5) \times SU(3) \times U(1)$ with the 63 and $O(10)$ into $O(6) \times O(4)$ with the 54. The addition of these non supersym-

metric terms in the Lagrangian allows us to select the most desirable one among the degenerate supersymmetric ground states.

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