THE WEAK INTERACTION AS AN INDIRECT
MANIFESTATION OF HYPERCOLOR

H. Fritzsch
CERN - Geneva
and
Max-Planck-Institut für Physik und
Astrophysik, München

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1. INTRODUCTION

Recently a large number of authors has been interested in constructing composite models of leptons and quarks. The idea is that the leptons and quarks are composed of several constituents which are bound together by a superstrong force. There exist various constraints on the sizes of leptons and quarks, e.g. the agreement between theory and experiment of the anomalous magnetic moment of the electron which imply that those sizes are less than about 10^{-17} cm.

If the leptons and quarks are composite objects, one may expect the same to be true for at least some of the intermediate (gauge) bosons. A possible point of view is to regard the fermions and the intermediate bosons as composite objects at a very high energy scale (of the order of the mass scale entering in the grand unification schemes ($\sim 10^{15}$ GeV) or more (see e.g. ref. (1)). In this case the interactions at relatively low energies (QCD, flavor interactions) can be interpreted as effective gauge theories; essentially no deviations from the standard pattern of the QCD and QED gauge theory framework are expected.

⁺ On leave from Sektion Physik, Universität München

Here we should like to explore another possible road. We shall suppose that the W bosons and the fermions are bound states, while the massless bosons (photon, gluons) are elementary^{2,3}). The masses of the W bosons are generated dynamically by the binding forces in much the same way the ρ meson mass is generated in QCD. In this approach the weak interactions are indirect manifestations of the strong binding forces inside the W boson. One is reminded of the situation which has evolved during the last 20 years with respect to the strong interactions. Many years ago the strong interactions were interpreted as a gauge theory in which the baryons played the rôle of the elementary fermions while the vector mesons (ρ , ...) were considered to be gauge particles. Today the situation has changed considerably. Both the baryons and the vector mesons are composite objects, consisting of quarks. The strong interactions between the nucleons are regarded as residual effects of the gluonic forces between the quarks.

Perhaps the weak interactions are of a similar nature, and the W bosons consist of constituents which are at the same time the building blocks of the fermions. Of course, we realize that the weak interactions differ in their properties substantially from the strong interactions. First of all, they violate parity. Moreover they are well described by the exchange of vector particles. No exchange forces generated by scalar or tensor particles are observed. Nevertheless we find it useful to explore the point of view outlined above in more detail. The main consequence of the schemes we shall discuss below is that there is no reason to believe that the predictions for the masses of the W and Z bosons made within the minimal $SU(2) \times U(1)$ model are valid. We expect these masses to be much lighter than expected within the standard model ($M_W \sim 100 \dots 160$ GeV, $M_Z \sim 120 \dots 350$ GeV).

2. COMPOSITE MODELS

We take the point of view that the leptons, quarks and W bosons are composed of constituents, while the photon and the gluons of QCD are elementary^{2,3}). In the absence of electromagnetism the global symmetry group of the weak interactions is $SU(2)$ (weak isospin). It will be shown later that the observed structure of the neutral current is obtained if one takes into account the mixing between the photon and the neutral $SU(2)$ boson W_3 . This mixing arises dynamically, due to the electromagnetic annihilation of the W_3 constituents. For this reason only models in which the W bosons consist of two constituents seem realistic. This excludes many composite models, including the rishon model⁴).

We shall assume that the underlying gauge symmetry is given by the group $SU(3)_c \times G_h \times U(1)_e$ (c: color, e: electric charge). The group G_h is the hypercolor gauge group describing the confining forces responsible for the binding of the hypercolored constituents.

The corresponding gauge theory is called QHD. For simplicity we shall use the hypercolor group $SU(n)$, where n is yet unspecified. The extension to other groups is easily made.

If we regard the weak bosons as bound states, several possibilities are open. For example, the weak bosons could be composed of hypercolored fermions or bosons; these fermions or bosons could be colored or color singlets. The weak bosons could be composed of two or more constituents. For reasons which will become clear afterwards, models in which the weak bosons consist of more than two constituents cannot be accepted, likewise models in which the weak bosons are composed of scalar or pseudoscalar bosons. We shall concentrate on schemes in which the weak bosons are composed of spin 1/2 fermions. Furthermore we shall mainly consider the lightest family of leptons and quarks, consisting of the (u,d) -pair and the (ν_e, e^-) -pair.

Two classes of models are possible:

- a) The W-bosons consist of hypercolored fermions, which are color-singlets.
- b) The W-bosons consist of hypercolored and colored fermions.

In both classes the W-bosons are, of course, hypercolor and color singlet bound states. For illustration we consider the following schemes, in which the fermions are composed of (pseudo) scalar and fermionic constituents (haplons). The underlying gauge group is

$$U(1)_e \times SU(3)_c \times G_H$$

(G_H : hypercolor gauge group).

The gauge theory QHD based on the group G_H (hypercolor gauge group) describes the dynamics of the haplon constituents bound together by the superstrong hypercolor forces. The QHD confinement parameter Λ_H is supposed to be of the order of a few hundred GeV ($\sim (G_F)^{-1/2}$). The gauge group G_H is not specified, but taken to be $SU(n)$.

Case A (as far as the quantum numbers of the constituents are concerned, this case has also been considered in ref. (5)).

	spin	el. charge	color	hypercolor
α	1/2	1/2	1	n
β	1/2	-1/2	1	n
x	0	1/6	3	$\bar{n}$
y	0	-1/2	1	$\bar{n}$

The simplest QHD singlets one can form are fermions of electric charges $(2/3, -1/3) = [(\alpha x), (\beta x)]$ and $(0, -1) = [(\alpha y), (\beta y)]$ identified with $(u, d), (\nu_e, e^-)$ respectively.

Scheme B²⁾

	spin	el. charge	color	hypercolor
α	1/2	-1/2	3	n
β	1/2	+1/2	3	n
x	0	-1/6	3	$\bar{n}$
y	0	1/2	$\bar{3}$	$\bar{n}$

This scheme has the special feature that all haplons carry electric charge, color and hypercolor. The simplest QHD singlets are

$$\nu_e = (\bar{\alpha} \bar{y})_1 \quad u = (\bar{\alpha} \bar{x})_3$$

$$e^- = (\bar{\beta} \bar{y})_1 \quad d = (\bar{\beta} \bar{x})_3$$

(the index denotes the color of the state). Both in scheme A and B there exist vector bosons composed of the fermions $(\bar{\alpha}\beta, \bar{\beta}\alpha, \dots)$. Those are interpreted as the carriers of the weak interactions. The global symmetry group SU(2) generated by the haplon doublet $\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$ is identified with the weak isospin.

The observed parity violation of the weak interaction can be accommodated in two different ways:

1. The bound state structure discussed above is assumed to be valid only for the lefthanded fermions.
2. Both the lefthanded and righthanded fermions are bound states, however the Λ_H parameter of the righthanded fermions is larger than the Λ_H parameter of the lefthanded ones, i.e. two QHD group are needed ($G_H(L), G_H(R)$). In this case the global symmetry groups are weak interactions is $SU(2)_L \times SU(2)_R$; the W-bosons coupling to the righthanded fermions are heavier than those which couple to the lefthanded fermions.

Both in the schemes A and B the weak interaction is an effective interaction of the Van der Waals type, generated by the superstrong QHD force. The universality of the weak interaction between leptons and quarks follows from the global SU(2) symmetry in the α - β -space.

The charge assignments made above are not unique. However we find the assignment chosen in both schemes A and B as particularly interesting; note that one of the scalar haplons has charge $1/6$. The fermionic haplons α and β have the charges $\pm 1/2$. This implies in particular that the electric charge is a pure isovector in the weak isotopic space spanned by the (α, β) -doublet.

One could also consider models in which the leptons and quarks are composed of three fermions bound together by a force based on the group $SU(3)_H$. As an illustrative example we consider the following model, based on the group

$$G = SU(3)_H \times SU(3)_C \times U(1)_e.$$

All are fermions, which transform under $SU(3)_H \times SU(3)_C$ as $(3_H, 3_C)$ or $(3_H, \bar{3}_C)$.

We need three different types of constituents:

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} 3_H, 3_C \\ 3_H, 3_C \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3_H, 3_C \\ 3_H, 3_C \end{pmatrix}$$

$$a = (3_H, 3_C).$$

The leptons and quarks are represented as

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} = \begin{pmatrix} \alpha \times a \\ \beta \times a \end{pmatrix}$$

$$\begin{pmatrix} u \\ d \end{pmatrix} = \begin{pmatrix} \alpha \times y \\ \beta \times y \end{pmatrix}$$

(only hypercolor singlets and color singlets / color triplets are taken into account).

Again the weak isospin is the group spanned by the doublet (α, β) . The W-bosons are composed of α and β .

However in this model a serious problem arises - its global symmetry is too large. For example we can easily enlarge the global symmetry from $SU(2)$ to $SU(3)$, generated by the triplet (α, β, a) . As a result we obtain too many new leptons and quarks which are degenerate in mass with the observed leptons/quarks. Such objects are not observed.

It can easily be seen that this disease is not specific to the model outlined above, but affects all models in which the leptons and quarks are composed of three spin 1/2 constituents. For this reason we believe that models in which the leptons and quarks are composed of three spin 1/2 hypercolored constituents are not realistic.

3. THE CHARGED WEAK CURRENT

It is assumed that the spectral functions of the weak currents in QHD are qualitatively similar to the ones in QCD. At low energies they are dominated by the lowest lying pole, and at high energies (energies large compared to Λ_H) they can be described by a continuum of "weak quanta" (this term is borrowed from ref. (6)), i.e. a continuum of $\bar{\alpha}\alpha$, $\bar{\alpha}\beta$ $\bar{\beta}\alpha$, or $\bar{\beta}\beta$ - pairs. The main difference between our approach and the so-called "standard model" of the weak interaction lies in the strength of the W-lepton or W-quark vertex. In the standard model the coupling constant describing this vertex is given by $e/\sin\theta_W$ (θ_W : $SU(2) \times U(1)$ mixing angle, e : electric charge). In our approach the weak interaction is an effective interaction of the van der Waals type, generated by the superstrong QHD force. The universality of the weak interaction follows from the global $SU(2)$ symmetry in the α - β -space. The strength g of the W-quark or W-lepton vertex has nothing to do with the electric charge e , since the electromagnetic interaction is an elementary interaction (like the QCD and QHD interaction). We expect the strength of the W-fermion vertex g to be of similar order as the strength of the p-nucleon vertex ($g \approx 1$) (in this respect our approach is similar to the one discussed in ref. (3)). Due to the relation $g^2/8M_W^2 = G_F/\sqrt{2}$ implying $M_W = g \cdot 123 \text{ GeV}$ the mass of the weak boson may well be much larger than expected in the standard $SU(2) \times U(1)$ gauge theory (in the case $g = 1$ the W-mass is 123 GeV).

4. THE NEUTRAL CURRENT AND THE SIZE OF THE W-BOSON

In the absence of mixing with the electromagnetic current there exist two neutral current channels, the isovector channel described by $(\bar{\alpha}\alpha - \bar{\beta}\beta)/\sqrt{2}$, and the isoscalar channel described by $(\bar{\alpha}\alpha + \bar{\beta}\beta)/\sqrt{2}$. Especially one expects that the isoscalar spectral function is dominated at low energies by the lowest lying meson with the quantum numbers $(\bar{\alpha}\alpha + \bar{\beta}\beta)/\sqrt{2}$. The observed neutral current is a pure isovector, if we neglect for a moment the mixing with the electromagnetic current. The experimental data allow at most a 10 % isoscalar contribution to the neutral current⁷⁾, i.e. the isoscalar

boson W_0 must be much heavier than the isovector boson W_3 (or the Z-boson). The isoscalar boson has the internal quantum numbers of the vacuum, i.e. its mass term may receive contributions from the hypergluon annihilation channel. The situation is quite similar to the one in QCD. Here the η' -meson is about 6 times heavier than the π_0 -meson. It is generally assumed that the source of the mass gap are the gluons of QCD. The isoscalar quark configuration $(\bar{u}u + \bar{d}d) / \sqrt{2}$ can mix with gluonic configurations. This implies that the isoscalar meson is heavier than the isovector meson⁸⁾.

Our approach makes sense only if something similar happens in QHD for the weak bosons. The hypercolor annihilation channels must lead to a rise of the mass of the isoscalar weak boson relative to the mass of the isovector boson. For our further discussion we shall assume that this is indeed the case.

The experimental data on the neutral current interaction require a mixing between the photon and the W_3 boson (the neutral, isovector partner of W^+ and W^-), which in the standard $SU(2) \times U(1)$ scheme is caused by the spontaneous symmetry breaking.

Within our approach this mixing is due the $W_3 - \gamma$ transitions, generated dynamically like the $\rho - \gamma$ transitions in QCD (for an early discussion, based on vector meson dominance, see ref. (9)). The magnitude of $\sin^2 \theta_W$ is directly related to the strength of the $\gamma - W_3$ transition. The latter is determined by the electric charges of the W-constituents and by the W wave function near the origin.

We suppose that in the absence of electromagnetism the weak interactions are mediated by the triplet (W^+ , W^- , W^3), where $M(W^+) = M(W^-) = M(W^3) = 0$ (Λ_H).

After the introduction of the electromagnetic interaction the photon and the W^3 -boson mix. We denote the strength of this mixing by a parameter λ , following ref. (9), which is related to g (W-fermion coupling constant) and the effective value of $\sin^2 \theta_W$

$$\sin^2 \theta_W = \frac{e}{g} \cdot \lambda$$

Furthermore one has:

$$M_W = g \cdot 123 \text{ GeV}$$

$$M_Z^2 = \frac{M_W^2}{1 - \lambda^2}$$

The mixing parameter λ is determined by the decay constant F_W (or f_W) of the W-boson, which we define in analogy to the decay constants of the ρ_0 -meson (F_ρ , f_ρ respectively):

$$\langle 0 | j_\mu^3 | W^3 \rangle = \langle 0 | \frac{1}{2} (\bar{\alpha} \gamma_\mu \alpha - \bar{\beta} \gamma_\mu \beta) | W^3 \rangle = \epsilon_\mu M_W^2 / f_W = \epsilon_\mu M_W F_W$$

One finds:

$$\lambda = \frac{e}{F_W} = e \cdot \frac{F_W}{M_W}$$

In the table we have displayed the numerical values for F_W as a function of g . F_W is a rather sensitive function of g ; for $g = 0.75$ one finds $F_W = 166$ GeV; for $g = 1.2$ one obtains $F_W = 425$ GeV¹⁰).

Table (all masses and energies in GeV; we have used $\sin^2 \theta_W = 0.22$ and $n = 4$)

g	0.75	0.9	1	1.1	1.2
λ	0.54	0.65	0.73	0.80	0.87
M_W	92	111	123	135	148
M_Z	110	146	179	225	300
F_W	166	239	295	357	425
Λ_W	76	103	123	144	166
x	0.82	0.93	1.0	1.06	1.12

In order to study the physics of the bound state structure of the weak bosons in more detail we express the decay constant F_W in terms of the bound state wave function of the weak bosons. We use a non-relativistic wave function, which, of course, cannot be but a very crude approximation.

One has, leaving out irrelevant Lorentz indices in case A:

$$| W^3 \rangle = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{n}} \sum_{i=1}^n (\bar{\alpha}_i \alpha_i - \bar{\beta}_i \beta_i) \phi(x)$$

($\phi(x)$ coordinate space wave function, i : hypercolor index, n : number of hypercolors)

In case B one finds

$$|W^3\rangle = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{n}} \frac{1}{\sqrt{3}} \sum_{i=1}^n \sum_{j=1}^3 (\bar{\alpha}_{i,j} \alpha_{i,j} - \bar{\beta}_{i,j} \beta_{i,j}) \phi(x)$$

(i: color index). The current matrix element can be written as

$$\langle 0 | j_\mu^3 | W^3 \rangle = \epsilon_\mu \sqrt{n} \cdot \gamma \sqrt{2M_W} \cdot \phi(0) = \epsilon_\mu \cdot M_W F_W$$

$$F_W = \sqrt{n} \cdot \gamma \cdot \sqrt{2/M_W} \phi(0)$$

where $\gamma = 1$ in case A and $\gamma = \sqrt{3} = \sqrt{n_c}$ in case B.

$$\begin{aligned} \sin^2 \theta_W &= \frac{e^2}{g} \sqrt{n} \cdot \gamma \sqrt{2/M_W^3} \phi(0), \\ &= e^2 / g \cdot F_W / M_W. \end{aligned}$$

e.g. $\sin^2 \theta_W$ is proportional to the coordinate space wave function of the W-boson at the origin. Taking for example a constant coordinate space wave function, the inverse radius Λ_W of the W-boson and $\phi(0)$ are related: $|\phi(0)|^2 = (4/3\pi)^{-1} \cdot \Lambda_W^3$.

It is useful to introduce the dimensionless quantity $x = (\Lambda_W / M_W)$, and one obtains

$$\sin^2 \theta_W = e^2/g \cdot \sqrt{2n} \gamma \left(\frac{4}{3\pi}\right)^{-1/2} x^{3/2}.$$

For example for $g = 1$, $\gamma = \sqrt{3}$ (case B) and $n = 4$ one finds $x = 1.0$, i.e. $\Lambda_W \approx M_W$. Unless n is very large ($n > 10$), x is not much smaller than one.

It is instructive to compare the structure of the W-bosons with the one of the ρ -meson. In the ρ -case one has

$$\lambda_\rho = \frac{e}{f_\rho} = 0.054,$$

taking $f_\rho = 5.6$. This leads to

$$x_\rho = \Lambda_\rho / m_\rho \approx 0.28.$$

Thus the structure of the ρ -meson differs substantially from the structure of the W-boson. (If the W were similar to the ρ , one would find $\sin^2 \theta_W = 0.016$.)

If the bound state dynamics of the W-bosons were similar to the quark dynamics inside the ρ -meson, the effective $SU(2) \times U(1)$ mixing parameter $\sin^2\theta_W$ would be more than an order of magnitude smaller than observed. In order to accommodate the observed value of $\sin^2\theta_W$, the decay constant F_W and the inverse size Λ_W must be of the same order as the mass of the W-boson. This suggests a similarity of the W bound state dynamics to the bound state dynamics of the π -meson in QCD. In the case of the π -meson one has $F_\pi \approx M_\pi$; the inverse radius of the π -meson, defined analogously to Λ_W , and the pion mass are of the same order, while in the case of the ρ -meson the radius is about four times larger than the inverse mass. One may conclude that the mass of the W-boson is anomalously small, compared to its inverse size.

The pion decay constant is about equal to the Λ -parameter of QCD. Thus the corresponding parameter in the hypercolor dynamics Λ_H is expected to be of the same order as the W-mass, i.e. 100 ... 160 GeV.

Thus far we have assumed that the W-bosons consist of spin 1/2 constituents. In some models spin zero constituents are used as W-constituents (see e.g. ref.(3)). We should like to argue that such models can be ruled out, on the basis of the following argument. The effective $SU(2) \times U(1)$ mixing parameter $\sin^2\theta_W$ is proportional to the wave function at the origin $\phi(0)$. If the W-bosons consist of spin zero constituents, the coordinate space wave function must be a p-wave, i.e. $\phi(0) = 0$. Thus in a nonrelativistic approach one finds $\sin^2\theta_W = 0$. Even if we allow for fairly large relativistic corrections, it seems impossible to us to obtain the observed large value of $\sin^2\theta_W$ in such models.

5. EXOTIC PARTICLES

If the leptons and quarks are composed of spin zero and spin 1/2 haplons, one expects new particles (exotic quarks, leptons, new types of W bosons) to exist and eventually to show up in the experiments. For example, in case B there exist color sextet quarks as well as color octet analogs of the leptons, whose effective masses should be larger than 50 GeV (see ref. (2) and ref. (11)).

6. THE FLAVOR PROBLEM

Thus far our considerations were based on the first family of leptons and quarks. An interesting way to interpret the other families is to suppose that they are excitations of the first family involving QHD-gluons. For example in case B the muon could be interpreted as the state $(\bar{B}\bar{y}h)$ where h denotes a QHD-gluon. The τ -lepton would be the state $(\bar{B}\bar{y}hh)$, etc. The QHD interaction causes transitions between these states, which lead to the phenomenon of weak-interaction mixing. Orbital excitations of the lepton or quark configurations are expected to have a mass of order Λ_H and are not considered here.

In such an approach the problem of stability of the "heavy" leptons μ, τ against radiative decay arises. Since no new quantum numbers are introduced, a decay like $\mu \rightarrow e\gamma$ can occur. The hypergluon turns itself into a photon, and a hypercolor rearrangement takes place. It could well be that such a rearrangement is highly suppressed, and a stability rule similar to the "Zweig rule" in QCD is at work. In any case the interpretation of the higher flavors as hypercolor excitations involving the QHD gluons seems a possible way to resolve the flavor problem.

7. FINAL COMMENTS

Here we have argued that the weak interactions could be a Van der Waals - type interaction, i.e. an indirect manifestation of QHD. If this is true, one expects that the weak interactions look at high energies much different than previously thought. In particular the W boson as well as the Z boson are heavier than expected within the $SU(2) \times U(1)$ gauge model. The width of the Z boson is relatively large, e.g. in the case B one expects $\Gamma(Z_0) \approx 10 \dots 20$ GeV. The leptonic branching ratio $B(Z_0 \rightarrow e^+e^-, \mu^+\mu^-, \dots)$ is much smaller than expected within the $SU(2) \times U(1)$ theory¹¹⁾. Thus it will be fairly difficult to observe the weak bosons in the p-p collider experiments now under way at CERN.

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