X-RAY EMISSION FROM PLANETARY NEBULAE
I. SPHERICALLY SYMMETRIC NUMERICAL SIMULATIONS

MATTHIAS STUTE AND RAGHVENDRA SAHAI
Jet Propulsion Laboratory, California Institute of Technology, 4800 Oak Grove Drive, Pasadena, CA 91109, USA

ABSTRACT

The interaction of a fast wind with a spherical Asymptotic Giant Branch (AGB) wind is thought to be the basic mechanism for shaping Pre-Planetary Nebulae (PPN) and later Planetary Nebulae (PN). Due to the large speed of the fast wind, one expects extended X-ray emission from these objects, but X-ray emission has only been detected in a small fraction of PNs and only in one PPN. Using numerical simulations we investigate the constraints that can be set on the physical properties of the fast wind (speed, mass-flux, opening angle) in order to produce the observed X-ray emission properties of PPNs and PNs. We combine numerical hydrodynamical simulations including radiative cooling using the code FLASH with calculations of the X-ray properties of the resulting expanding hot bubble using the atomic database ATOMDB. In this first study, we compute X-ray fluxes and spectra using one-dimensional models. Comparing our results with analytical solutions, we find some agreements and many disagreements. In particular, we test the effect of different time histories of the fast wind on the X-ray emission and find that it is determined by the final stage of the time history during which the fast wind velocity has its largest value. The disagreements which are both qualitative and quantitative in nature argue for the necessity of using numerical simulations for understanding the X-ray properties of PNs. The X-ray luminosity in the 0.2–10 keV range (1.24–62天文单位) covered by the CHANDRA/ACIS instrument shows a very slow decrease with time over the range of evolutionary ages explored in our models (up to 3750 years). Furthermore, investigating the emission in other wavelength ranges, we find that most of the luminosity emerges at longer wavelengths ($\lambda > 140$天文单位) from the cooler outer edge of the hot bubble. We apply our spherical models to the objects BD+30°3639 and NGC 40. We find that the model values of the X-ray temperature and luminosity for these objects are significantly higher than observed values and discuss several mechanisms for resolving the discrepancies.

Subject headings: circumstellar matter — hydrodynamics — ISM: jets and outflows — planetary nebulae: general — stars: mass loss — X-rays: ISM

1. INTRODUCTION

The shaping of Pre-planetary nebulae (PPN) and Planetary Nebulae (PN) is believed to result from the interaction of a fast, collimated post-AGB wind blowing into the slow, dense wind emitted during the AGB phase, followed by an isotropic tenuous wind during the PN phase (Sahai & Trauger 1998). These interacting winds result in an expanding shell of shocked matter which forms the PPN and later the PN. Due to the large speed of the fast wind (from few $100$ km s$^{-1}$ up to $2000$ km s$^{-1}$), one expects extended X-ray emission to be produced in PPNs (where the shaping mainly occurs) and PNs.

However, X-ray emission was only detected in 3 of 60 PNs observed with ROSAT, followed by a few more from CHANDRA and XMM (e.g. Guerrero et al. 2005). In the case of PPNs, there is so far only one confirmed X-ray detection (Sahai et al. 2003), although many have been observed with CHANDRA. X-ray emission should be present in principle in all of these systems – hence the low detection rate implies that it is obviously below the detection limit.

The general problem of understanding the formation and shaping of PNs has been addressed analytically and numerically (see Balick & Frank 2002, and references therein). In principle, one of the most direct probes of the fast wind and the interaction process which drives PN formation is the X-ray emission. It may help in answering several important questions – e.g. what are the physical properties of the fast wind (speed, mass-flux, opening angle – and the time histories of these parameters). Such studies can also help in identifying the nebular regions that are mainly responsible for the X-ray emission for comparison with recent X-ray images of PNs obtained with CHANDRA and XMM (e.g. Kastner et al. 2000, 2003; Guerrero et al. 2005).

X-ray emission as a probe was first exploited by Zhekov & Perinotto (1996), who derived an analytical, self-similar, spherically symmetric model, taking into account the effects of thermal conductivity. Very recently, Akashi et al. (2006, hereafter ASB06) presented new analytical models with radiative cooling, but without heat conduction. Although analytical modeling is potentially a powerful tool for understanding of the X-ray emission from PNs and its dependence on fundamental physical parameters, it necessitates making certain assumptions which may or may not be completely valid. For example, a major problem with such analytical models is, that they are not able to treat all the effects (hydrodynamics, radiative cooling, heat conduction) self-consistently. In this paper, we have therefore performed numerical simulations using the code FLASH in which we vary the basic parameters of the fast and slow wind over an extensive parameter grid and compute the X-ray emission as a function of time.
these parameters. A major goal of our work here is to test the main results from ASB06’s analytical modeling and verify their general conclusions. We have therefore, as in ASB06, not included heat conduction in our treatment. However, we discuss the heat conduction process and how it may be inhibited by magnetic fields which are likely to be present in PNs. For example, the lack of spherical symmetry in most PNs has been explained by some authors as resulting from the presence of magnetic fields. In this paper, in which we have carried out spherically symmetric simulations of interacting winds, is a first in a series of papers in our quest to understand X-ray emission from PPNs and PNs.

The rest of the paper is organized as follows. In §2, we describe our numerical model. In §3, we present the results of our one-dimensional simulations including the structure and X-ray properties of the expanding bubble and their dependency on the parameters of the fast wind. In §4, new effects from the computation of a spherically symmetrical model on a two-dimensional grid are shown. In §5, we apply our models to two PNs, BD+30°3639 and NGC 40, and examine several mechanisms for resolving the discrepancies between the models and observed values of the X-ray temperature $T_X$ and luminosity $L_X$ for these objects. We compare in detail our results with those of ASB06 in §6 and discuss the still unsolved problem of fitting $T_X$ and $L_X$ in PNs in §7. Finally in §8, we present our conclusions.

2. THE NUMERICAL METHOD

2.1. Initial and boundary conditions

Using FLASH, we solve the following set of the differential equations of ideal hydrodynamics

$$
\frac{\partial \rho}{\partial t} + \nabla (\rho \mathbf{v}) = 0
$$

$$
\frac{\partial (\rho \mathbf{v})}{\partial t} + \nabla (\rho \mathbf{v} \otimes \mathbf{v}) = -\nabla p
$$

$$
\frac{\partial e}{\partial t} + \nabla (e \mathbf{v}) = -p \nabla \mathbf{v} - \nabla^2 \Lambda(T)
$$

$$
p = (\gamma - 1) e.
$$

where $\rho$ is the gas density, $p$ the pressure, $e$ the internal energy density, $\mathbf{v}$ the velocity and $\gamma$ the ratio of the specific heats at constant pressure and volume which is set to $\gamma = 5/3$. We include radiative losses using the cooling function $\Lambda(T)$ of Sutherland & Dopita (1993) in the temperature range between $10^4$ and $10^6$ K and the low temperature function of Dalgarno & McCray (1972) for $T < 10^4$ K.

A spherically symmetric AGB wind is implemented with

$$
\rho_s = \frac{\dot{M}_s}{4 \pi r^2 v_s} \equiv \rho_{01} r^{-2}
$$

and

$$
\vec{v}_s = v_s \hat{e}_r.
$$

The temperature of the AGB wind is set to 10 K. In this $r^{-2}$-density profile, we let a collimated fast wind (CFW) plow from the inner radial boundary with the following parametrization:

$$
\rho_l = \frac{\dot{M}_l}{4 \pi r^2 v_l K} \exp \left(-\frac{\theta}{\theta_1}^2\right)
$$

and

$$
\vec{v}_l = v_l \exp \left(-\frac{\theta}{\theta_1}^2\right) \vec{e}_r.
$$

$\theta_1$ is the opening angle of the CFW. $K$ is a normalization factor calculated by

$$
K = \int_0^{\pi/2} \exp \left(-2\frac{\theta}{\theta_1}^2\right) \sin \theta d\theta.
$$

For a temperature of the CFW, we assume $10^4$ K. The simulations are performed in spherical coordinates with the dimensions of $r = 5 \times 10^{10} - 2.5 \times 10^{17}$ cm and $\theta = 0 - \pi/2$ in the case of the two-dimensional runs. Mirror boundaries are used along the symmetry axis and the equatorial plane and outflow conditions at the upper radial boundary.

2.2. Model parameters

The models which are presented in this paper are spherical ones without the $\theta$ dependency in eqns. 4 – 5 and $K = 1$. Preliminary results of models with $\theta$ dependency (i.e. incorporating collimated fast winds) were published in (Stute & Sahai 2006). Detailed results are deferred to a future publication. The parameters of the models are then the mass outflow rate and velocity of the fast and the slow wind, respectively. For these parameters we adopted the same ranges as in the models of ASB06 for ease of comparison (see Table 1). Thus we performed three sets of runs with $\dot{M}_s$ of $3 \times 10^{-6}$ (runs A), $7 \times 10^{-6}$ (runs B) and $3 \times 10^{-5}$ M$_\odot$ yr$^{-1}$ (runs C) and within each set we varied the velocity of the fast wind from 300 to 700 km s$^{-1}$ (e.g. the 5 in C5 means $v_f = 500$ km s$^{-1}$). The velocity of the slow wind was set to 10 km s$^{-1}$ and $M_l$ was chosen such that $M_l v_f = M_s v_s$. We expanded these sets while applying our models to observations.

2.3. Calculating the X-ray properties

Using the radial density and temperature profiles from the hydrodynamical simulations, we determine the expected X-ray flux. We have used the atomic database ATOMDB with IDL including the Astrophysical Plasma Emission Database (APED) and the spectral models output from the Astrophysical Plasma Emission Code (APEC) (Smith et al. 2001) to calculate the emissivity. The default abundances in ATOMDB, i.e. 14 elements (H, He, C, N, O, Ne, Mg, Al, Si, S, Ar, Ca, Fe, Ni) with solar abundances of Anders & Grevesse (1989) are used. The energy range is divided into bins of 0.01 keV. We compute the spectrum and the total flux in X-rays as a function of evolutionary time for each of our models.

X-ray emission in an interacting winds model, in principle, will arise from gas with a large range of temperatures above $10^4$ K. We calculate the X-ray emission between 0.01 - 10 keV. The lower energy boundary is determined by the limitations of ATOMDB. We have divided this range into three bins. The high energy bin (0.2 – 10 keV or 1.24 – 62 Å) represents the energy range covered by the ACIS instrument and (almost) that of HETG (0.4 – 10 keV) on CHANDRA. We define a medium energy bin (0.09 – 0.2 keV or 62 – 137 Å), since the energy ranges of the EPIC instrument on XMM (0.1 – 12 keV)
and of LETG on CHANDRA (0.09 – 3 keV) extend to lower energies, and a low energy bin between 0.01 – 0.09 keV (137 – 1180 Å) which is sensitive to gas with temperatures $\lesssim 10^6$ K. In Fig. 1, we show how the X-ray emissivity varies as a function of temperature in the three energy bins we have defined above. It is interesting to note that gas with a temperature lower than $2 \times 10^6$ K contributes only marginally in the high energy bin which is probed by the ACIS instrument on CHANDRA (Fig. 1, bottom).

### 3. ONE-DIMENSIONAL SIMULATIONS

3.1. Structure

The profiles of density and temperature as a function of radius (Fig. 2; runs B3, B5 and B7 at an age of 390 years) show four different regions: (i) the unshocked fast wind, (ii) the shocked fast wind (hot bubble), (iii) the shocked slow wind forming a dense shell and (iv) the unshocked slow wind. The reverse shock separates region (i) from (ii). The large jump in density and temperature between regions (ii) and (iii) represents the contact discontinuity. The outer shock separates region (iii) from (iv). Due to the interaction of the fast and the slow wind, both regions of shocked matter are heated to high temperatures, but the dense shell is highly radiative and cools quickly. The evolution of the shell normally has the following stages: first there is free expansion, if cooling sets in there is then the radiative, momentum-driven bubble and finally the adiabatic, energy-driven bubble.

Depending on the velocity of the fast wind, namely if it is larger or smaller than a critical velocity

$$v_{cr} = \left( \frac{\mathcal{L}}{6 \pi \rho_0} \right)^{1/3} \left( \frac{1}{3} \frac{M_t}{M_s} \frac{v_f^2}{v_s} \right)^{1/3}, \quad (7)$$

the end stage is then the momentum-driven or energy-driven bubble (Koo & McKee 1992a,b), where $\mathcal{L} = 1/2 M_t v_f^2$ is the kinetic luminosity of the fast wind. This critical velocity is independent of the cooling function in the case of a constant mass outflow rate of the fast wind and a $r^{-2}$ density profile (Koo & McKee 1992b). As the fast wind is always faster than $v_{cr}$ in our models, the bubble is expected to go from free expansion

### Table 1

<table>
<thead>
<tr>
<th>run</th>
<th>$M_t$</th>
<th>$v_f$</th>
<th>$M_s$</th>
<th>$v_s$</th>
<th>$v_{cr}$</th>
<th>$R_{cd}/t$</th>
<th>$R_{Md}/t$</th>
<th>$I$</th>
<th>$v_{ASB}$</th>
<th>$v_{cd,sim}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A3</td>
<td>$1 \times 10^{-7}$</td>
<td>300</td>
<td>$3 \times 10^{-6}$</td>
<td>10</td>
<td>21.54</td>
<td>27.51</td>
<td>9.58</td>
<td>0.033</td>
<td>28.6</td>
<td>25.16</td>
</tr>
<tr>
<td>A5</td>
<td>$6 \times 10^{-8}$</td>
<td>500</td>
<td>$3 \times 10^{-6}$</td>
<td>10</td>
<td>25.54</td>
<td>32.62</td>
<td>9.8</td>
<td>0.02</td>
<td>32.6</td>
<td>28.47</td>
</tr>
<tr>
<td>A7</td>
<td>$4.3 \times 10^{-8}$</td>
<td>700</td>
<td>$3 \times 10^{-6}$</td>
<td>10</td>
<td>25.54</td>
<td>36.62</td>
<td>9.66</td>
<td>0.014</td>
<td>35.6</td>
<td>30.92</td>
</tr>
<tr>
<td>B3</td>
<td>$2.3 \times 10^{-7}$</td>
<td>300</td>
<td>$3 \times 10^{-6}$</td>
<td>10</td>
<td>21.54</td>
<td>27.51</td>
<td>9.58</td>
<td>0.033</td>
<td>28.6</td>
<td>25.16</td>
</tr>
<tr>
<td>B4</td>
<td>$1.75 \times 10^{-7}$</td>
<td>400</td>
<td>$7 \times 10^{-6}$</td>
<td>10</td>
<td>23.71</td>
<td>30.28</td>
<td>9.75</td>
<td>0.025</td>
<td>30.8</td>
<td>25.70</td>
</tr>
<tr>
<td>B5</td>
<td>$1.4 \times 10^{-7}$</td>
<td>500</td>
<td>$7 \times 10^{-6}$</td>
<td>10</td>
<td>25.54</td>
<td>32.62</td>
<td>9.8</td>
<td>0.02</td>
<td>32.6</td>
<td>26.96</td>
</tr>
<tr>
<td>B6</td>
<td>$1.17 \times 10^{-7}$</td>
<td>600</td>
<td>$7 \times 10^{-6}$</td>
<td>10</td>
<td>27.17</td>
<td>34.7</td>
<td>10.03</td>
<td>0.017</td>
<td>34.2</td>
<td>28.10</td>
</tr>
<tr>
<td>B7</td>
<td>$1 \times 10^{-7}$</td>
<td>700</td>
<td>$7 \times 10^{-6}$</td>
<td>10</td>
<td>28.58</td>
<td>36.49</td>
<td>9.66</td>
<td>0.014</td>
<td>35.6</td>
<td>29.10</td>
</tr>
<tr>
<td>C3</td>
<td>$1 \times 10^{-6}$</td>
<td>300</td>
<td>$3 \times 10^{-5}$</td>
<td>10</td>
<td>21.54</td>
<td>27.51</td>
<td>9.58</td>
<td>0.033</td>
<td>28.6</td>
<td>26.60</td>
</tr>
<tr>
<td>C5</td>
<td>$6 \times 10^{-7}$</td>
<td>500</td>
<td>$3 \times 10^{-5}$</td>
<td>10</td>
<td>25.54</td>
<td>32.62</td>
<td>9.8</td>
<td>0.02</td>
<td>32.6</td>
<td>25.05</td>
</tr>
<tr>
<td>C7</td>
<td>$4.28 \times 10^{-7}$</td>
<td>700</td>
<td>$3 \times 10^{-5}$</td>
<td>10</td>
<td>28.57</td>
<td>36.47</td>
<td>9.66</td>
<td>0.014</td>
<td>35.6</td>
<td>26.88</td>
</tr>
<tr>
<td>A10</td>
<td>$3 \times 10^{-8}$</td>
<td>1000</td>
<td>$3 \times 10^{-6}$</td>
<td>10</td>
<td>32.18</td>
<td>41.10</td>
<td>9.90</td>
<td>0.01</td>
<td>36.56</td>
<td></td>
</tr>
<tr>
<td>B10</td>
<td>$7 \times 10^{-8}$</td>
<td>1000</td>
<td>$7 \times 10^{-6}$</td>
<td>10</td>
<td>32.18</td>
<td>41.10</td>
<td>9.90</td>
<td>0.01</td>
<td>33.43</td>
<td></td>
</tr>
<tr>
<td>D7</td>
<td>$1 \times 10^{-6}$</td>
<td>700</td>
<td>$7 \times 10^{-5}$</td>
<td>10</td>
<td>28.58</td>
<td>36.49</td>
<td>9.66</td>
<td>0.014</td>
<td>25.99</td>
<td></td>
</tr>
<tr>
<td>E10</td>
<td>$2.4 \times 10^{-6}$</td>
<td>1000</td>
<td>$5 \times 10^{-4}$</td>
<td>10</td>
<td>25.19</td>
<td>32.17</td>
<td>6.88</td>
<td>0.007</td>
<td>27.03</td>
<td></td>
</tr>
<tr>
<td>B5 (2D)</td>
<td>$1.4 \times 10^{-7}$</td>
<td>500</td>
<td>$7 \times 10^{-6}$</td>
<td>10</td>
<td>25.54</td>
<td>32.62</td>
<td>9.8</td>
<td>0.02</td>
<td>22.4</td>
<td></td>
</tr>
</tbody>
</table>

**Note.** — The first five columns give the parameters of our models: the name of the run, the mass outflow rate and velocity of the fast and the slow wind, respectively. The next four columns give results of the analytical solution of Koo & McKee (1992a,b): the critical velocity, the velocity of the contact discontinuity in the energy-driven case and in the momentum-driven case and the constant $I$. The velocity used by ASB06 following a formula in Volk & Kwok (1985) is given in the column before last, the last column shows the measured velocity of the contact discontinuity $v_{cd,sim}$ in the simulations.

![Fig. 1](image-url)

**Fig. 1.** — Top: X-ray emissivity between 0.01 – 10 keV (solid), 0.09 – 10 keV (dash-dotted) and 0.2 – 10 keV (dashed) derived from the ATOMDB as a function of the plasma temperature; bottom: X-ray emissivity in the three different energy bins 0.01 – 0.09 keV (solid), 0.09 – 0.2 keV (dash-dotted) and 0.2 – 10 keV (dashed).
phase directly to the energy-driven phase. This transition occurs in our models within a short period of time (about few years), after which the velocity of the contact discontinuity reaches a stable value characteristic of an energy-driven shell. In the momentum-driven case, the position of the shell would be (Koo & McKee 1992a,b)

\[ R_{cd}^M = \frac{T}{1 + \frac{2}{\Gamma M s}} \approx \left( \frac{M f v f v s}{M s} \right)^{1/2} t, \]

with

\[ T = \frac{\frac{L}{2 \pi \rho v_0}}{\frac{\rho v_0}{\Gamma}} \left( \frac{M f v f v s}{M s} \right)^{1/2}. \]

The resulting velocity of about 10 km s\(^{-1}\) (Table 1) are significantly lower than those found in our simulations. In the energy-driven, adiabatic case, the analytical solutions of the position of the shell, \(R_{cd}\), is

\[ R_{cd}^E = \left( \frac{\Gamma_{rad} \xi L}{3 \rho v_0} \right)^{1/3} t = \left( \frac{2 \pi \Gamma_{rad} \xi M f v f^2 v s}{3 M s} \right)^{1/3} t, \]

with \(4 \pi \Gamma_{rad} \xi = 4.165\) (Koo & McKee 1992a,b). Here the ambient shock is assumed to be adiabatic. Assuming a radiative ambient shock, \(4 \pi \Gamma_{rad} \xi = 2.0\) and the velocity of the shell reduces to \(v_{cr}\). \(\Gamma_{rad}\) is the fraction of the injected energy in the bubble which is radiated away, \(\xi\) is a numerical constant of order unity (Koo & McKee 1992b). The velocity of the contact discontinuity \(v_{cd, sim}\) in our simulations is within the range of velocities between the limiting values given by the radiative and adiabatic approximations – \(v_{cr}\) and \(R_{cd}^E / t\) – for the dense shell of Koo & McKee (1992a,b) (see Table 1).

The density and temperature of the hot bubble are in good agreement with results from the self-similar formulation of Chevalier & Imamura (1983), although radiative cooling emphasizes the density and temperature jumps close to the contact discontinuity. For example in our model B5, the density is about \(3 \times 10^{-22}\) g cm\(^{-3}\), i.e. a number density of about 200 cm\(^{-3}\), and the temperature is \(5 \times 10^6\) K. Our density is slightly lower than the value derived from the self-similar formulation of 350 cm\(^{-3}\) – this leads to a reduced cooling efficiency, and thus our temperature is slightly higher than the value from the self-similar model of \(3.4 \times 10^6\) K.

The temperature of the hot bubble is given in Table 2. We calculated an X-ray weighted average temperature as defined in ASB06. It is only a function of the velocity of the fast wind, ranging from \(1.5 \times 10^6\) K for \(v_f = 300\) km s\(^{-1}\) to \(2.2 \times 10^7\) K for \(v_f = 1000\) km s\(^{-1}\). The mass loss rate of the slow wind is fairly unimportant, i.e., the temperature is almost identical, e.g., in the models A3, B3 and C3. The temperature is roughly given by

\[ T_{hb} = \frac{1.4 \times 10^6 K}{(v_f / 300 \text{ km s}^{-1})^2} \]

in which the dependence on only \(v_f\) is as expected from the Rankine-Hugoniot conditions (e.g. Landau & Lifshitz 1959). As the bubble evolves, the temperature of the hot bubble remains fairly constant with time.

3.2. X-ray properties

We now present the properties of the X-ray emission from our models and their dependence on the physical parameters of the fast wind.

3.2.1. The X-ray luminosities

Several clear trends in the X-ray luminosity are visible between the different sets of simulations. We show the X-ray luminosity in the high energy bin between 0.2 – 10 keV, \(L_{X,\text{ACIS}}\), as a function of the evolution time for all models in Fig. 3. Comparing models from different sets with fixed \(M_s\) which have the same fast wind velocity, we find that the higher the mass outflow rate of the fast wind is, the higher \(L_{X,\text{ACIS}}\) is. For example, at an age of 470 years, in runs A3, B3 and C3 \(L_{X,\text{ACIS}}\) is \(0.1 \times 10^{32}\) erg s\(^{-1}\), \(0.4 \times 10^{32}\) erg s\(^{-1}\) and \(2.4 \times 10^{32}\) erg s\(^{-1}\), respectively. In these runs, the fast wind velocity is 300 km s\(^{-1}\) and \(M_f\) is \(10^{-7}\), \(2.33 \times 10^{-7}\) and \(10^{-6}\) M\(_\odot\) yr\(^{-1}\), respectively.

Comparing the curves within each panel (each panel represents a fixed \(M_f\) and thus a constant \(M_f v_f\)), \(L_{X,\text{ACIS}}\) decreases with increasing fast wind velocity. This is because the correspondingly lower mass outflow rate of the fast wind\(^1\) leads to lower densities in the shocked fast wind and therefore to reduced X-ray emission.

We can also investigate the dependence of \(L_{X,\text{ACIS}}\) on \(v_f\), when \(M_f\) is constant. Using pairs of our runs, as

\(^1\) A result of the assumption \(M_f v_f = M_s v_s\)


Table 2

<table>
<thead>
<tr>
<th>run</th>
<th>$R_{cd}$ (10$^{16}$ cm)</th>
<th>$R_{cd}/v_{ASB}$ (yrs)</th>
<th>$L_{ASB}$ (erg s$^{-1}$)</th>
<th>$L_{x, ACIS}$ (erg s$^{-1}$)</th>
<th>$T_{hb}$ (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A3</td>
<td>3.4</td>
<td>377</td>
<td>$1.3 \times 10^{31}$</td>
<td>$4.33 \times 10^{31}$</td>
<td>$1.5 \times 10^{6}$</td>
</tr>
<tr>
<td>A5</td>
<td>3.9</td>
<td>379</td>
<td>$2.0 \times 10^{33}$</td>
<td>$2.48 \times 10^{33}$</td>
<td>$4.5 \times 10^{6}$</td>
</tr>
<tr>
<td>A7</td>
<td>4.3</td>
<td>383</td>
<td>$1.3 \times 10^{31}$</td>
<td>$1.20 \times 10^{31}$</td>
<td>$8.8 \times 10^{6}$</td>
</tr>
<tr>
<td>B3</td>
<td>3.2</td>
<td>365</td>
<td>$6.9 \times 10^{44}$</td>
<td>$1.64 \times 10^{44}$</td>
<td>$1.5 \times 10^{8}$</td>
</tr>
<tr>
<td>B4</td>
<td>3.5</td>
<td>360</td>
<td>$1.48 \times 10^{32}$</td>
<td>$3.0 \times 10^{6}$</td>
<td></td>
</tr>
<tr>
<td>B5</td>
<td>3.7</td>
<td>361</td>
<td>$9.48 \times 10^{33}$</td>
<td>$4.9 \times 10^{6}$</td>
<td></td>
</tr>
<tr>
<td>B6</td>
<td>3.8</td>
<td>352</td>
<td>$6.84 \times 10^{33}$</td>
<td>$7.1 \times 10^{6}$</td>
<td></td>
</tr>
<tr>
<td>B7</td>
<td>4.0</td>
<td>356</td>
<td>$7.6 \times 10^{34}$</td>
<td>$4.31 \times 10^{34}$</td>
<td>$9.6 \times 10^{6}$</td>
</tr>
<tr>
<td>C3</td>
<td>3.0</td>
<td>332</td>
<td>$&lt; 2.0 \times 10^{34}$</td>
<td>$1.01 \times 10^{34}$</td>
<td>$1.7 \times 10^{6}$</td>
</tr>
<tr>
<td>C5</td>
<td>3.4</td>
<td>330</td>
<td>$1.4 \times 10^{33}$</td>
<td>$5.03 \times 10^{32}$</td>
<td>$5.3 \times 10^{6}$</td>
</tr>
<tr>
<td>C7</td>
<td>3.7</td>
<td>329</td>
<td>$1.3 \times 10^{33}$</td>
<td>$2.40 \times 10^{32}$</td>
<td>$1.1 \times 10^{6}$</td>
</tr>
<tr>
<td>A10</td>
<td>4.7</td>
<td>$-$</td>
<td>$2.84 \times 10^{39}$</td>
<td>$1.8 \times 10^{4}$</td>
<td></td>
</tr>
<tr>
<td>B10</td>
<td>4.4</td>
<td>$-$</td>
<td>$1.00 \times 10^{33}$</td>
<td>$2.0 \times 10^{7}$</td>
<td></td>
</tr>
<tr>
<td>D7</td>
<td>4.0</td>
<td>$-$</td>
<td>$9.05 \times 10^{32}$</td>
<td>$1.1 \times 10^{7}$</td>
<td></td>
</tr>
<tr>
<td>E10</td>
<td>3.0</td>
<td>$-$</td>
<td>$2.34 \times 10^{33}$</td>
<td>$2.2 \times 10^{7}$</td>
<td></td>
</tr>
</tbody>
</table>

Note. — The X-ray luminosity $L_{x, ACIS}$ is in the high-energy bin 0.2 – 10 keV at an age of 470 years in our simulations and results $L_{ASB}$ from ASB06 (from epochs where the bubbles have equal size in our models and those of ASB06, since the velocity of the contact discontinuity is slightly different in our models compared to ASB06); X-ray weighted average temperature of the hot bubble $T_{hb}$, defined in ASB06 at an age of 470 years.

e.g. runs A3 and B7 (in which $M_f = 10^{-7}$ M$_{\odot}$ yr$^{-1}$) or C3 and D7 (M$_f = 10^{-6}$ M$_{\odot}$ yr$^{-1}$), we can see that the luminosities are almost the same in each pair (Table 2), even though, due to the different velocities, the temperatures in the hot bubble are very different. This is a somewhat surprising result, but which can be understood in terms of the important physical quantities determining the X-ray luminosity. These can be the volume, density and temperature of the hot bubble. The volumes of the hot bubble in run C3 and D7 are about 6.84 x 10$^{49}$ cm$^3$ and 2.2 x 10$^{49}$ cm$^3$, respectively (derived from $R_{cd} = 3.0 \times 10^{16}$ cm and $R_{as} = 2.2 \times 10^{16}$ cm for C3 and $R_{cd} = 3.9 \times 10^{16}$ cm and $R_{as} = 2.2 \times 10^{16}$ cm for D7). The mean (radially averaged) density in the hot bubble for C3 (D7) is 1.66 x 10$^{-21}$ g cm$^{-3}$ (6.43 x 10$^{-22}$ g cm$^{-3}$), the emissivity at the mean temperature of the hot bubble $-1.73 \times 10^{6}$ K (1.06 x 10$^{6}$ K) is $3.0 \times 10^{-10}$ erg s$^{-1}$ cm$^{-2}$ (2.63 x 10$^{-12}$ erg s$^{-1}$ cm$^{-2}$), thus the luminosity is $9.7 \times 10^{32}$ erg s$^{-1}$ (8.6 x 10$^{32}$ erg s$^{-1}$). These estimates are in good agreement with the more accurate values of the luminosity in Table 2. In the other pair A3 and B7, the differences between the estimates using radially averaged quantities and the values in Table 2 are somewhat larger – the luminosities in runs A3 and B7 are 4.9 x 10$^{32}$ erg s$^{-1}$ and 3.9 x 10$^{32}$ erg s$^{-1}$, respectively, compared to 4.3 x 10$^{32}$ erg s$^{-1}$ in both cases in Table 2. This is because the density and temperature gradients across the hot bubble are larger than in the first pair and therefore also the errors, we introduce by the averaging process.

In summary, the weak dependence of the X-ray luminosity, $L_{x, ACIS}$, on $v_f$ has two reasons: i) the emissivity in the high energy bin is fairly flat over the range of temperatures in our models (Fig. 1) and ii) the volume of the hot bubble depends weakly on $v_f$ ($R_{cd} \sim v_f^{-1/3}$) due to our assumption of $M_f$ = $M_{ac}v_f$. In Fig. 4, we illustrate that $L_{x, ACIS}$ depends much more strongly on $M_f$ than on $v_f$ by plotting the values for all computed models (therefore the full range of $v_f$). The data can be reasonably well fitted by the curve $L_{x, ACIS} \sim M_f^{1/2}$ — the scatter of the individual data points above and below this curve lies within a factor of 2. Interestingly, even model E10 where $M_f v_f = 0.5 M_{ac} v_f$ fits the $L_{x, ACIS} \sim M_f^{1/2}$ relationship.

We now show that this variation of $L_{x, ACIS}$ with $M_f$ can be partially understood by considering the dependencies of the volume ($V$), temperature ($T_{hb}$), and density of the hot bubble ($\rho_{hb}$) on $M_f$ and $v_f$ across and within the different sets of models. We utilize analytical estimates of these dependencies based on the self-similar solutions without radiative cooling (see §3.1), therefore we expect some differences between the dependencies derived below and those actually found from our modeling (which takes radiative cooling effects into account). The volume of the hot bubble depends on the radius of the reverse shock ($R_{as} = [3 M_f v_f/(4 M_{ac})]^{1/2} t = \sqrt{3/4} v_f t$) and the contact discontinuity (given by eqn. 7). In the range of $T_{hb}$ in our models ($1.5 \times 10^{6} - 2 \times 10^{7}$ K), the emissivity changes only within a factor of 1.5 (Fig. 1) and therefore we assume it to be constant. The density of the hot bubble can be approximated as constant with radius (Fig. 2) and equal to the density of the fast wind given by eqn. (4) at the reverse shock scaled by a constant compression factor of 4, valid for an adiabatic shock.

In general, $L_{x, ACIS}$ is a function of the 4 parameters in our models $- v_f, M_f, v_h$ and $M_{ac}$. Since $v_h$ is kept constant in all our models and we assume $M_f v_f = M_{ac} v_f$, the number of independent parameters is only two — we select $v_f$ and $M_f$. By holding each of these constant in turn one can get insights into the behavior of $L_{x, ACIS}$ as a function of $M_f$.

First we keep $v_f$ constant, i.e. we compare models from different sets with the same $v_f$. Then the radius of the contact discontinuity is constant, thus also the volume. Hence $\rho_{hb} \sim M_f$ and $\rho_{hb} \sim M_f^2$. Therefore we expect $L_{x, ACIS}$ to vary as $M_f^2$. In our models, the runs with the same $v_f$ follow $L_{x, ACIS} \sim M_f^{1.35}$, which is primarily caused by a departure from the analytical estimate of the model (radially-averaged) density dependency, $< \rho > \sim M_f^{0.75}$.

Next we keep $M_{ac}$ constant, i.e. we compare mod-
els within each set, thus \( v_f \dot{M}_f \) is constant. Hence \( R_{cd} \propto v_f^{1/3} \propto \dot{M}_f^{-1/3} \) and therefore \( V \propto \dot{M}_f^{-1} \). The density \( \rho_{\text{h}b} \propto \dot{M}_f^2 \) and therefore \( \rho_{\text{h}b} V \propto \dot{M}_f^3 \), implying \( L_{x,\text{ACIS}} \propto \dot{M}_f^3 \). This expectation is supported by the steep increase in \( L_{x,\text{ACIS}} \) with \( \dot{M}_f \) which we see in sets A and B for models with the lower values of \( \dot{M}_f \), i.e. with the larger values of \( v_f \) (500 to 1000 km s\(^{-1}\)). For our models with lower values of \( v_f \), however, the increase is not as steep, due to two effects: (i) deviations from the analytical dependencies as well as (ii) the replacement of the radial integral for \( L_{x,\text{ACIS}} \) with the product of radial-averages. For example, as a result of effect (i) in our models B3 and B4 we find \( R_{\text{en}} \propto \dot{M}_f^{0.35}, R_{cd} \propto \dot{M}_f^{0.28}, V \propto \dot{M}_f^{-1.21}, \rho_{\text{h}b} \propto \dot{M}_f^{1.84} \). Furthermore the emissivity in run B3 in 30% lower compared to that in B4, equivalent to \( \Lambda \propto \dot{M}_f^{-0.9} \). The net result is that

\[ L_{x,\text{ACIS}} \sim \rho_{\text{h}b}^2 V \Lambda \sim \dot{M}_f^{2.6}. \]

As a result of effect (ii), the radial integral becomes increasingly overestimated by the product of radially-averaged quantities for models with the lowest values of \( v_f \), where the radial gradients of these quantities are the largest (e.g. compare run B3 and B5 in Fig. 2). Hence the expected dependence of \( L_{x,\text{ACIS}} \) on \( \dot{M}_f \) will be even shallower as observed.

The X-ray luminosities in our different models do not change strongly with time as the bubble evolves, after an initial phase of increasing luminosity (e.g. a period of \( \sim 100 \) years in run C3). The length of the initial phase of rising luminosity is primarily given by the cooling time in the hot bubble. Once a gas parcel has cooled below 10\(^6\) K, it does not contribute to the luminosity \( L_{x,\text{ACIS}} \) of the hot bubble anymore. Within each set, the lower the value of \( v_f \), the lower the temperature and the higher the density in the hot bubble, thus the shorter the cooling time. Hence in these cases, the length of the initial phase is the highest. We discuss the weak dependence of \( L_{x,\text{ACIS}} \) on time and the initial phase of rising luminosity and compare our results with those of ASB06 in §6.

The relation \( L_{x,\text{ACIS}} \sim \dot{M}_f^{3/2} \) shown in Fig. 4 appears to hold for ages of PNs up to \( \sim 2000 \) years, presumably because of the similar weak decay of \( L_{x,\text{ACIS}} \) with time in all models.

Up to now, we only took into account luminosities in the high energy bin 0.2 – 10 keV, which can be observed with the ACIS instrument. An inspection of the X-ray luminosity emitted in the lower energy bins in our models may be of potential use in identifying new observational probes of cooler gas in the hot bubble near its interface with the dense shell. Plotting the X-ray emissivity and luminosity in our three energy bins (e.g. for run B5), we find that most of the emission from the main body of the hot bubble, i.e. excluding its edges, is radiated in the high energy bin. However, the total energy radiated by the main body of the hot bubble in these three energy bins \( L_{\text{h}b,m} (1.4 \times 10^{32} \text{ erg s}^{-1}) \) is significantly smaller than the rate at which mechanical energy is pumped.
into the system by the fast wind, $L_{\text{fw}} = 1.1 \times 10^{34}$ erg s$^{-1}$. This mechanical energy is partly converted into the mechanical energy of the dense shell, which increases at a rate given by

$$L_{\text{ds}} = \frac{1}{2} \frac{M_s v_s^3}{v_b} \approx 4.6 \times 10^{33} \text{ erg s}^{-1},$$

and the remaining energy goes into the hot bubble. The energy input into the hot bubble $L_{\text{hb}}$ is about $L_{\text{fw}} - L_{\text{ds}} = 6 \times 10^{33}$ erg s$^{-1}$, from of which $L_{\text{ad}} = \frac{1}{3} \frac{M_s v_{\text{cd}}^3}{v_s} \approx 3 \times 10^{33}$ erg s$^{-1}$ is required for the adiabatic expansion of the hot bubble. Therefore the energy radiated by the hot bubble, $L_{\text{hb}} - L_{\text{ad}} = 3 \times 10^{33}$ erg s$^{-1}$, is significantly larger than $L_{\text{hb,m}}$ and this difference must be radiated by the edges. This is qualitatively demonstrated by a jump in the cumulative luminosity function, $C_x(r)$ (i.e., the luminosity emitted by material inside a given radius $r$), across the contact discontinuity. There are spikes in the emissivity in the low energy bin at both edges of the hot bubble (Fig. 5, top) due to the presence of gas with intermediate temperatures $\lesssim 10^5$ K. However, only the outer edge produces a jump in $C_x(r)$ for the low energy bin, because it has a significantly higher emission measure (see Fig. 2, top).

Hence the maximum fractional contribution to the X-ray luminosity emitted in the full energy range (0.01 – 0.09 keV) lies in the low energy bin and comes, not from the main body of the hot (5 × 10$^6$ K) bubble, but from its somewhat cooler (10$^4$ – 10$^5$ K) edges. The gas in the dense shell is too cool ($< 10^4$ K) to emit any X-ray radiation. The structure of the hot bubble is similar in the high and medium energy bins, however, it is significantly limb-brightened in the low energy bin.

A plot of the luminosity in the high and medium energy bins as a function of the evolution time (Fig. 6) shows that these are roughly constant with time. Note, that since most of the energy in the 0.01 – 0.09 keV range is emitted by only a few grid cells in our models, we do not have reliable quantitative estimates of the luminosity in the low energy bin.

### 3.2.2. The X-ray spectra

In this section, we investigate the X-ray spectra from our models and how they are affected by different physical parameters. For this purpose, the model spectra have been convolved with the effective area of the ACIS S instrument (Fig. 7). We assume a distance of 1 kpc. Our spectra are presented with a resolution of 0.01 keV. The default abundances in ATOMDB (see §2.3) are used.

As the temperature of the hot bubble is only a function of the fast wind velocity, the spectra from, e.g., the models A5, B5 and C5, look almost identical. The differences between each set of models are in the absolute values of the flux, not the relative fluxes of different lines in the spectra.

Within each set, as the value of $v_f$ changes, the spectra also change dramatically (Fig. 8). At moderate velocities of the fast wind of 300 km s$^{-1}$, only a few lines with energies up to 1 keV are present. Increasing the fast wind velocity (and with it the temperature of the hot bubble) leads to the emergence of lines at higher energies,
so that at and above velocities of 600 km s$^{-1}$ one can see a peak of blended lines in the 0.8 – 1.1 keV range with an increasingly dominant contribution from iron lines. In the model B10, the continuum contributes to an energy range from 0.3 to above 2 keV.

4. TWO-DIMENSIONAL SPHERICAL MODELS

In one-dimensional simulations, which we have discussed so far, instabilities and turbulence cannot occur. Instabilities may, however, be quite important in explaining microstructure in PNs, such as the observations of globules in a few PNs (Huggins & Frank 2005). Therefore, we also calculated a two-dimensional spherical model$^2$ with the parameters of run B5 to examine the presence of such instabilities and how they affect the structure of the hot bubble and the dense shell and the resulting X-ray properties.

The velocity of the shell is similar to the result of the one-dimensional simulation, but somewhat lower – the value is 22.4 km s$^{-1}$ compared to about 27 km s$^{-1}$. As the expansion in the energy-driven phase is mainly due to the pressure in the hot bubble, the reduction of the shell velocity indicates that the pressure is reduced in the two-dimensional run. The pressure in the one-dimensional run is $2.5 \times 10^{-7}$ dyn and $1.47 \times 10^{-7}$ dyn in the two-dimensional run. The latter is reduced by about one third, which leads to a reduction of the shell velocity of 20%, as $v_{\text{shell}} \sim \sqrt{p}$, i.e. a reduction from 27 to 22 km s$^{-1}$ as seen in the data. In Fig. 9, we show plots of the temperature and density as a function of radius at an age of 288 years of the one-dimensional run and an azimuthal average of the two-dimensional run – as

---

$^2$ i.e. a two-dimensional coordinate system and spherical initial conditions
the temperature is identical, but the density is lower, the pressure is lower in the latter. The reduction of the pressure also produces a smaller extent of the hot bubble (i.e., the radial distance from the reverse shock to the contact discontinuity).

The transition from a one-dimensional to a two-dimensional model introduces instabilities (Rayleigh-Taylor-instabilities), as shown in Fig. 10. Due to these instabilities, some fraction of the total energy is redirected into the kinetic energy of non-radial motion. This fraction is typically a few percent and can be high as up to 40% locally in the transition layer between the hot bubble and the dense shell. Therefore less energy is available for the kinetic energy of radial motion of the dense shell and the internal energy of the hot bubble. Another effect triggered by instabilities is the mixing of material in the hot bubble and the dense shell near the contact discontinuity which leads to a lowered temperature there. The consequent decrease in cooling time thus reduces the pressure and hence the extent of the hot bubble.

The reduced extent of the hot bubble results in a reduced luminosity $L_x,\text{ACIS}$ in the energy bins between 0.09–0.2 keV and 0.2–10 keV as a function of the evolution time in the one-dimensional and two-dimensional runs B5.

5. COMPARISON WITH OBSERVATIONS OF APPARENTLY ROUND PNS

In this section, we apply our one-dimensional models to two PNs which show apparently round morphology. Rather than computing new 2D runs, we used 1D runs in our modeling of BD+30°3639 and NGC 40, since other uncertainties, as, e.g., those due to uncertain abundances, are much larger than the differences between the 1D and 2D runs. We use measured values of the fast wind properties (velocity, mass outflow rate) estimated from optical lines and derive the corresponding properties of the slow wind (using eqns. 7 and 10) to fit the observed expansion velocity of the shell. In these simulations, we do not assume $\dot{M}_f v_f = \dot{M}_s v_s$. Then we choose the evolutionary age in our simulation, where the size...
of the model dense shell is equal to the observed size of the PN, and compare the temperature of the hot bubble and the X-ray luminosity with values inferred from available X-ray observations. Thus we constrain our models using four observed properties of PNs – $v_{\text{exp}}$, $r_{\text{shell}}$, $T_x$ and $L_x$ – whereas ASB06 take $T_x$, $L_x$ and the dynamic age ($r_{\text{shell}}/v_{\text{exp}}$) into account while comparing their models with the X-ray properties of PNs and ignore the measured values of the fast wind properties.

5.1. BD+30°3639

BD+30°3639 shows well-resolved, extended X-ray emission (Kastner et al. 2000) which lies within the interior of the shell of ionized gas seen in optical images. The extent of the shell in CHANDRA images is approximately 5°×4°. Assuming a distance of 1 kpc (Kastner et al. 2000), the radius is therefore about 3.3 × 10^{16} cm. The fast wind velocity is about 700 km s^{-1} (Leuenhagen et al. 1996) and the mass loss rate of the fast wind is about 10^{-6} M\odot yr^{-1} (Soker & Kastner 2003). The average expansion velocity derived from [OIII] and [NII] emission lines is 25.5 km s^{-1}. Kastner et al. (2000) give a CHANDRA ACIS spectrum of BD+30°3639 between 0.3 – 1.7 keV and derive a total luminosity of 1.6 × 10^{32} erg s^{-1} after fitting the spectrum with a variable-abundance MEKAL model. The deduced emission-region temperature is 2.7 × 10^{6} K.

A new simulation D7 was performed using $M_\star = 7 \times 10^{-5} M\odot$ yr^{-1}. The observed size of the shell is reached at an age of 390 years. The model expansion velocity is 25.9 km s^{-1}, which is in good agreement with the observations\(^3\). However, both the temperature of the hot bubble (1.1 × 10^{7} K) and the luminosity (9.2 × 10^{32} erg s^{-1} in the full range of ACIS of 0.2–10 keV, 7.8 × 10^{32} erg s^{-1} in the observed energy range) are too high in our model.

The model spectrum (Fig. 12, top) is different from the observed spectrum. In the observations, there is a strong, broad peak at about 0.4 keV, which is not apparent in any of our model spectra. Furthermore there is a narrower, less prominent peak at about 0.9 keV in the observed spectrum, however, our model spectrum shows a strong, broad peak at about 0.9 keV (only visible in the models with a fast wind velocity above 600 km s^{-1}). The strongest lines in the region around the peak at 0.9 keV are iron lines (Fig. 12, top). This suggests that the iron abundance (and perhaps also those of other elements) in BD+30°3639 are very different from the solar abundances (Anders & Grevesse 1989), which we have used in our models including run D7.

5.1.1. The effects of uncertain abundances on X-ray emission properties

The abundance problem may be especially important for objects where the central star is of late [WC] type, as in BD+30°3639 and NGC 40. In these objects, the abundances in the fast wind can be very different from those of the dense shell. The former, however, are very important for the X-ray properties, as the hot bubble consists of material ejected in the fast wind. As mixing of material from the dense shell into the hot bubble may occur, the abundances in the dense shell also may be relevant for the X-ray properties. Such mixing has been inferred in the hydrogen-deficient PN Abell 30 by Chu et al. (1997).

An inspection of published values of abundances BD+30°3639 shows that the abundances, both in the fast wind and the dense shell, are in general poorly determined. Aller & Hyung (1995) estimated the following nebular abundances: C and N are solar, O is roughly solar, Ne is depleted (although they only give a lower limit) from a high spectral-resolution optical spectrum of BD+30°3639. Bernard-Salas et al. (2003) determined that C is enhanced by a factor of 2, N and O are roughly solar, Ne is enhanced by a factor of 1.7, using IR spectra and UV observations.

The abundances in the fast wind are determined by Leuenhagen et al. (1996) by fitting optical spectra. They find an upper limit of the hydrogen abundance and that helium, carbon and oxygen are very prominent as expected. Arnaud et al. (1996) fit ASCA observations with two sets of abundances: (i) a solar C and half-solar N abundance, 0.2 solar for Ne, O and Fe are depleted; (ii) C is enhanced by a factor of 354, N and Ne are enhanced by a factor of 10, O is roughly solar and no iron. Kastner et al. (2000) concluded that C is enhanced, Ne is roughly solar and N and O are depleted, from a fit of its CHANDRA ACIS spectrum. Maness et al. (2003) revisited the CHANDRA observations of Kastner et al. (2000) and showed that the fast wind abundances (the second set of abundances in Arnaud et al. 1996) can fit the observations much better than the nebular abundances of Aller & Hyung (1995). Furthermore they find that an enhanced Ne abundance relative to solar is required. Georgiev et al. (2006), re-investigating the same CHANDRA observations, also fit the spectrum with two sets of abundances: (i) C is enhanced by a factor of 3.7, Ne, N and O are depleted, only Si, Ca and Ni of the 7 heavier elements are solar, the others (Mg, S, Ar, Fe) are not existent; (ii) C is enhanced by a factor of 350, N and Ne are enhanced by a factor of 20, O is roughly solar, 7 heavier elements up to Ni are solar. In summary, the available data can be fit with a wide range of abundances, although there seem to be general agreement that carbon is enhanced and oxygen is either depleted or solar.

In order to demonstrate the dramatic effect of changing elemental abundances on X-ray emission properties, we have recalculated the spectrum from our model D7 using the elemental composition determined by Kastner et al. (2000) (i.e. only H, He, C and Ne), but with abundance ratios as in the Sun). The spectrum has the form as shown in Fig. 12 (bottom), which is closer to the observed shape. The total luminosity in the range of 0.3 – 1.7 keV is then reduced to 1.75 × 10^{32} erg s^{-1}, which is now very close to the observed value of 1.6 × 10^{32} erg s^{-1}.

However, an important caveat to note here is that the cooling function depends on the abundances. For example, at temperatures $\gtrsim 10^{6}$ K the most important coolant is iron (see Fig. 18 in Sutherland & Dopita 1993). Hence an accurate calculation of X-ray properties as a function of abundances would require us to abandon the use of a cooling function, replace it with a set of rate equations for the important species and calculate the resulting cooling

\(^3\) It is interesting to note that $M_\star v_\star = M_\text{sh} v_\text{sh}$ is satisfied in this object.
for all transitions. This, however, would need a substantially increase in the computational effort, which is beyond the scope of this paper.

Such an effort, however, might be warranted for modeling the high resolution spectrum of BD+30°3639, recently taken using the LETG on CHANDRA, which shows abundance ratios of C/O $\sim$ 20, N/O $\lesssim$ 1, Ne/O $\sim$ 4 and Fe/O $\lesssim$ 0.1 (Kastner et al. 2006). The first step towards self-consistency of the cooling treatment would then be to modify the cooling function taking into account the effects of lower metallicities as in Fig. 13 in Sutherland & Dopita (1993).

5.2.

NGC 40

Another example of a PN, which is detected in X-ray and is almost round, is NGC 40 (Montez et al. 2005). Compared to BD+30°3639, the fast wind velocity is significantly higher in this object (1000 km s$^{-1}$); the mass loss rate of the fast wind is $2.4 \times 10^{-6}$ M$_{\odot}$ yr$^{-1}$ (Leuenhagen et al. 1996). The observed radius of the shell is 2.0,000 AU at an estimated distance of $\sim$ 1 kpc. The average expansion velocity from [OIII] and [NII] emission lines is 27.5 km s$^{-1}$. The measured X-ray luminosity is $1.5 \times 10^{40}$ erg s$^{-1}$ in the range of 0.3–1 keV and the inferred temperature of the X-ray emitting gas is $1.5 \times 10^{6}$ K.

This object was modeled with a new run E10. We chose a mass loss rate of the slow wind of $5 \times 10^{-4}$ M$_{\odot}$ yr$^{-1}$ to adjust the expansion velocity of the dense shell in our model to the observed value$^4$. The expansion velocity is 27.03 km s$^{-1}$, which is very close to the observed one, as expected by our choice of the parameters. The observed size of the bubble was reached in our model after 3775 years, thus NGC 40 is much older than BD+30°3639.

We find from our model that the temperature of the hot bubble is $2.2 \times 10^{5}$ K and the luminosity is $2.4 \times 10^{43}$ erg s$^{-1}$ in the full range of ACIS (0.2–10 keV) and $7.6 \times 10^{42}$ erg s$^{-1}$ in the observed energy range. If, as in the case of BD+30°3639, we restrict the composition to only H, He, C, N and O, the above luminosities are reduced to $1.5 \times 10^{43}$ erg s$^{-1}$ and $6.4 \times 10^{42}$ erg s$^{-1}$, respectively. These values are still too high (by a factor of 400) to explain the observations.

5.3. The effects of different time histories of the fast wind

The previous sections show that in our models of both objects, BD+30°3639 and NGC 40, $T_\alpha$ and $L_\alpha$ are higher than the observed values. Noting that $T_\alpha$ is most sensitive to $v_f$ (§3.1 and eqn. 11), we investigate whether better fits to the data could be obtained by assuming that the value of $v_f$ was lower in the past (thus implying that both objects are now being observed in a phase where an even faster wind has emerged producing the line profiles analyzed by Leuenhagen et al. (1996) for deriving $v_f$). We therefore compare two models, one with an ultra-fast wind of 1000 km s$^{-1}$ (run B10) and the second in which the fast wind has a velocity of 300 km s$^{-1}$ for the first 100 years followed by the ultra-fast wind (run B3+10). The mass loss rate of the slow wind was $7 \times 10^{-6}$ M$_{\odot}$ yr$^{-1}$. The momentum discharge $M_\odot v_f = M_\odot v_s$ is again kept constant.

We find that, in the second model, the contact discontinuity between the ultra-fast and the fast wind (CD$_2$) reaches the original contact discontinuity between the fast and the slow wind (CD$_1$) in 7 years and, after a short period of complex interaction, produces a hot bubble with a structure very similar to that in the other model (Fig. 13). The radial extent of the hot bubble is slightly smaller. The final expansion velocity of the shell is the same in both models (33 km s$^{-1}$).

We extract the luminosities at early ages in the evolution of these models (i.e., soon after emergence of the fast wind), when the size of the shell in each model has a value of 2000 AU. The corresponding ages are 340 (in run B10) and 370 years (run B3+10). The simulated luminosities in the full range of ACIS are similar – $8.9 \times 10^{40}$ erg s$^{-1}$ (run B10) and $9.3 \times 10^{42}$ erg s$^{-1}$ (run B3+10). The temperatures of the hot bubble are $2.1 \times 10^6$ K in both cases. Due to the slightly smaller extent of the hot bubble in run B3+10, the temperature gradient is slightly steeper. Although the spectra are very similar, a few small differences occur due to a larger quantity of cooler gas in the run B3+10: enhanced emission is seen in the following lines at 0.22, 0.29, 0.36, 0.43 and 0.56 keV, which are lines of Si IX, C V, C VI, N VI and O VII. These results show that, once CD$_2$ merges with CD$_1$, neither the expansion velocity of the shell nor the total X-ray luminosity or spectrum distinguishes between the very different time histories of the fast wind.

Observable differences are only present in the short period of time between the emergence of the ultra-fast

$^4$ In this simulation, $M_\odot v_f$ is not equal to $M_\odot v_s$, but lower by a factor of 2.
wind and the merger of \( \text{CD}_2 \) with \( \text{CD}_1 \). We now have to check if it is likely that a specific PN is observed in this period. By using the radius of the dense shell at the time of emergence of the ultra-fast wind\(^5\) and noting that \( \text{CD}_2 \) must expand faster than the fast wind, we can estimate a maximum time until the merger \( t_{\text{coll}} \). Thus, for example, in our simulation B3+10, where the radius of \( \text{CD}_1 \) is about \( 6.3 \times 10^{15} \) cm after 100 years (Fig. 13),

\[
t_{\text{coll}} = \frac{6.3 \times 10^{15} \text{ cm}}{300 \text{ km s}^{-1}} = 6.65 \text{ years}
\]

We derive \( t_{\text{coll}} \) for BD+30\(^{\circ}\)3639 and NGC 40, using the observed sizes of the two objects (\( 3.3 \times 10^{16} \) cm for BD+30\(^{\circ}\)3639 and \( 3 \times 10^{17} \) cm for NGC 40), and 300 km s\(^{-1}\) for the minimum velocity of \( \text{CD}_2 \) since the temperature of the hot bubble is better fitted by such a fast wind velocity (eqn. 11). We find values of \( t_{\text{coll}} \) of about 36 for BD+30\(^{\circ}\)3639 and 320 years for NGC 40. Since both values are a small fraction (one tenth) of the total age of the bubble in each object, it is very unlikely that the observations of Leuenhagen et al. (1996) caught these two objects in the special period after the emergence of the ultra-fast wind and before the merger of \( \text{CD}_2 \) with \( \text{CD}_1 \).

In summary, we conclude that the utilization of a smaller value of \( v_f \) in the past will not help in explaining the low values of \( L_x \) and \( T_x \) observed in objects like BD+30\(^{\circ}\)3639 and NGC 40.

5.4. The importance of heat conduction and its inhibition by magnetic fields
Heat conduction across the contact discontinuity results in a cooler and denser hot bubble. Zhukov & Perinotto (1996) give analytical estimates for the importance of heat conduction, calculating the ratio of the thermal conductivity time and the evolution time, claiming that heat conduction is important right from the beginning of the fast wind-slow wind interaction. However, they ignore the effects of radiative cooling, which are considered by Soker (1994). By comparing the energy loss due to radiation and due to heat conduction from the hot bubble into the dense shell, Soker (1994) finds that radiative losses become significant compared to heat conduction after a certain timescale $t_{\text{cool}}$ (his eqn. 6), when the conduction front thickness (which increases with time) reaches a certain value. He derives this timescale by using the growth rate of the conduction front from a model by Balbus (1986). However, both Soker (1994) and Balbus (1986) assume that the pressure of the hot gas is constant with time. Since the pressure of the hot bubble is expected to vary strongly with time in PNe ($\sim t^{-2}$), the formulation of Soker (1994) may not be valid.

The heat conduction front goes through three different stages. In the first stage, mass from the cold shell is evaporated into the hot bubble. This results in the hot bubble mass being dominated by the evaporating gas from the dense shell (Zhukov & Perinotto 1996). Thus in a PN in this stage the X-ray spectrum should reflect the abundances of the dense shell and not those of the shocked fast wind. This is also true for the second, quasi-static stage. In the third stage, i.e., “condensation”, the region of intermediate temperature expands into the region of the initially hot gas (Borkowski et al. 1990). In the last two stages radiative losses dominate heat conduction. Soker & Kastner (2003) claim that in BD+30°3639 the inferred depleted abundance of oxygen, based on the modeling of the CHANDRA ACIS spectrum by Kastner et al. (2000), suggests that the X-ray emitting conduction front is in one of the first two phases, i.e. located within the dense shell, making the implicit assumption that oxygen is depleted in the dense shell. However, we do not think that their claim is justified as the oxygen abundance seems to be roughly solar in the nebula, i.e. the dense shell ($\S$5.1.1). The abundances as inferred from the new high-resolution X-ray spectra definitely show a very large overabundance of carbon (as expected in the fast wind from the [WC] central star of this object) in contrast to the carbon abundance of the dense shell ($\S$5.1.1). If heat conduction is operating in this object, it can not have evaporated a large amount of mass from the dense shell into the hot bubble. Alternatively, heat conduction is inhibited by magnetic fields in this object.

Magnetic fields, which are believed to play an important role in collimating the fast outflow, may reduce thermal heat conduction across field lines (e.g. Soker 1994). Following Borkowski et al. (1990), the conductivity coefficient can be written

$$\kappa = \frac{C T^{5/2}}{1 + (n \tan \theta_e/n_b)^{5/2}}$$  \hspace{1cm} (13)

Using the expression for the gyroradius $\delta = \sqrt{2 T m_e/(e B)}$ with the electron mass $m_e$ and the magnetic field strength $B$ (Spitzer 1962), one can show that a very weak magnetic field of the order of 0.1 $\mu G$ is sufficient to reduce the conductivity (Soker 1994). Chevalier & Luo (1994) find in the models of magnetized stellar wind bubbles that the tangential magnetic field component in the shocked fast wind is larger than the radial component by several orders of magnitude, thus heat conduction is inhibited in radial direction.

Heat flux inhibition by electromagnetic instabilities can also occur along the field lines, if $\beta = 8 \pi p/B^2 \gg 1$ (Levinson & Eichler 1992), which gives a maximum magnetic field

$$B \ll \sqrt{8 \pi p} = \sqrt{8 \pi n_k T} =$$

$$= 5.88 \mu G \left( \frac{n}{100 \text{cm}^{-3}} \right)^{1/2} \left( \frac{T}{10^6 \text{K}} \right)^{1/2}$$  \hspace{1cm} (14)

When the magnetic field is strong, the suppression is small (Pistinifer & Eichler 1998).

Several lines of evidence support the presence of magnetic fields in PNe and therefore the inhibition of heat conduction is not unlikely. For example, magnetic fields in PPNs have been inferred from the presence of polarized OH and H$_2$O masers (e.g. Vlemmings et al. 2006) and have been detected in the central stars of PNe (Jordan et al. 2005). Magnetic fields are also considered to be the main agent for producing bipolarity in PNe (García-Segura et al. 2005).

6. COMPARISON WITH ANALYTICAL RESULTS

We compare our results now with the analytical results of ASB06. A basic advantage of an analytical approach is that it allows us to explore a large part of the parameter in a short time. However, the analytical approach requires certain assumptions which at best may be only partially valid. Numerical simulations like those which we have carried out in this paper, although much more time consuming, are thus necessary for testing the limitations of the analytical approach as well as making realistic models of real objects.

ASB06 calculated the structure of the hot bubble and the dense shell using self-similar solutions as in Chevalier & Imamura (1983). They computed the X-ray luminosity between 0.2 – 10 keV (now called high energy bin) by taking into account only those gas parcels in which the cooling timescale (from the cooling function of Sutherland & Dopita 1993) is larger than the evolution timescale of the flow. They derived the X-ray emissivities from the APEC database (Smith et al. 2001) and integrated the emission measure over the region between the inner shock and the contact discontinuity. Their results can now be compared with those from our simulations.

We find a different temporal behavior of the luminosity in the high energy bin, $L_{x,\text{ACIS}}$, compared to the result of ASB06 – whereas $L_{x,\text{ACIS}}$ in our models increases slightly over the first few 100 years and decreases slowly ($\sim t^{-0.35}$) after that, in ASB06’s models it decays with time as $\sim t^{-1}$. These differences probably arise because their cooling treatment is not fully self-consistent. They do not follow the evolution of the gas parcels with radiative cooling, as of course is done in our numerical simulations. Furthermore the temporal behavior of the luminosity in ASB06’s models depends on the specific criteria for removing cooled regions (see their Fig. 9).

The initial phase of low $L_{x,\text{ACIS}}$ is of significantly smaller duration in our models as compared to ASB06 – e.g. in model C3, it covers a range of only 100 years
in our models, whereas in ASB06 it is about 700 years. This difference is relevant to the understanding of X-ray emission from PPNs where the ages of the nebulae are typically \(\sim\) few \(\times\) 100 years. The length of the phase of increasing luminosity is dependent on the parameters of our models – the lower the velocity and the higher the mass outflow rate of the fast wind, the longer this phase. Both trends are qualitatively consistent with the results of ASB06.

After the initial phase, a little step in the luminosity occurs (e.g. in run B3 at an age of about 230 years in Fig. 3). Shortly before this transition, oscillations of density, pressure and velocity occur in the hot bubble. Frank & Mellema (1994) have also found oscillations in their numerical simulations and suggest that these occur when energies, e.g. the total and the internal energy, are almost equal. In our case, the internal energy is about 96\% of the total energy. However, whether these oscillations are only symptoms of, or the reason for, the transition, is unclear. Investigating the age at which the steps occur as a function of the parameters of the fast wind, we find that \(t_{\text{step}} \sim \dot{M}_{\text{f}}^{0.37} v_\text{f}^{0.75}\). Although we can not say as yet if the oscillations are a numerical artefact or a real physical effect, these scalings might help in eventually finding an answer.

The temporal behavior of \(L_{x,\text{ACIS}}\) in the models of ASB06 and our models can be easily understood as follows. In the self-similar models of ASB06, the radius of the contact discontinuity and that of the reverse shock are proportional to \(t\), i.e. the volume of the hot bubble varies as \(t^3\). The density decreases as \(t^{-2}\), hence the emission measure \(n^2 V\) varies as \(t^{-1}\). As the temperature in the hot bubble remains roughly constant, the emissivity \(\Lambda\) of the hot bubble gas is also constant and therefore the total luminosity (\(\Lambda n^2 V\)) varies as \(t^{-1}\). In comparison, the empirical scaling laws in our runs are \(r_{\text{rs}} \sim \dot{M}_{\text{f}}^{0.36} v_\text{f}^{0.75}\), \(r_{\text{cd}} \sim t^{1.08}\), \(V = (r_{\text{cd}}^3 - r_{\text{rs}}^3) \sim t^{3.35}\), \(<\rho > \sim t^{-1.86}\) and \(<\rho >^2 V \sim t^{-0.37}\), resulting in \(L_{x,\text{ACIS}} \sim t^{-0.37}\). The scaling laws of density and volume of the hot bubble imply that the total mass of shocked fast wind material increases as about \(\sim t^{1.5}\), whereas the mass input by the fast wind increases only as \(\sim t\), since the mass outflow rate is kept constant. This difference in the time dependence reflects the evolution of the hot gas in the shocked fast wind under the influence of radiative cooling. In the beginning, only a small fraction of the mass injected by the fast wind is at high temperatures in the hot bubble, most of it lies near the contact discontinuity at low temperatures. This is because, due to radiative cooling, large parts of the shocked fast wind collapse towards the contact discontinuity. With increasing age, the density decreases and this effect becomes weaker, progressively increasing the fraction of gas injected by the fast wind which remains at high temperatures after being shocked.

We find that the higher the mass outflow rate of the fast post-AGB wind, the higher \(L_{x,\text{ACIS}}\) is (§3.2), in accord with the results of ASB06. However, within each set of our runs with constant \(\dot{M}_f v_f\), \(L_{x,\text{ACIS}}\) decreases for increasing \(v_f\) – this behavior is different from the results of ASB06 in which the models with \(v_f = 500\) km s\(^{-1}\) (A5, B5, C5) always have the highest \(L_{x,\text{ACIS}}\) value within each set of constant \(\dot{M}_f v_f\).

We can also compare the absolute luminosities in our models (which lie in the range \(10^{30} – 10^{33}\) erg s\(^{-1}\)) with those from ASB06\(^6\) (see Table 2). We find good agreement (i.e. within a factor of 2–3) for the runs in sets A and B, however, in set C the derived \(L_{x,\text{ACIS}}\) values of ASB06 are higher (for example, a factor of 3 higher in run C5 and a factor of 5 higher in run C7). In addition, within each set (i.e. with constant \(\dot{M}_f v_f\)), in ASB06 the values of \(L_{x,\text{ACIS}}\) lie within a factor < 2 of each other, whereas in our models \(L_{x,\text{ACIS}}\) values cover a larger range of 4–10.

ASB06 conclude that their models can fit the X-ray properties and the dynamical ages of specific PNs, including BD+30°3639 and NGC 40 (e.g. Fig. 5 in ASB06). We think this conclusion is problematic for the following reasons: first ASB06 use smaller values of the fast wind velocities than those which have been derived from measurements (Leuenhagen et al. 1996). They find that their best fits to BD+30°3639 and NGC 40 are obtained with models C3 and B4, respectively, for which the fast wind velocities are too low compared with observations (300 and 400 km s\(^{-1}\) compared to 700 and 1000 km s\(^{-1}\), respectively), and justify their choice by claiming that the details of the late-time evolution of the fast wind are not important to the X-ray emission. They state that the X-ray emission predominantly emanates from wind segments of the hot bubble which were expelled early, i.e. with a moderate velocity, and over a relatively short time (few \(\times\) 100 years). Hence the X-ray emission of the hot bubble is determined by the moderate fast wind and the subsequent time history is unimportant; however, we find that the reverse is true — the X-ray emission is determined by the final stage of the time history during which the fast wind velocity has its largest value.

Second, their claimed agreement between models and data appears to break down, when we consider in detail the model and observed values of all three parameters \((L_x, T_x\) and age) together. Thus, although BD+30°3639 is well fitted by C3 in the \(\log(L_x)–\text{age}\) plane (Fig. 5 of ASB06), in the \(\log(L_x)–\log(T_x)\) plane (Fig. 6 of ASB06), it is far removed from model C3 and lies close to model B6. NGC 40 (not shown in the \(\log(L_x)–\text{age}\) plane), is located at a position in the \(\log(L_x)–\log(T_x)\) plane which appears to lie on an extrapolation of the curve for model B4 representing very late ages. However, taking into account the declining luminosity with increasing age and therefore size, this location would be reached at a size of the dense shell which is a factor 7 larger than observed.

7. FITTING \(T_X\) AND \(L_X\) IN PNS – AN UNSOLVED PROBLEM?

In section 5, we have found that using the values of \(v_{\text{exp}}, r_{\text{shell}}, \dot{M}_f\) and \(v_f\) in BD+30°3639 and NGC 40 as inferred from observations, our models produce values of \(T_x\) and \(L_{x,\text{ACIS}}\) which are much higher than the observed values. We explored three different mechanisms for reducing \(T_x\) and \(L_{x,\text{ACIS}}\). First, noting that the central stars in both objects are of [WC] type, deviations

\(^6\) For this comparison, we have used bubbles with the same size rather than those with the same age, since the velocity of the contact discontinuity in our models is slightly different compared to ASB06. The velocities used by ASB06 follow from a cubic formula (Volk & Kwok 1985, also Table 1).
Fig. 14.— Plots of the mean density $<\rho>$ of the hot bubble, the radius of the reverse shock $r_{rs}$ and the contact discontinuity $r_{cd}$, the volume of the hot bubble $V$ and of the X-ray luminosity $L_{x,ACIS}$ in run E10 as a function of time; overplotted are the scaling laws for these variables ($<\rho> \sim t^{-1.86}, r_{rs} \sim t^{0.98}, r_{cd} \sim t^{1.08}, V = (r_{cd}^3 - r_{rs}^3) \sim t^{3.35}$), which – together with the temperature (not shown, as constant with time) – determine the X-ray luminosity $L_{x,ACIS}$.
of the abundances from the solar values were tested as a way of reducing $L_{x,ACIS}$. In the case of BD+30°3639, this mechanism appears to be promising, but needs to be tested using a cooling function consistent with the non-solar abundances. In the case of NGC 40, although $L_{x,ACIS}$ was reduced, the discrepancy between the model and the observed value remains too large. The second mechanism, in which the fast wind had a slower velocity initially, was also not effective. Thirdly, heat conduction, which has been discussed previously as a possible mechanism, does lower $T_x$, but leads to a large increase in $L_{x,ACIS}$ (Schönberner et al. 2006) due to an increase in the density of the hot bubble. We also found that, even though $L_{x,ACIS}$ is reduced in models computed on a two-dimensional grid, the reduction is not adequate.

Since $T_x$ ($L_{x,ACIS}$) depends sensitively on $v_f$ ($M_t$), is it possible that the values of these parameters inferred from observations by Leuenhagen et al. (1996) are too high? Since $v_f$ is directly measured from Doppler shifts of absorption features, the uncertainties in determining $v_f$ are likely to be small. The value of $M_t$ is of course model-dependent and is also affected by the assumed distance as $M_t \sim d^{3/2}$ (Leuenhagen et al. 1996) – because of the distance dependency, $L_{x,ACIS}$ should vary with distance as $\sim d^9/4$, since in our models $L_{x,ACIS} \sim M_t^{3/2}$ (Fig. 4). $L_{x,ACIS}$ in our model is also dependent on time and therefore on the size of the bubble. Assuming the dependency $L_{x,ACIS} \sim t^{-0.37}$ (see §6), we get $L_{x,ACIS} \sim T^{-0.37} \sim d^{-0.37}$. Combining both dependencies, the ratio of $L_{x,ACIS}$ in our models to the observed value varies as $\sim d^{2.25-0.37/2} = d^{-0.12}$ – because of this weak dependency, distance uncertainties cannot be invoked to reduce the discrepancies between the model and observed values of $L_{x,ACIS}$. Using the dependency $L_{x,ACIS} \sim M_t^{3/2}$ (Fig. 4), $M_t$ would have to be reduced by a factor of 50 from the value inferred by Leuenhagen et al. (1996) in order to achieve consistency between the model and the observed value of $L_{x,ACIS}$ in NGC 40. Of course, the problem of the model $T_x$ being too high would still remain. In summary, the problem of hydrodynamical models for the formation of PNs producing values of $T_x$ and $L_x$ which are too high compared to observed values remains unsolved.

8. CONCLUSION

We computed the structure and evolution of, and X-ray emission from, numerical simulations of one-dimensional interacting winds models and compared the results with different analytical models. The kinematics and the evolution of our bubbles and the structure of the flows agrees well with analytical results. Comparing the X-ray properties of the hot bubbles in our models with the analytical results of ASB06, we find some agreements and many disagreements. The disagreements which are both qualitative and quantitative in nature argue for the necessity of using numerical simulations for understanding the X-ray properties of PNs.

We further investigated the luminosities in the energy ranges other than the 0.2 – 10 keV range covered by ACIS. We find that most of the X-ray flux may emerge at energies less than 0.09 keV (wavelengths above 137 Å), although we do not have reliable quantitative estimates of this flux from our models. At these wavelengths, commonly labeled as extreme and far UV, observations of PNs with FUSE (900 – 1200 Å), EUVE (70 – 760 Å), IUE (1150 – 3200 Å) and GALEX for even longer wavelengths (1350 – 2800 Å) would therefore be quite important. Unfortunately, photospheric emission from the hot central star easily blends with the emission from the PN. Instruments with high spatial resolution are needed.

We also investigated the X-ray spectra from our models, which are not addressed by the analytical studies. We showed that the shapes of the spectra are diagnostic of the fast wind velocity which determines the temperature of the hot bubble.

We applied our spherical model to the objects BD+30°3639 and NGC 40. We find in both cases that the simulated temperatures and luminosities of the hot bubble are much higher compared to the observations. While investigating these discrepancies, we identified several general issues which have to be considered for correctly modeling the X-ray emission of PNs.

The first one is the importance of an independent knowledge of the abundances – especially in objects with [WC]-type central stars, where the abundances of the hot bubble (shocked fast wind) can be dramatically different from those in the dense shell (shocked slow wind). We could roughly reproduce the observed luminosity in BD+30°3639 by simply (i.e. in a non-self-consistent manner) removing several heavy elements including iron in the calculations of the X-ray properties. The treatment of cooling, however, has to be done self-consistently, as the cooling function itself is highly sensitive to abundances (e.g. iron).

A second issue is the possible dependence of $L_x$ on $T_x$ on the time history of the fast wind velocity. Noting that $T_x$ is most sensitive to $v_f$, we investigated whether $L_x$ and $T_x$ could be lowered by assuming that the value of $v_f$ was lower in the past, but found that such a time history will not help in explaining the observed low values of $L_x$ and $T_x$. The X-ray emission is determined by the final stage of the time history during which the fast wind velocity has its largest value. Although we have calculated only one case of wind evolution with a particular jump in the fast wind properties, the claim of ASB06, that the X-ray emission of the hot bubble is determined by the moderate fast wind and that the subsequent time history is unimportant, is not supported by our results.

We discussed the importance of heat conduction on lowering the temperature of the hot bubble, and its possible inhibition by magnetic fields. As the presence of magnetic fields is supported both by observations and theoretical considerations, heat conduction is likely inhibited. Even fields of the order of $\mu G$ are sufficient to inhibit heat conduction across (and along) the field lines. Since the X-ray spectra strongly indicate a very large overabundance of carbon in the hot bubble (as expected in the fast wind from the [WC] central star of this object, but much higher than the carbon abundance of the dense shell), if heat conduction is operating in this object, it can not have evaporated a large amount of mass from the dense shell into the hot bubble. Alternatively, heat conduction is inhibited by magnetic fields in this object.

We also calculated a two-dimensional spherical model to examine the presence of instabilities and how they affect the structure of the hot bubble and the dense shell.
and the resulting X-ray properties. Due to these instabilities, some fraction of the total energy is redirected into the kinetic energy of non-radial motion, which leads to slower expansion velocities of the shell (by about 20%) and a reduced luminosity between 0.2 – 10 keV (by $\sim 1.6$), although the temperature of the hot bubble does not change significantly.

From our detailed modeling, we conclude that the problem of hydrodynamical models for the formation of PNs producing values of $T_x$ and $L_x$ which are too high compared to observed values remains unsolved. The simulations in our study are the first step towards modeling PNs and PPNs detected in X-rays which show bipolar or asymmetric structures with varying degrees of collimation. Our preliminary results show that higher collimation leads to lower X-ray luminosities (Stute & Sahai 2006). Detailed results will be presented in a future publication.

We thank V. Dwarkadas for help related to the use of the FLASH code and D. Schönberner for fruitful discussions during the IAU 234 conference. We acknowledge the comments and suggestions by the referee, Noam Soker, which improved the manuscript. The software used in this work was in part developed by the DOE-supported ASC / Alliance Center for Astrophysical Thermonuclear Flashes at the University of Chicago. This work was partially funded by NASA/CHANDRA grants GO3-4019X and GO4-5163Z, and NASA/STScI grant HST-GO-10317.01-A. The research described in this publication was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration.

REFERENCES

Anders, E., Grevesse, N. 1989, GeCoA 53, 197
Huggins, P. J., Frank, A. 2005, poster #08.11, American Astronomical Society Meeting 207

Landau, L. D., Lifshitz, E. M., Course of Theoretical Physics, Volume 6, Fluid Mechanics, (Pergamon Press, 1959)
Soker, N. 1994, AJ 107, 276
Spitzer, L. 1962, Physics of Fully Ionized Gases (New York: Interscience)
Sutherland, R. S., Dopita, M. A. 1993, ApJS 88, 253