Chromomagnetic instability in two-flavor quark matter at nonzero temperature

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We calculate the effective potential of the 2SC/g2SC phases including vector condensates \(\langle gA_z^A \rangle\) and \(\langle gA_z^8 \rangle\) and study the gluonic phase and the single plane-wave Larkin-Ovchinnikov-Fulde-Ferrell state at nonzero temperature. Our analysis is performed within the framework of the gauged Nambu–Jona-Lasinio model. We compute potential curvatures with respect to the vector condensates and investigate the temperature dependence of the Meissner masses squared of gluons of color 4–7 and 8 in the neutral 2SC/g2SC phases. The phase diagram is presented in the plane of temperature and coupling strength. The unstable regions for gluons 4–7 and 8 are mapped out on the phase diagram. We find that, apart from the case of strong coupling, the 2SC/g2SC phases at low temperatures are unstable against the vector condensation until the temperature reaches tens of MeV.

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I. INTRODUCTION

Sufficiently cold and dense quark matter has a rich phase structure; during the last decade, significant advances have been made in our understandings of color superconductivity \[1\]. At asymptotically large quark density, studies using the perturbative one-gluon exchange interaction, directly based on first principles of QCD, are reliable and have clarified the nature of the pairing dynamics of quarks. However, if the color-superconducting phase is realized in nature, it appears in the interior of compact stars. The density regime of interest is, therefore, up to a few times nuclear density. The investigation of the ground state of quark matter in this moderate density regime under conditions relevant for the bulk of the ground state of quark matter in this moderate density regime is therefore still an open question.

Resolving the chromomagnetic instability and clarifying the nature of ground state are pressing issues in the study of color superconductors \[13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23\]. So far, a single plane-wave Larkin-Ovchinnikov-Fulde-Ferrell (LOFF) state \[13, 24\] and a gluonic phase \[16\] have been proposed as candidates for the solution to the instability in two-flavor color superconductor. It is interesting to note that both phases are equivalent to the conventional 2SC/g2SC phases with vector condensates [i.e., \(\langle A_k^3 \rangle \neq 0\) for the single plane-wave LOFF state and \(\langle A_3^3 \rangle, \langle A_3^6 \rangle, \langle A_3^8 \rangle \neq 0\) for the gluonic phase (without loss of generality)]. The neutral single plane-wave LOFF state is indeed free from the instability in the weak-coupling regime \[13, 17\], but still suffers from the instability related to gluons 4–7 in the intermediate- and strong-coupling regimes \[17\]. In addition, it has been indicated that a LOFF state with many plane waves is energetically more favored than the single plane-wave LOFF state \[5\]. (Note that it has recently been demonstrated that, in the three-flavor case, a LOFF state with realistic crystal structures has much lower free energy than the single plane-wave LOFF state \[6\].)

On the other hand, the gluonic phase could resolve the instability associated with gluons 4–7 in the intermediate- and the strong-coupling regimes \[16, 22\]. However, its dynamics has been clarified only around the critical point. It should be mentioned that a mixed phase \[25\] and, in the three-flavor case, phases with spontaneously induced meson supercurrents \[26\] are also candidates for the solution to the chromomagnetic instability.

Most of the studies of the chromomagnetic instability have been restricted to zero temperature. However, in order to look at its relevance for the phase diagram, one has to investigate the nature of the instability at...
nonzero temperature. To our knowledge the instability is weakened by thermal effects and should finally vanish at sufficiently high temperature [11, 12, 27, 28].

In this work, we calculate the effective potential of the 2SC/g2SC phases including the vector condensates \langle gA_8^0 \rangle and \langle gA_8^8 \rangle. The Meissner masses squared in the 2SC/g2SC phases are computed from the curvature of the effective potential with respect to the vector condensates. The potential curvature in the direction of \langle gA_8^0 \rangle and \langle gA_8^8 \rangle corresponds to the Meissner mass squared of gluons of color 4–7 and 8, respectively. We also present the phase diagram in the plane of temperature and diquark momentum cutoff \( \Lambda = 653.3 \) MeV. This specific choice does not affect the qualitative features of the present analysis.

In \( m_{\nu,\alpha} \), the elements of the diagonal matrix of quark chemical potentials \( \mu \) are given by

\[
\begin{align}
\mu_{sr} &= \mu_{ug} = \bar{\mu} - \delta \mu, \\
\mu_{dr} &= \mu_{dg} = \bar{\mu} + \delta \mu, \\
\mu_{ub} &= \bar{\mu} - \mu_8, \\
\mu_{db} &= \bar{\mu} + \delta \mu - \mu_8.
\end{align}
\]

In a gauge theory, the electron chemical potential \( \mu_e \) and the color chemical potential \( \mu_8 \) are induced by the dynamics of gauge fields [27]. The Maxwell and the Yang-Mills equations with the requirement of vanishing photon- and gluon-tadpole diagrams ensure electric and color neutrality. In NJL-type models, on the other hand, one has to impose the neutrality conditions by adjusting \( \mu_e \) and \( \mu_8 \) [30]. In what follows, we neglect the color chemical potential because it is suppressed in the 2SC/g2SC phases, \( \mu_8 \sim \mathcal{O}(\Delta^2/\bar{\mu}) \ll \Delta \).

In Nambu-Gor’kov space, the inverse full quark propagator \( S^{-1}(\vec{p}) \) is written as

\[
S^{-1}(\vec{p}) = \left( \begin{array}{c}
S_0^+^{-1} \\
\Phi^-
\end{array} \right),
\]

with

\[
\begin{align}
(S_0^+)^{-1} &= \gamma^\mu p_\mu + (\bar{\mu} - \bar{\mu}_3) \gamma_0 + g \gamma^\mu A_\mu T^a, \\
(S_0^0)^{-1} &= \gamma^\mu p_\mu - (\bar{\mu} - \bar{\mu}_3) \gamma_0 - g \gamma^\mu A_\mu T^a T^a.
\end{align}
\]

and

\[
\Phi^- = -i \varepsilon^{b} \gamma_5 \Delta, \quad \Phi^+ = -i \varepsilon^{b} \gamma_5 \Delta.
\]

Here \( \tau^3 = \text{diag}(1, -1) \) is a matrix in flavor space. Following the usual convention, we choose the diquark condensate to point in the third direction in color space. In this work, we are interested in the Meissner masses squared of gluons of color 4–7 and 8, it is sufficient to study the case of the nonvanishing vector condensates \( \langle gA_8^0 \rangle \neq 0 \) or \( \langle gA_8^8 \rangle \neq 0 \) [17].

In the one-loop approximation, the effective potential of two-flavor quark matter (without electrons) is given by

\[
V(\Delta, \langle gA_8^0 \rangle, \delta \mu, \mu, T) = \frac{\Delta^2}{4G_F} - \frac{T}{2} \sum_{n=-\infty}^{\infty} \int_{-\Lambda}^{\Lambda} \frac{d^3p}{(2\pi)^3} \times \ln \det S^{-1}(i\omega_n, \vec{p}),
\]

where \( \omega_n = (2n + 1)\pi T \) are the Matsubara frequencies and \( \alpha \in (6, 8) \). Note that the dependence on the vector condensates enters through the covariant derivative in the quark propagator [26].

The Meissner mass squared in the 2SC/g2SC phases can be calculated from

\[
m_{M,\alpha}^2 = \frac{\partial^2 V}{\partial \langle gA_8^0 \rangle^2} \bigg|_{\langle gA_8^0 \rangle = 0}.
\]
However, in order to obtain meaningful results, we have to subtract ultraviolet divergences from Eq. (9) (see below).

### B. Meissner mass squared of gluons of color 4–7

In this subsection we investigate the gluonic phase and the Meissner mass of the 6th gluon at nonzero temperature. Let us first note that, neglecting terms of order $O(\mu^2/\Lambda^2)$ and $O(\Delta^2/\mu^2)$, one can show that the potential curvature at zero temperature is [16, 22]

$$m_{M,6}^2 = \frac{\bar{\mu}^2}{6\pi^2} \left[ 1 - \frac{2\delta\mu^2}{\Delta^2} + 2\frac{\delta\mu\sqrt{\delta\mu^2 - \Delta^2}}{\Delta^2} \theta(\delta\mu - \Delta) \right]. \quad (10)$$

The result is consistent with the Meissner mass squared for gluons 4–7 derived within the hard-dense-loop (HDL) approximation [9]. However, this is not the case for the Meissner mass squared calculated directly from Eq. [9]. The potential curvature suffers from ultraviolet divergences $\propto \Lambda^2$ and therefore we have to subtract them.

At zero temperature the subtraction term is given by

$$\frac{\partial^2 V(0, B, 0, 0, 0)}{\partial B^2} \bigg|_{B=0} = -\frac{\Lambda^2}{3\pi^2}, \quad (11)$$

where $B \equiv (gA_\mu^6)$. (In this paper, we shall refer to the phase where $B \neq 0$ as the gluonic phase.)

At nonzero temperature, on the other hand, the subtraction term should depend on temperature. Indeed one finds

$$\frac{\partial^2 V(0, B, \delta\mu, \mu, T)}{\partial B^2} \bigg|_{B=0} = \frac{\Lambda^2}{12\pi^2} \sum_{e_1,e_2,e_3=\pm} e_1 N_F(e_1\Lambda + e_2\bar{\mu} + e_3\delta\mu), \quad (12)$$

where $N_F(x) = 1/(e^x/T + 1)$, otherwise the Meissner mass in the normal phase assumes an unphysical positive value.

Let us note that, in the case of $T \to 0$, Eq. (12) is in exact agreement with Eq. (11). It should be also noted that the temperature dependence in Eq. (12) is made redundant when $\Lambda \gg T$. Thus, the temperature dependence of Eq. (12) is a cutoff artifact indeed.

In order to investigate the phase transition from the gluonic phase to the 2SC/g2SC phases, we define the following normalized effective potential, that corresponds to the subtraction (12),

$$\Omega_B \equiv V(\Delta, B, \delta\mu, \mu, T) - V(0, B, \delta\mu, \mu, T). \quad (13)$$

It should be noted that, even if $T \to 0$ or $\Lambda \gg T$, the effective potential (13) itself is no longer coincident with that in our previous paper [22]:

$$\Omega'_B = V(\Delta, B, \delta\mu, \mu, T) - V(0, B, 0, 0, 0). \quad (14)$$

If we introduce a cutoff large enough to neglect the temperature dependence in Eq. (12), we can use the mass subtraction (11) and, therefore, the potential subtraction (14). (Of course, strictly speaking, such a cutoff should be infinite.) However, we wish to study the qualitative properties of the chromomagnetic instability in the phase diagram at moderate density regime within the gauged NJL model (i.e., nonrenormalizable four-fermi interactions), so we shall use the finite cutoff and, hence, the normalization (13). Fortunately, we did not find a qualitative difference between $\Omega_B$ and $\Omega'_B$. Here we quote quantitative differences between these two normalizations at zero temperature: $B$ determined from Eq. (13) around the gapless onset could be 40% larger as compared to that in Ref. [22] and, accordingly, the free energy gained by $B \neq 0$ could be 35% larger.

Figure 1 shows the Meissner mass squared of the 6th gluon, which is given by

$$m_{M,6}^2 = \frac{\partial^2 \Omega_B(\Delta, B, \delta\mu, \mu, T)}{\partial B^2} \bigg|_{B=0}. \quad (15)$$

At $T = 0$, we see the manifestation of the chromomagnetic instability at all values below $\Delta/\delta\mu = \sqrt{2}$. Note that the critical point of the instability ($\Delta/\delta\mu_c$) is somewhat lower than $\sqrt{2}$. This is because our model parameters do not correspond to the HDL limit, so the contribution from subleading logarithms $\sim \Delta^2 \ln(\Lambda/\Delta)$ is not negligibly small. We find, however, that the behavior of the Meissner mass squared is qualitatively consistent with that derived by using the HDL approximation [9].

As $T$ is increased, due to thermal smoothing effects, the characteristic kink at $\Delta/\delta\mu = 1$ is smeared and the Meissner mass tends to approach its value in the normal phase. However, its temperature dependence (at
fixed $\Delta/\delta \mu$ is non-monotonic. One can also see that the Meissner mass squared at small but nonzero values of $\Delta/\delta \mu$ remains negative even at high temperatures. At $T \approx \delta \mu/2 \approx 40$ MeV, the chromomagnetic instability related to gluons 4–7 finally disappears and the Meissner mass squared turns positive at all values of $\Delta/\delta \mu$. As $T$ increases further, the Meissner mass squared begins to go down and approaches zero.

In Figs. 2 and 3 we plot the effective potential measured with respect to the 2SC/g2SC phases at $B = 0$ as a function of $B$ for several temperatures.

Figure 2 shows the case of $\Delta/\delta \mu = 1$ (i.e., at the onset of the g2SC phase). At $T = 0$, one clearly sees that the gluonic phase at $B \neq 0$ is energetically more favored than the g2SC phase at $B = 0$. As $T$ is increased, the vacuum expectation value (VEV) of $B$ continuously goes to zero and the free energy gained by the gluonic phase also decreases to zero. We observe that the second-order phase transition from the gluonic phase to the g2SC phase occurs at $T \approx 30$ MeV. It can be also seen that the potential curvature at $B = 0$ is negative at $T = 0$, corresponding to the negative Meissner mass squared in the g2SC phase at $T = 0$. The potential curvature grows with increasing temperature (within the interval $T = 0 \to 30$ MeV) and its temperature dependence agrees with the result shown in Fig. 1.

In Fig. 3, the same plot is shown for the case of $\Delta/\delta \mu = 0.7$. At this value of $\Delta/\delta \mu$, the temperature dependence of the Meissner mass squared is not monotonic (see Fig. 1). One can in fact see that, as temperature grows, the potential curvature at $B = 0$ first drops and then goes up.

On the other hand, the free energy gain monotonically decreases with increasing temperature. We again find that the second-order phase transition at $T \approx 35$ MeV.

Here, we would like to make some comments. The phase transition from the gluonic to the 2SC/g2SC phases is, presumably, of second order. (Evaluating the effective potential at small $\Delta/\delta \mu$’s with sufficient accuracy is not easy and hence we cannot exclude the possibility of a first-order transition.) If so, the Meissner mass squared in the 2SC/g2SC phase could be a criterion for choosing the energetically favored phase without calculating the free energy.

In Fig. 4 we plot the temperature dependence of the critical point $(\Delta/\delta \mu)_c$ where the Meissner mass becomes negative. It is seen that $(\Delta/\delta \mu)_c$ continuously goes to zero at $T \approx \delta \mu/2$. We have checked the robustness of this relation varying model parameters and found that it works well as long as $\bar{\mu}$ is not too small. One can also see that $(\Delta/\delta \mu)_c$ grows slightly at low temperatures. Using huge values of $\Lambda$ and $\bar{\mu}$, we confirmed that this behavior remains true even in the HDL limit.

C. Meissner mass squared of gluons of color 8

We now turn to the 8th gluon. In order to investigate the single plane-wave LOFF state, we use the following gauge transformation described in Ref. [17]. Using the gauge transformation $\psi \to \psi' = \exp(-i\vec{q} \cdot \vec{x})\psi$, one can show that the single plane-wave LOFF state whose gap parameter spatially oscillates like $\Delta(x) = \Delta \exp(2i\vec{q} \cdot \vec{x})$...
is equivalent to the 2SC/g2SC phases with the Abelian vector condensate \( q \equiv \langle gA^8_z \rangle / (2\sqrt{3}) \).

The same argument for the normalization of the effective potential that we made in the previous subsection holds also for the 8th gluon. Therefore the normalized effective potential of the single plane-wave LOFF state

is equivalent to the 2SC/g2SC phases with the Abelian vector condensate \( q^2 = \langle gA^8 \rangle / (2\sqrt{3}) \).

The same argument for the normalization of the effective potential that we made in the previous subsection holds also for the 8th gluon. Therefore the normalized effective potential for the single plane-wave LOFF state

at nonzero temperature is

\[
\Omega_q = V(\Delta, q, \delta \mu, \mu, T) - V(0, q, \delta \mu, \mu, T). \tag{16}
\]

where \( q = \langle gA^8_z \rangle / (2\sqrt{3}) \). Here, we chose the third component of \( \vec{q} \) without loss of generality. The Meissner mass squared in the 2SC/g2SC phases is then given by

\[
m^2_{M,8} = \frac{1}{12} \frac{\partial^2 \Omega_q(\Delta, q, \delta \mu, \mu, T)}{\partial q^2} \bigg|_{q=0}. \tag{17}
\]

We have checked that the subtraction term,

\[
\frac{1}{12} \frac{\partial^2 V(0, q, \delta \mu, \mu, T)}{\partial q^2} \bigg|_{q=0}, \tag{18}
\]

agrees with \(-\Lambda^2/(9\pi^2)\) when \( T \to 0 \) or \( \Lambda \gg T \).

In Fig. 5, we plot \( m^2_{M,8} \) as a function of \( \Delta/\delta \mu \) for several temperatures. We see that \( m^2_{M,8} \) at \( T = 0 \) shows different behavior from that obtained by using the HDL approximation \[9\]: in particular, in the 2SC phase, \( m^2_{M,8} \) is not constant and there exists an enhancement in its values (the value of \( m^2_{M,8} \) in the HDL approximation should be \( 2/3 \) in Fig. 5). This is because, in Fig. 4, contributions from subleading logarithms have not been subtracted from the potential curvature. However, the behavior is qualitatively consistent with the HDL result and, furthermore, we observe a negative infinite Meissner mass squared at the gapless onset \( \Delta/\delta \mu = 1 \).

At \( T > 0 \), the negative infinite Meissner mass squared at the gapless onset is smeared and \( m^2_{M,8} \) tends to approach zero. However, the temperature dependence is
measured with respect to the 2SC/g2SC phases at $q = 0$ as a function of $q$ for $T = 0$ MeV (solid), 10 MeV (dotted), 20 MeV (dashed), and 30 MeV (dot-dashed). We used the $T$-independent gap $\Delta = 1.05\delta\mu$ and the same values of $\bar{\mu}$ and $\delta\mu$ as in Fig. 1.

non-monotonic. We also find a temperature-induced instability in a narrow region above $\Delta/\delta\mu = 1$. (In Fig. 4 one can see that the critical point for the 8th gluon grows at low temperatures. We shall take a closer look at this problem later.) These same results has been observed in Ref. [11]. Like in the case for gluons 4–7, $m_{M,8}^2$ at small $\Delta/\delta\mu$'s remains negative at high temperatures, but the instability related to the 8th gluon disappears at $T \simeq \delta\mu/2$.

In Figs. 6 and 7 we illustrate the effective potential measured with respect to the 2SC/g2SC phases at $q = 0$ as a function of $q$ for several temperatures.

Figure 6 corresponds to the gapless onset $\Delta/\delta\mu = 1$. At $T = 0$, the LOFF state at $q \simeq 70$ MeV is energetically more favored state than the g2SC phase and the potential curvature at $q = 0$ has a cusp, corresponding to a negative infinite Meissner mass squared at $\Delta/\delta\mu = 1$. (The result is consistent with that reported in Ref. [19].) The cusp at $q = 0$ is immediately smeared by temperature effects and the Meissner mass squared takes negative nonzero values. As $T$ is increased, both the VEV of $q$ and the free-energy gain by the LOFF state are decreased. At $T \simeq 30$ MeV, we find the second-order phase transition from the LOFF state to the g2SC phase. The temperature dependence of the potential curvature at $q = 0$ is consistent with the results shown in Fig. 5.

Figure 7 shows the temperature dependence of the effective potential at $\Delta/\delta\mu = 1.05$, slightly above the gapless onset. In this case, as mentioned earlier, one finds the instability only at $T > 0$. In terms of the free energy, the reason for the temperature-induced instability can be interpreted as follows. At $\Delta/\delta\mu = 1.05$, the Meissner mass squared in the 2SC phase is positive at $T = 0$ (see Fig. 4). One can see that the potential curvature at $q = 0$ is indeed positive at $T = 0$. However, as is clear from Fig. 4 there exists an energetically more favored state at $q \neq 0$, the LOFF state. Therefore, the 2SC phase is only metastable, though the Meissner mass squared is positive in this phase. (The metastable 2SC phase, separated from the LOFF state by a potential hump, exists in the region $1 < \Delta/\delta\mu \lesssim 1.08$.) As $T$ grows, the potential hump is smoothed out and, at a certain small temperature, the 2SC phase becomes unstable against the formation of the LOFF state. Specifically the Meissner mass squared turns negative at $T \simeq 2$ MeV. At $T \simeq 19$ MeV, we find a second-order phase transition from the LOFF state to the 2SC phase. The temperature dependence of the effective potential clearly explains the induced instability. However, it is fair to say that we have found an unexpected growth of the critical point also for the 8th gluon: $(\Delta/\delta\mu)_e$, grows to $\Delta/\delta\mu \simeq 1.09$.

Before concluding this subsection, we would like to mention the order of the phase transition LOFF $\rightarrow$ 2SC/g2SC. We found a second-order phase transition in a wide range of $\Delta/\delta\mu$. However, it is not easy to determine the order of the transition at small $\Delta/\delta\mu$ and around $\Delta/\delta\mu \simeq 1.08$. Hence, we do not exclude the possibility of a first-order transition.

**D. Phase diagram in $T$-$\Delta_0$ plane**

In order to see the consequences of the present analysis, we study the phase diagram of a two-flavor color superconductor. To this end, we first solve the gap equation,

$$\frac{\partial \Omega_{2SC/g2SC}}{\partial \Delta} = 0,$$

and the neutrality condition,

$$\frac{\partial \Omega_{2SC/g2SC}}{\partial \mu_e} = 0,$$

where the effective potential of the neutral 2SC/g2SC phases $\Omega_{2SC/g2SC}$ is given by

$$\Omega_{2SC/g2SC} = -\frac{1}{12\pi^2} \left( \mu_e^4 + 2\pi^2 T^2 \mu_e^2 + \frac{7\pi^4}{15} T^4 \right) + V(\Delta, 0, \delta\mu, \mu, T).$$

Here, the contribution from electrons has been added to Eq. 8.

In the left panel of Fig. 8 we illustrate the phase diagram of a neutral two-flavor color superconductor in the plane of $T$ and $\Delta_0$, where $\Delta_0$ is the value of the 2SC gap at $\delta\mu = 0$ and at $T = 0$ (i.e., the parameter $\Delta_0$ corresponds to the diquark coupling strength). The result is plotted for $\mu = 400$ MeV, which is a value typical...
FIG. 8: Left: The phase diagram of a neutral two-flavor color superconductor in the plane of temperature and $\Delta_0$. The solid (dashed) line denotes the critical line of the phase transition between the normal quark phase and the g2SC phase (the g2SC phase and the 2SC phase). The results are plotted for $\mu = 400$ MeV. Middle: The same as the left panel, but the unstable region for gluons 4–7 is depicted by the region enclosed by the thick solid line. Right: The same as the left panel, but the unstable region for the 8th gluon is depicted by the region enclosed by the thick solid line.

for the interior of compact stars. The solid (dashed) line denotes the critical line of the phase transition between the normal quark phase and the g2SC phase (the g2SC phase and the 2SC phase).

Let us have a look at main features of the phase diagram. One sees that qualitative features of the left panel of Fig. 8 are consistent with the $T$-$\mu$ phase diagram in the literature.

In the weak-coupling regime $77$ MeV $< \Delta_0 < 92$ MeV, the Fermi momentum mismatch is too large for these coupling strengths for diquark pairing and the system is in the normal quark (NQ) phase at $T = 0$. At $T > 0$, the mismatch of the Fermi surfaces is thermally smeared and, then, it opens the possibility of finding the g2SC phase (see, for example, Fig. 1 in the first paper in Ref. [3] and Fig. 4 in the third paper in Ref. [4]).

In the intermediate coupling regime, $92$ MeV $< \Delta_0 < 134$ MeV, the g2SC phase is realized at $T = 0$. For relatively strong coupling, $110$ MeV $< \Delta_0 < 134$ MeV, the g2SC phase at low temperature is replaced by the 2SC phase at intermediate temperature. At higher temperature, the 2SC phase is replaced by the g2SC phase again. It is known that this unusual behavior happens in the intermediately coupled two-flavor quark matter. For a detailed discussion, see the second paper in Ref. [7].

For strong coupling, $\Delta_0 > 134$ MeV, the gap $\Delta$ increases and the 2SC phase is accordingly favored at $T = 0$. At higher temperatures, however, $\Delta$ is decreased by thermal effects and the g2SC phase becomes possible (cf. Fig. 6 in the third paper in Ref. [3]).

Let us now take into account the chromomagnetic instability. Combining Eqs. (19) and (20) with Eqs. (15) and (17), we calculate the Meissner masses squared and map out the unstable regions on the phase diagram. (It should be remembered that we do not solve the gap equations for $\Delta$ and $\langle g A^\mu \rangle$ and the neutrality condition for $\mu_e$ self-consistently.)

FIG. 9: The Meissner masses squared of gluons of color 6 (solid) and 8 (dotted) [divided by $\bar{\mu}^2/(6\pi^2)$] as a function of $\Delta_0$ for $T = 0$ MeV (upper panel) and $T = 10$ MeV (lower panel). The results are plotted for $\mu = 400$ MeV.
The region enclosed with the solid thick line in the middle panel of Fig. 8 corresponds to the unstable region where gluons 4–7 have a negative Meissner mass squared. In this unstable region, therefore, the gluonic phase should be realized by a nonvanishing VEV of \(\langle g A^6_z \rangle\). At \(T = 0\), we see the manifestation of the instability in the region \(92\ \text{MeV} < \Delta_0 < 162\ \text{MeV}\) (see the upper panel of Fig. 9). (Note that the upper boundary of this unstable region, \(\Delta_0 = 162\ \text{MeV}\), is lower than that quoted in Refs. 17, 22. The reason for this discrepancy is the following: in order to find the unstable window, we have calculated the Meissner mass squared directly, whereas they used the relation \((\Delta/\delta\mu)_{c} = \sqrt{2}\), which is derived by using the HDL approximation. Of course, the discrepancy is nothing but a cutoff artifact.) At low temperatures, the whole g2SC phase and a part of the 2SC phase suffer from the instability. At \(T \approx 20\ \text{MeV}\), the instability related to gluons 4–7 is washed out and the phase transition is most probably of second order. (The unstable region should disappear at \(T \approx \delta\mu/2\) along the thin solid line (see Fig. 4). As a check, let us assume \(\mu_e \approx 90\ \text{MeV}\), which roughly corresponds to the value of \(\mu_e\) at the zero-temperature edge of the g2SC window. Then we see that the relation yields \(T \approx 20\ \text{MeV}\) indeed.)

In the right panel of Fig. 8, the unstable region for the 8th gluon is depicted by the enclosed region. (The region in which the 2SC phase is metastable is not shown in this figure.) In this region, the LOFF state is favorable to cure the instability related to the 8th gluon.

At \(T = 0\), as it should be, only the g2SC phase suffers from the instability. At \(T > 0\), however, the critical point shifts to larger \(\Delta_0\)'s and the unstable region penetrates into the 2SC phase. (This is nothing but the temperature-induced instability.) While the temperature-induced instability in the 2SC phase disappears at \(T \approx 16\ \text{MeV}\), the unstable region in the g2SC phase remains until \(T \approx 21\ \text{MeV}\).

Overlapping the middle and the right panels of Fig. 8, one can see that, apart from the case of strong coupling, the 2SC/g2SC phases at low temperature are unstable. In particular, the g2SC phase suffers from a severe instability related to both gluons 4–7 and 8. (In Fig. 10, we illustrate the temperature dependence of the Meissner masses squared \(m^2_{M,6}\) and \(m^2_{M,8}\) for the cases of \(\Delta_0 = 80, 110, 140\ \text{MeV}\). It is clear that at least one of the Meissner masses in the 2SC/g2SC phases is imaginary at low temperatures and both Meissner masses squared become positive at \(T \approx 20\ \text{MeV}\).) For strong couplings \(\Delta_0 \gtrsim 162\ \text{MeV}\), the system gets rid of the chromomagnetic instability. Note that one still finds a stable g2SC phase in the high-temperature region in the phase diagram, though the gapless structure is not significant at high temperature.

**III. SUMMARY AND DISCUSSION**

We studied the chromomagnetic instability in two-flavor quark matter at nonzero temperature. We use the gauged NJL model and first analyzed the curvature of the effective potential. Then, a temperature-dependent subtraction for the Meissner masses squared was introduced, so that magnetic gluons remain unscreened in the
normal phase. As mentioned earlier, this temperature dependence is indeed the cutoff artifact. In this work, we studied the properties of the chromomagnetic instability in the intermediate and strong coupling regimes, using a phenomenological four-fermi interaction. Therefore, we used a temperature-dependent subtraction and, accordingly, the ad hoc normalization of the effective potential. It should be emphasized that, for our standard value of the NJL cutoff, the temperature dependence of the subtraction term is not negligibly small actually.

We calculated the temperature dependence of the Meissner masses squared as a function of $\Delta/\delta \mu$ and found that, at $T \simeq \delta \mu/2$, the instability related to gluons 4–7 and 8 is washed out at all values of $\Delta/\delta \mu$. We also confirmed the temperature-induced instability (i.e., the growth of the critical point at $T > 0$) for the 8th gluon. In order to look at the temperature dependence of the Meissner masses squared, we computed not only the potential curvature but also the effective potential itself as a function of the vector condensates $\langle g A^z \rangle$ and $\langle g A^z \rangle$. Evaluating the effective potential played a crucial role for understanding the temperature-induced instability for the 8th gluon. By comparing the free energies of the 2SC phase and the single plane-wave LOFF state, we clarified that the induced instability mainly arises from the fact that the 2SC phase in the region slightly above the gapless onset is only metastable at $T = 0$.

We also presented the phase diagram in the $T-\Delta_0$ plane and mapped out the unstable regions for gluons 4–7 and 8 on the phase diagram. We found that, apart from the case of strong coupling, the 2SC/g2SC phases at low temperatures $\lesssim 20$ MeV suffer from a severe instability related to both gluons 4–7 and 8 and a large region in the g2SC phase should be replaced by the vector condensed phases.

In calculating the effective potential, we did not enforce the neutrality condition. Hence, the effective potential shown in Figs. 4 and 5 might be altered by the neutrality constraint. The result for the LOFF state (Fig. 7) should be taken as an indication that the neutral 2SC phase is not stable against the formation of the LOFF state even before $\Delta/\delta \mu$ reaches the gapless onset and that a first-order transition (2SC $\leftrightarrow$ LOFF) occurs at a certain value of $\Delta/\delta \mu > 1$. However, they must remain true even if we take into account the neutrality condition. At $T = 0$, a neutral LOFF state was studied by solving the gap equations for $\Delta$ and $g$ and the neutrality condition for $\mu_z$, self-consistently and it was revealed that the LOFF state is indeed favored over the 2SC phase even above the gapless onset (the edge of the LOFF state with the 2SC phase was determined to be $\Delta_0 = 137$ MeV [17]). In addition, Fig. 1 in Ref. [17] indicates such a first-order transition actually happens. At $T > 0$, although we did not examine the low-temperature effective potential of the LOFF state, it is likely that the first-order transition (2SC $\leftrightarrow$ LOFF above the gapless onset) takes place as long as $T$ is not too high.

Let us look at the right panel of Fig. 8 again. (We should recall that the region where the 2SC is metastable is not depicted in Fig. 8.) As we mentioned above, there exists a window where the LOFF state is energetically more favored than the 2SC phase, even though $m_{1,8}^2 > 0$ in the 2SC phase. The actual phase boundary (2SC $\leftrightarrow$ LOFF), as a consequence, shifts to larger $\Delta_0$'s when we solve the set of equations self-consistently and take account of the metastability of the 2SC phase. In addition, the NQ phase in the weak coupling regime will be replaced by the neutral LOFF state. At $T = 0$, in fact, it was found that the neutral LOFF state exists in the window $63 \text{ MeV} < \Delta_0 < 137$ MeV and that it is more stable than the NQ phase in whole this window [17]. The weakly coupled LOFF state survives at nonzero temperature and, presumably, undergoes a phase transition into the g2SC phase or the NQ phase [27]. Then, we conclude that, apart from the case of strong coupling, the low-temperature ($\lesssim 20$ MeV) region of a chromomagnetically stable, non color-flavor-locked phase has a completely different structure from known phase diagrams.

The most interesting remaining task is now clear: we have to take into account all the possible gluonic condensates and calculate the free energy in a self-consistent manner. (In particular, we have a limited knowledge of the gluonic phase [16, 22]. While suggestive, it is not sufficient for drawing a conclusion about the phase structure of the gluonic phase.) The resulting gluonic-condensed phase will be free from the chromomagnetic instability. Here, it should be noted that the study of the effective potential shows that the Meissner masses squared itself cannot be a criteria for choosing the ground state, in other words, the small-$\langle A^z \rangle$ expansion of the effective potential does not work. It is also interesting to make a free energy comparison between the gluonic-condensed phase and the LOFF state with realistic crystal structures. Finally, the gluonic-condensed phase has not been studied in the three-flavor case, so an extension to the realistic three-flavor quark matter would have important implications for the physics of compact stars.

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Note added. While writing this paper, I learned that an overlapping study was recently done by L. He, M. Jin, and P. Zhuang [27]. I am grateful that they made the results of their study available to me.


