Experiment and theory in the Casimir effect

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Casimir effect is the attractive force which acts between two plane parallel, closely spaced, uncharged, metallic plates in vacuum. This phenomenon was predicted theoretically in 1948 and reliably investigated experimentally only in recent years. In fact, the Casimir force is similar to the familiar van der Waals force in the case of relatively large separations when the relativistic effects come into play. We review the most important experiments on measuring the Casimir force by means of torsion pendulum, atomic force microscope and micromechanical torsional oscillator. Special attention is paid to the puzzle of the thermal Casimir force, i.e., to the apparent violation of the third law of thermodynamics when the Lifshitz theory of dispersion forces is applied to real metals. Thereafter we discuss the role of the Casimir force in nanosystems including the stiction phenomenon, actuators, and interaction of hydrogen atoms with carbon nanotubes. The applications of the Casimir effect for constraining predictions of extra-dimensional unification schemes and other physics beyond the standard model are also considered.

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I. FORCE ARISING FROM THE VACUUM

In 1948 H. B. G. Casimir \[1\] made a prediction that two large, neutral, parallel conducting plates separated by a distance \(z\) in vacuum attract each other with the force per unit area

\[
P(z) = \frac{F(z)}{S} = -\frac{\pi^2 \hbar c}{240 z^4}.
\]

Here \(\hbar\) is the Planck constant, \(c\) is the velocity of light, and \(S\) is the area of the plates. Figure 1 illustrates configuration giving rise to this force which was named after Casimir. The Casimir force is a quantum phenomenon (because it depends on \(\hbar\)) and also the relativistic one (because it depends also on \(c\)). In classical electrodynamics, there is no net force acting between uncharged conducting plates. Thus the Casimir force is very unusual. All forces which we know from both classical and quantum physics depend on some charges or interaction constants. For example, electric force acts between charged bodies and depends on their charges. The gravitational force depends on the masses (i.e., the gravitational charges) of interacting bodies. Forces acting between elementary particles depend also on the constants of weak and strong interactions. But the above expression for the Casimir force per unit area of the plates (i.e., for the pressure) does not depend on any interaction constant. It depends on only the fundamental constants \(\hbar\) and \(c\) and on the separation distance \(z\) which is the geometrical parameter. The magnitudes of the Casimir pressure are characteristic for macroscopic rather than for microscopic scales. For example, at a separation \(z = 1 \mu m\) it holds \(P = 1.3\) mPa which is not a small pressure as one could expect for a quantum phenomenon.

Physical explanation given by Casimir to his effect is connected with the concept of quantum vacuum. According to quantum field theory, “empty” space is in fact filled with zero-point (or vacuum) oscillations of all frequencies. These oscillations are sketched in figure 2, left. The total energy of the vacuum oscillations is equal to infinity. This, however, does not create a problem. The point is that in all fields of physics (with exception of gravitation) the energies are defined up to an additive constant. It is generally believed that all physical energies are measured from the top of the infinite vacuum energy in empty space. Because of this, when calculating the physical energy of some quantum process, one arrives first at an infinite result and then makes it finite by subtracting the infinite vacuum energy in empty space.

Using this procedure, Casimir calculated the total energy of vacuum oscillations between two parallel plates made of ideal metal and spaced a distance \(z\) apart. This calculation took into account that on metal surfaces the tangential component of electric field (it is parallel to the surface) vanishes. This boundary condition selects the vacuum oscillations shown in figure 2, right. The selected oscillations have roots on the plate surfaces and their total energy is equal to infinity. However, when we subtract from this infinity the infinite energy of vacuum oscillations in empty space, a finite energy which depends on \(z\) is found. The Casimir force is just the negative derivative of the above obtained finite energy with respect to separation.

In essence, the Casimir force can be understood in a unified way with the well known van der Waals force. The van der Waals force acts between two neutral atoms or molecules separated by a distance which is rather small but much
larger than the atomic dimensions. For nonpolar molecules, which have no intrinsic dipole moments, the expectation values for their dipole moment operators are zero. The van der Waals force arises in second order perturbation theory from the fluctuating dipole-dipole interaction. It is caused by the dispersions of dipole operators, i.e., by quantum fluctuations which create the instantaneous dipole moments in atoms and nonpolar molecules. Thus, the van der Waals force is a quantum phenomenon and depends on $\hbar$. This force acts also between an atom and a macroscopic body and between two closely spaced macroscopic bodies because each atom of one body interacts with each atom of another. Regarding vacuum oscillations it can be said that there is a fluctuating electromagnetic field both inside the condensed bodies and also in the gap between them.

If the separation between two atoms, an atom and a macroscopic body or between two macrobodies is large enough, the retardation of the electromagnetic fluctuating interaction comes into play due to the finiteness of the velocity of light. In this regime, the van der Waals force depends on both $\hbar$ and $c$ and is in fact the generalization of the above discussed above Casimir force, arising due to boundary conditions on ideal metal plates, for the case of real material bodies. (Notice that for ideal metals which have infinite conductivity the above dependence $P \sim z^{-4}$ is applicable at all separations, but for real bodies in a nonrelativistic regime of short separations $P \sim z^{-3}$ holds.) Thus, both the van der Waals and Casimir forces are caused by quantum fluctuations. Historically, the name “van der Waals” was usually connected with the nonrelativistic case and the name “Casimir” (or “Casimir-Polder” for atom-atom or atom-wall interactions) with the relativistic case. The generic name for both kinds of interactions is “dispersion forces”.

In 1956 E. M. Lifshitz developed a more general theory of the van der Waals and Casimir forces acting between two thick parallel plates (semispaces) with plane boundary surfaces (see figure 3). In this theory the plate materials are considered as continuous media described by the frequency-dependent dielectric permittivity $\varepsilon(\omega)$ and characterized by randomly fluctuating sources of the electromagnetic field. On the boundary surfaces electromagnetic fields obey the familiar boundary conditions of classical electrodynamics, i.e., that the tangential components of electric field $E$ and magnetic induction $B$ are continuous as well as the normal components of magnetic induction and electric displacement $D$ (only nonmagnetic materials are considered). The Lifshitz formula for the dispersion force takes into account not only vacuum fluctuations but also thermal photons when the material of the plates is in thermal equilibrium at some nonzero temperature $T$ (in Casimir formula the ideal metal plates are supposed to be at zero temperature). The dispersion force is given in the following symbolic form:

$$F(z, T) = \sum_{l=0}^{\infty} \int_{0}^{\infty} dk R_l(k, i\xi_l, r_P, r_{\perp}).$$

Here $R_l$ is some fixed function which depends on the magnitude of wave vector projection in the plane of plates $k$, the so called Matsubara frequencies $\xi_l = 2\pi k_B T l / \hbar$, where $k_B$ is the Boltzmann constant, and on the reflection coefficients $r_{P,\perp} = r_{P,\perp}(i\xi_l, k)$ for two independent polarizations of electromagnetic field (indices $\parallel$ and $\perp$ mean that $E$ is parallel and perpendicular to the plane of incidence, respectively). These reflection coefficients are in fact the familiar Fresnel’s coefficients but calculated on the imaginary frequency axis. They account for reflections of electromagnetic oscillations on the boundary surfaces of the plates (see figure 3). The Lifshitz theory describes both the van der Waals and Casimir forces (at small and large separations, respectively), and also the transition region from one type of dispersion force to another. In the original Lifshitz paper this theory was applied to dielectrics and the reflection coefficients were expressed in terms of dielectric permittivities of the plate material $\varepsilon(i\xi_l)$. In succeeding years the Lifshitz theory has found a lot of applications in the investigation of the van der Waals and Casimir forces (see, e.g., reviews and monographs). The Casimir effect is an interdisciplinary subject and attracts considerable attention in elementary particle physics, gravitation and cosmology, condensed matter physics and atomic physics. Below we discuss only a few selected topics in this wide field of research which are of great promise in both fundamental physics and nanotechnology but present severe problems in understanding and interpretation.

II. HOW TO MEASURE THE CASIMIR FORCE?

The measurement of the Casimir force is a very complicated problem because it is very difficult to hold macroscopic bodies at a separation of 1 $\mu$m or less (for small test bodies at larger separations, force becomes smaller than the sensitivity of most measuring devices). There are several basic challenges that need to be overcome for the measurement of the Casimir force. The surface of any real material is not perfectly shaped. It is covered by roughness, chemical impurities and dust particles, thus, making surface properties different of bulk material properties. Next, given that the force has a very strong dependence on separation, precise and reproducible measurement of separation between the two surfaces is required. It is not an easy matter to measure separations below one micrometer with sufficient precision. Furthermore, the electrostatic force due to residual electric charges on the surface and potential differences should be kept negligible.
The first attempt to measure the Casimir force between metal plates was undertaken by Sparnaay [11] using a force balance based on a spring balance. Measurements indicated the presence of an attractive force, but its magnitude was found with about 100% error which prevented a good comparison with Casimir prediction. In succeeding years many measurements of the van der Waals and Casimir force between both metal and dielectric bodies with plane, spherical and cylindrical surfaces was performed (see review [1]). Notable improvements have been made in resolution of the above-mentioned problems. As a result, the experimental error of force measurement was decreased to about 50%. The slow progress was caused by outstanding difficulties in the measurement of small forces and separations between closely spaced rough surfaces.

The first in the modern stage of Casimir force measurements, was performed by Lamoreaux in 1997 [12]. In this experiment the Casimir force between a gold coated spherical lens and flat plate was measured using a balance based on the torsion pendulum. Several important improvements have permitted to achieve higher accuracy than in all previous experiments. First, the residual potential difference between grounded surfaces was compensated with application of voltage to the lens. The lens was moved towards the plate by application of voltage to the piezo. The displacement was measured with a laser interferometer with an error of 10 nm. The calibration of the setup was done by measuring the electrostatic force due to different applied voltages at large separations where the Casimir force is very small.

The approximate theoretical expression for the Casimir force acting between a perfectly shaped plate and a sphere made of ideal metals at zero temperature is given by

\[ F(z) = \frac{\pi^3 \hbar c R}{360 z^4}, \]

where \( R = 12.5 \text{ cm} \) is the sphere radius. The geometrical corrections to this expression are less than \( z/R \) and, thus, it works good at separations \( z \ll R \) (a condition well satisfied in experiment [12] with large supply). The experimental data were found to be in agreement with theory at separations of about 1 \( \mu \text{m} \) with a 5 – 10% error. This was a major achievement in comparison with the previous measurements. However, no evidences of the corrections to the Casimir force due to finite conductivity (i.e., nonideality) of metal films, surface roughness and nonzero temperature were reported (the experiment was performed at laboratory temperature of about 300 K). Above all the Lamoreaux experiment has attracted much experimental and theoretical attention to the Casimir effect by applying modern measurement techniques and opening new opportunities for more precise tests of the theory.

A radically new approach to the precision measurements of the Casimir force was proposed by Mohideen and his collaborators [13, 14]. They have employed the atomic force microscope (a micromechanical device for the investigation of the surface with high resolution) to perform a set of most definitive experiments on the Casimir effect. The schematic diagram of the typical experiment for measuring the Casimir force using an atomic force microscope is shown in figure 4. The Casimir force acts between a relatively large polystyrene sphere of about 200 \( \mu \text{m} \) diameter and a sapphire disk both coated by metallic layer (Al in the first experiment and Au in the next). Instead of the usually used sharp tip, here a sphere is attached to the cantilever of an atomic force microscope which flexes under the influence of the force. The flexing of the cantilever leads to the deflection of the laser beam which is registered by the photodiodes A and B. The separation distance between a plate and a sphere can be changed with the help of the piezo. The calibration of the spring constant of the cantilever and determination of the absolute separations between a sphere and a plate was performed by means of application different voltages to the plate while the sphere was grounded. For this purpose the electric force between the two bodies was repeatedly measured at separations larger than 2 \( \mu \text{m} \), where the Casimir force is negligibly small, and compared with exact theoretical force-distance relation derived for sphere-plate configuration in classical electrodynamics. In the most refined experiment in [21], the absolute error of separation measurements as small as 0.8 nm was achieved. The roughness peaks are often higher than 0.8 nm. This does not prevent a precise determination of separations which are measured between zero roughness levels. The absolute error of force measurements was about 8.5 pN = 8.5 \times 10^{-12} \text{ N} at 95% confidence which results in relative error of 1.75% at the shortest separation of 62 nm.

High precision achieved in the measurements of the Casimir force by means of an atomic force microscope placed more stringent requirements upon the theoretical computations. To compare more precise experimental results with theory, it was necessary to take into account the nonideality of a metal covering the test bodies (i.e., the fact that electromagnetic oscillations penetrate into the metal up to the “skin depth”). It was necessary also to account for the surface roughness (in the experiments by Mohideen et al. it was carefully investigated by using the atomic force microscope with a sharp tip). This was done in [22] where the Casimir force, acting between a sphere and a disk, was calculated including of both finite conductivity and roughness corrections. In figure 5 the measured Casimir force as a function of plate-sphere separation is shown as open squares. The theoretical force with corrections due to surface roughness and finite conductivity is shown by the solid line, and without any corrections by the dashed line. From figure 5 it is clearly seen that the measurements of the Casimir force by means of an atomic force microscope are sensitive enough to demonstrate the role of corrections due to the finite conductivity of a metal and surface roughness.
The quantification of errors and precision in the Casimir force measurements by means of an atomic force microscope was performed in [23]. Special attention was paid to the sample-dependent variations of the optical properties, needed for computations using the Lifshitz formula, due to the presence of grains. Many other factors which influence the theoretical result were also carefully analyzed. At the shortest separation of 62 nm the relative theoretical error of about 1.7% was found. With the increase of separation, the theoretical error decreases. Within the limits of both experimental and theoretical errors, i.e., on the level of 1–2% at the shortest separations depending on the chosen confidence probability, very good agreement between experiment and theory was demonstrated.

The Casimir force discussed above acts perpendicular to the surfaces. A new interesting physical phenomenon discovered in [14,20] is the lateral Casimir force. This may arise when the bodies are asymmetrically positioned or their properties are anisotropic. The lateral Casimir force also originates from the modification of electromagnetic vacuum oscillations by material boundaries [24]. In [14,20] it was measured between a plate and a sphere both covered by the aligned sinusoidal corrugations of equal periods but different amplitudes. The schematic of the experiment is shown in figure 6. The first sphere is attached to the cantilever of an atomic force microscope. The second sphere is imprinted with the corrugations and interacts with the corrugated plate. The addition of the first sphere is needed to isolate the laser light from the region between the two corrugated surfaces. Unlike figure 4, the corrugated plate is mounted vertically in order to suppress the effect of the ordinary (normal) Casimir force.

Theory predicts that the lateral Casimir force should have a sinusoidal dependence on the phase difference (lateral displacement) between corrugations on the sphere and a plate. This was confirmed experimentally in [14,21]. In figure 7 the average measured lateral Casimir force is shown by solid squares as a function of lateral displacement. The solid line is the theoretical prediction. The lateral force amplitude $3.2 \times 10^{-13} \text{N}$ is determined with rather large error of about 24% because it is much smaller than the normal force.

Several more experiments on the measurement of the Casimir force were performed in the last few years. Among them one should mention the experiment [25] making use of the original Casimir configuration, i.e., two plane parallel metal plates. It is really hard to maintain plates parallel at a micrometer separation. This partly accounts for not too high precision of this experiment (the original Casimir formula for ideal metals was confirmed in the limits of 15% error). We will return to other experiments on the Casimir force in the following sections devoted to the puzzle of the thermal Casimir force and applications to nanotechnology.

### III. PUZZLE OF THE THERMAL CASIMIR FORCE

As was mentioned in section 1, Casimir obtained his famous formula by assuming that the temperature of the plates is equal to zero. In [26] the case when ideal metal plates are at some nonzero temperature $T$ was considered in the framework of thermal quantum field theory with appropriate boundary conditions. It was shown that at short separations (or, equivalently, at low temperatures) the corrections to the original Casimir result due to nonzero temperature are negligibly small. On the contrary, if separations between plates are several micrometers (or the temperature is high enough) the temperature correction becomes large. At room temperature $T = 300 \text{K}$ and plate separations larger than 6 μm the total Casimir pressure is already equal to the thermal term

$$P(z) = -\frac{k_B T}{4\pi z^3}\zeta(3),$$

where $\zeta(x)$ is the Riemann zeta function and $\zeta(3) \approx 1.202$. This behaviour is usually called “the classical limit” because it is determined by thermal photons $27$. All these results, obtained for the ideal metals at $T \neq 0$ from the thermal quantum field theory, follow also from the Lifshitz formula if one uses what is known as Schwinger’s prescription [28]. According to this prescription, one should take limit $\varepsilon \to \infty$ in the Lifshitz formula first and then perform summation in $l$. As a consequence, for both reflection coefficients at zero frequency it follows

$$r_{||}(0,k) = r_{\perp}(0,k) = 1$$

and the results of thermal quantum field theory are reobtained.

In the beginning of 2000, the rapid progress in experiment raised a question: What is the dependence of the Casimir force on the temperature for the case of real metal boundaries? At first the answer seemed simple. There was an opinion shared by many physicists that it can be simply solved by the substitution of the dielectric permittivity of a metal into the reflection coefficients, whereupon they are substituted into the Lifshitz formula. This was done in [29] and in [30,31] using different models for the dielectric permittivity of a metal and created a puzzle which is not completely resolved up to the present despite repeated attempts of several research groups in different countries. The basic facts of the problem are as follows.
In [29] the Drude model was used where the dielectric permittivity of a metal depends on the frequency as \( \varepsilon_D(\omega) \sim \omega^{-1} \). From this it follows

\[
 r_{\parallel}(0, k) = 1, \quad r_{\perp}(0, k) = 0
\]

in drastic contradiction with the above result obtained for ideal metals. At this point it is necessary to explain why these zero-frequency reflection coefficients that are of prime physical interest. The thing is that at large separations (or, alternatively, at high temperatures) the zero-frequency term of the Lifshitz formula alone determines the total magnitude of the Casimir force, the other terms being exponentially small. Thus, in [29], instead of the classical limit, as obtained for ideal metals, it follows

\[
 P(z) = -\frac{k_B T}{8\pi z^4}\zeta(3).
\]

This is a counter intuitive result because one may use plates made of metals of higher and higher conductivity (i.e., approaching to the case of the ideal metal), but nevertheless the Casimir pressure were only one half the magnitude found between ideal metals. Paper [29] (and supporting papers [32, 33]) also predict relatively large thermal corrections to the Casimir pressure between real metals at short separations. These corrections are about 500 times greater than between ideal metals.

Unlike paper [29], papers [30, 31] have used the plasma model dielectric function with \( \varepsilon_p(\omega) \sim \omega^{-2} \). This leads to quite different result for the reflection coefficients at zero frequency

\[
 r_{\parallel}(0, k) = 1, \quad r_{\perp}(0, k) = \frac{\sqrt{c^2k^2 + \omega_p^2} - ck}{\sqrt{c^2k^2 + \omega_p^2} + ck},
\]

where \( \omega_p \) is the plasma frequency. When real metal properties are approaching the ideal one, it holds \( \omega_p \to \infty \), and \( r_{\perp}(0, k) \) approaches unity, i.e., to the value valid for ideal metals. Thus, at large separations papers [30, 31] predict the same classical limit as was obtained for ideal metals from the thermal quantum field theory which is in accordance with physical intuition. Furthermore, at short separations papers [30, 31] predict only small thermal corrections to the Casimir pressure in qualitative agreement with the case of ideal metals.

Although both the above approaches to the thermal Casimir effect between real metals were in sharp mutual contradiction, over a period of time there were no decisive theoretical or experimental evidences against one or the other. Things have changed after the publication of papers [34, 35] where it was proved that the approach using the Drude model violates the fundamentals of thermodynamics. It is common knowledge that the thermal Casimir pressure is the negative derivative of the free energy per unit area with respect to separation. The negative derivative of the same free energy with respect to temperature is the entropy of a fluctuating field per unit area of plates. As was shown in [34, 35], in the case of perfect metallic crystal lattices with no impurities the entropy of a fluctuating field at zero temperature, computed using the Drude dielectric function, is not equal to zero. Thus, the third law of thermodynamics (the Nernst heat theorem) is violated. On the contrary, if the dielectric function of the plasma model is used, the entropy at zero temperature is equal to zero in accordance with the Nernst heat theorem. The behaviour of the entropy as a function of temperature is shown in figure 8 for the case of gold plates spaced at a separation \( z = 1 \mu m \). Solid line is computed using the plasma model, and the dashed line, which violates thermodynamics, using the Drude model.

To avoid the contradiction with thermodynamics, paper [36] applied the Drude dielectric function to metallic lattices with impurities possessing some nonzero residual relaxation at zero temperature (for perfect lattices there is no relaxation at zero temperature). As a result, [36] arrived at the conclusion that entropy at zero temperature is equal to zero. This, however, does not solve the problem of the thermodynamic inconsistency of the approach using the Drude model, as perfect crystal lattices have a nondegenerate dynamic state of lowest energy and, thus, according to quantum statistical physics, the entropy at zero temperature must be equal to zero in this idealized situation. This property, however, is violated by the approach using the Drude model.

The severity of the problem is connected with the fact that the Drude model presents the same behaviour of \( \varepsilon_D(\omega) \sim \omega^{-1} \) at low frequencies, as given by the Maxwell equations, whereas the behaviour \( \varepsilon_p(\omega) \sim \omega^{-2} \) given by plasma model is characteristic only for the frequency region of infrared optics. If this is the case, the question arises on the correctness of extrapolation of the latter behaviour to zero frequency. In this respect a deeper physical understanding of what is meant by the “zero” frequency is required. According to one approach, we should extrapolate to zero the actual reflection properties of plate materials at very low, quasistatic frequencies. According to the other approach, the zero-frequency limit should not be understood literally because there are no static fields in the vacuum state and static-field fluctuations. More likely, it should be understood as a mathematical limit to zero from the region...
of characteristic frequencies giving the major contribution to the Casimir force. The uselessness of the Drude model for the Casimir effect then becomes clear because this model is the self-consistent solution of Maxwell equations with the real current of conduction electrons. This current is created through the incidence on the conductor of a real electromagnetic field. It also involves the electric resistance and heating of a metal, a phenomenon which cannot be caused by the zero-point, vacuum oscillations. On the contrary, in the infrared, frequencies are so high that they cannot cause real current (in this region electric current is pure imaginary). This is the reason why the plasma model is well adapted for use in the theory of the Casimir effect.

Recently the arguments against a literal understanding of the zero-frequency term in the Lifshitz formula received strong support from the investigation of the Casimir force between two dielectric plates. As was shown in paper, the Casimir entropy of a fluctuating field between dielectrics goes to zero when temperature vanishes if the plate materials are described by $\varepsilon(\omega)$ with some finite static value $\varepsilon(0)$. If, however, the dc conductivity of dielectrics is taken into account, this results in a violation of the Nernst heat theorem. Note that at nonzero temperature dielectrics really possess some small but nonzero conductivity at zero frequency. Then it becomes clear that real material properties at very low frequencies are in fact irrelevant to the fluctuating phenomena described by the Lifshitz formula.

Although the plasma model approach is free of contradictions with thermodynamics, it cannot be considered as a final resolution of the puzzle of the thermal Casimir force between real metals. The free electron plasma model does not take relaxation into account and is not applicable in the frequency regions of the anomalous and normal skin effects. Thus, strictly speaking, it should not lead to very exact results at separations between plates above 2 $\mu$m at room temperature. Below about 250 $\text{nm}$ it is also not so accurate because it disregards the interband transitions and other processes taken into account in the optical data for the complex index of refraction. A more universal description of the thermal Casimir interaction between real metals free of contradictions with thermodynamics is given in terms of the Leontovich impedance $Z(\omega)$. This description was elaborated in papers and is based on the Leontovich boundary condition imposed on metal surfaces

$$E_t = Z(\omega) [B_t \times n],$$

where $n$ is the unit vector normal to the surface and directed into a metal. This permits to obtain an alternative representation of the reflection coefficients in the Lifshitz formula without use of the dielectric permittivity. In the case of the normal and anomalous skin effect, the impedance approach leads to the same values, unity, for the reflection coefficients at zero frequency as was the case for ideal metals. In the region of the infrared optics, the impedance reflection coefficients at zero frequency are

$$r_\parallel(0, k) = 1, \quad r_\perp(0, k) = \frac{\omega_p - ck}{\omega_p + ck}.$$  

In the limit of ideal metals ($\omega_p \rightarrow \infty$), $r_\perp(0, k) = 1$ again. Within the separation region from 150 to 250$\text{nm}$, the impedance approach is more accurate than the plasma model approach. It, however, is also an approximation. The Leontovich boundary condition is valid with the proviso that $|Z| \ll 1$. Because of this, the impedance approach is not applicable at short separations between the plates below the plasma wavelength. Here, however, the thermal corrections are very small and the Lifshitz formula at zero temperature can be used to calculate the Casimir force.

Recently, in addition to important theoretical information, there have been experimental results which help to resolve the puzzle of the thermal Casimir force. In Refs. the results of two new precise experiments on the determination of the Casimir pressure between metal coated parallel plates by means of micromechanical torsional oscillator have been reported. In fact, in the experimental setup, shown schematically in figure 9, the Casimir force acts between a large sphere and a plane plate, similar to the measurements using an atomic force microscope. Here, however, the vertical separation $z$ between the sphere and the plate was varied harmonically with time. Due to the presence of the Casimir force, the resonant frequency of the oscillator has been changed and this change was measured. As is known from theory, the change in the resonant frequency is proportional to the derivative of the force acting between a sphere and a plate with respect to separation. This derivative, in its turn, is approximately proportional to the Casimir pressure $P$ between two parallel plates spaced at a separation $z$. (The error in the determination of $P$ by using this procedure is no larger than $z/R$, where $R$ is the sphere radius.)

Separations between the sphere and the plate were determined by using the formula

$$z = z_{\text{meas}} - (D_1 + D_2) - b\theta.$$  

Here $z_{\text{meas}}$ was measured interferometrically, $D_1 + D_2$ was found by the application of different voltages and measuring the electrostatic force, the lever arm $b$ was determined optically, and the rotation angle $\theta$ was determined by measuring the difference in capacitance between the right and left electrodes (see figure 9 for the definition of all quantities).
As a result, the absolute error in separation measurements was reduced to 0.6 nm. In a wide separation region from 170 to 300 nm the relative error in the Casimir pressure varied between 0.55 and 0.60% at 95% confidence probability. This is the distinctive feature of the experiment under consideration when compared to previous measurements where high precision was achieved only at the shortest separations.

The experimental results were compared with the theoretical computations of the Casimir pressure described above using the Leontovich impedance and the Drude model. Notice that these approaches are qualitatively different only at zero Matsubara frequency. As to the contributions from nonzero frequencies, both approaches find these using tabulated optical data extrapolated to low frequencies and obtain practically the same result. In figure 10, left we plot by dots the pressure differences \( P_{\text{th}} - P_{\text{exp}} \) versus separation, where \( P_{\text{th}} \) is computed using the impedance approach. In figure 10, right the differences \( P_{\text{th}} - P_{\text{exp}} \) are plotted where \( P_{\text{th}} \) is computed using the Drude model approach. Solid lines show the absolute errors of the pressure differences found at 95% confidence. From figure 10, left it follows that the impedance approach is consistent with data (the plasma model approach also turns out to be consistent with data within experimental and theoretical errors). From figure 10, right it is seen that the Drude model approach is excluded experimentally within the separation region from 170 to 700 nm at 95% confidence. In the separation region from 300 to 500 nm this approach is excluded by experiment at even higher 99% confidence. Thus, experimentally the puzzle of the thermal Casimir force between real metals is resolved in favour of the approaches with nonzero contributions of both polarizations of electromagnetic field at zero frequency. At the same time, the thermal effect predicted by these approaches is very small at short separations and it is yet to be measured.

### IV. CASIMIR EFFECT AND NANOSYSTEMS

The first paper anticipating the importance of the Casimir effect in nanosystems was published 20 years ago. However, only recently, owing to shrinking device dimensions to nanometers, the role of the Casimir forces in their performance and fabrication has been generally recognized. At separations below a few ten nanometers the Casimir force dominates over other forces. As a result, movable components in nanoscale devices fabricated at so short separations often stick together due to the strong Casimir and van der Waals attraction. This process is referred to as “stiction”. It may lead to the collapse of movable elements to the underlying substrate, permanent adhesion and other undesirable consequences. Together with the familiar capillary forces, the stiction leads to poor yield in the fabrication of micro- and nanomechanical systems. Thus, it would be of much promise to develop systems with zero or suppressed Casimir force. This aim can be achieved if one considers that the Casimir force in layered structures and in closed volumes can be not only attractive but also repulsive. Then, by using the appropriate design, one may achieve the equilibrium between the attractive and repulsive contributions, i.e., effectively obtain a nanosystem with zero Casimir interaction.

Although stiction is a harmful phenomenon, the Casimir force can also play a useful role in nanosystems actuating the nanofabricated silicon plate. This was first demonstrated in paper where the gold coated sphere was suspended above one side of the heavily doped polysilicon plate which can rotate around a thin road (a more advanced device of this type shown in figure 9 was later used in for the precision measurement of the Casimir force). When the sphere was moved closer to the plate, the Casimir force, acting on the plate, tilted it about its central axis towards the sphere. Thus, vacuum oscillations of the electromagnetic field led to the mechanical motion of the plate demonstrating the first micromechanical device driven by the Casimir force.

A similar device was used to demonstrate the influence of the Casimir force on the oscillatory behaviour of microsystems. The simple model of the Casimir oscillator is shown in figure 11. It consists of a movable metallic plate subjected to the restoring force of a spring obeying the Hooke’s law, and the Casimir force arising from the interaction with a fixed metallic sphere. The Hooke’s force is linear in a shift of the plate \( \Delta z \), whereas the Casimir force is strongly nonlinear. As a result, the potential energy of this microdevice possesses a local and a global minima separated by the potential barrier. The Casimir force changes the resonant frequency of oscillations around the local minimum, and makes oscillations anharmonic. This could be used in future micro- and nanoelectromechanical systems.

All precise measurements of the Casimir force discussed above were performed between metal coated test bodies. However, the most important materials extensively used in nanotechnology are semiconductors. The conductivity properties of semiconductors range from metallic to dielectric and have considerable opportunity for the control and modulation of the Casimir force. Measurement of the van der Waals and Casimir forces between dielectrics has always been a problem because of the localized electric charges on their surfaces and contact potential differences. Semiconductors with a relatively high conductivity have an advantage that their surfaces avoid accumulation of charges but, at the same time, show a typical dielectric dependence of the dielectric permittivity on frequency within a range of frequencies contributing to the Casimir force.

First measurement of the Casimir force between a large gold coated sphere and single crystal silicon plate was
performed by Mohideen et al. \cite{21} by using an atomic force microscope. The experimental results were compared with the Lifshitz theory at zero temperature. (At separations from 62 to 120 nm, where the smallest total experimental error from 0.87 to 5.3% is achieved, any approach to the calculation of the thermal corrections discussed above leads to a negligible effect far below the experimental precision.) In figure 12 we present the differences $F^{\text{th}} - F^{\text{exp}}$ between theoretical and mean experimental Casimir force versus separation. Solid lines show the 95% confidence intervals. As is seen from this figure, theory is in a very good agreement with experiment. Later it was demonstrated that by changing the conductivity properties of a semiconductor plate, either by doping or due to irradiation by laser light, it is possible to change the force-distance relation. This opens new opportunities for the modulation of the Casimir force in semiconductor micro- and nanodevices.

One more important application of the van der Waals and Casimir forces to nanosystems is connected with atom-surface interaction. This interaction attracted much attention during the past few years in connection with experiments on quantum reflection of ultracold atoms and Bose-Einstein condensation \cite{47, 48}. In paper \cite{49} the Lifshitz formula describing atom-surface interaction was generalized for the case of an atom interacting with a multi-wall carbon nanotube. These nanotubes are the nanosystems which can be modelled by a graphite cylindrical shell containing several concentric hexagonal layers. The study of the van der Waals and Casimir forces between atoms and carbon nanostructures has become urgent due to the potential use of single-wall nanotubes for hydrogen storage (see references in \cite{49}). It is common knowledge that the storage of hydrogen is the key problem in the hydrogen energetics which offers an alternative to petroleum. For this reason, any new hydrogen storage mechanism would be of much importance. The van der Waals and Casimir forces acting between hydrogen atoms or molecules and carbon nanostructures play a deciding role in absorption phenomena and until recently were practically unexplored. Calculations show \cite{49} that the location of hydrogen atoms and molecules inside a multi-wall carbon nanotube is energetically preferable as compared with the outside. This result is rather promising for the problem of hydrogen storage in carbon nanostructures and calls for further investigation.

V. CASIMIR EFFECT AS A TEST FOR NEW PHYSICS

Modern science is aware of four fundamental interactions: gravitational, electromagnetic, weak and strong. The gravitational interaction is described by the Einstein’s general relativity theory which contains the Newton’s law of gravitation as a particular case. The electromagnetic interaction is described by the Maxwell’s equations of classical electrodynamics and, including quantum effects, by quantum electrodynamics. The weak interaction is only quantum. It is described together with electromagnetic phenomena in the framework of the Weinberg-Salam theory of electroweak interactions. The strong interaction is described by quantum chromodynamics. There is also the so called standard model which provides an uniform description of the three interactions: electromagnetic, weak and strong. Many attempts were undertaken during the last decades to unify all four interactions. They, however, did not achieve much success. The test of many theoretical predictions would require construction of new accelerators of huge energies, unattainable in the foreseeable future.

Many extensions of the standard model, including supergravity and string theory, make use of an old idea that the true dimensionality of space-time is larger than four. It is supposed that the additional spatial dimensions are compactified at some length scale, which is so small that they do not influence our everyday life and even precise scientific experiments. For a long time it was generally believed that the compactification scale is on the order of the Planck length, i.e., $\sim 10^{-33}$ cm. The corresponding energy scale is of order $10^{19}$ GeV.

The situation changed dramatically with the proposal of unification models for which the compactification energy may be of order $1 \text{ TeV} \approx 10^{3}$ GeV \cite{50}. According to these models, the compactification scale of extra dimensions may be as large as a fraction of millimeter. Such “large” extra dimensions are consistent with observations if it is presumed that the usual physical fields of the standard model exist only in the ordinary four dimensional space-time whereas gravity alone propagates into the extra spatial dimensions. As a result, the Newtonian gravitational potential between two point masses $m_1$ and $m_2$ (atoms for instance) separated by a distance $r$ acquires a Yukawa correction

$$V(r) = -\frac{G m_1 m_2}{r} \left( 1 + \alpha_G e^{-r/\lambda} \right),$$

where $G$ is the gravitational constant, $\alpha_G$ is a dimensionless constant characterising the strength of the Yukawa force, and $\lambda$ is its range. It is significant that the Yukawa-type forces are also predicted in ways unrelated to extra-dimensional physics. For example, the Yukawa potential describes new forces generated by the exchange of light bosons of mass $\hbar/(\lambda c)$ which arise in many other extensions of the standard model \cite{51, 52}.

Interestingly, at short separations the gravitational experiments of Eötvös- and Cavendish-type do not impose strong constraints on the values of $\alpha_G$ and $\lambda$. The best constraints obtained from the gravitational experiments made to date are shown in figure 13 where the region of $(\lambda, \alpha_G)$ plane above the lines 1–7 is experimentally prohibited and
below these lines is permitted (see paper \[\text{53}\] where the references to all experiments used in this figure are presented). From figure 13 it is seen, for instance, that for the interaction range \(\lambda = 10^{-5}\) m the Yukawa-type correction with \(\alpha_G = 10^5\) is not prohibited experimentally. If such correction would exist, the corresponding force would be \(10^5\) times greater than the usual Newtonian gravitation at a separation \(r = 10^{-5}\) m. Thus, at such short separations gravity loses its role as the dominant force acting between non-magnetic electrically neutral bodies. For these separations, constraints on new forces and extra dimensions should be extracted from the measurements of the van der Waals and Casimir forces \[\text{51, 50}\].

The first constraint on new physics from the Casimir force measurements between metals was obtained \[\text{54}\] by analysing data of the torsion pendulum experiment \[\text{12}\]. This constraint is shown by line 7 in figure 13. For \(\lambda < 5.6 \mu m\) it becomes stronger than any constraint obtained from gravitational experiments. The rigorous approach to the constraining hypothetical forces from the measurements of the Casimir force was developed in \[\text{12, 53}\]. In the second improved experiment by means of a micromechanical torsional oscillator \[\text{12, 55}\], the differences between the theoretical Casimir pressures, calculated using the Leontovich impedance, and the experimental ones belong to the 95% confidence interval with a half-width \(\Delta_{\text{tot}}|P^\text{th}(z) - P^\exp(z)|\) shown by the solid lines in figure 10, left. The quantity \(\Delta_{\text{tot}}\) is in fact a characteristic value of the agreement between experiment and theory at a chosen confidence.

If in addition to the Casimir pressure there were some hypothetical pressure \(P^\text{hyp}(z)\) due to extra dimensions or exchange of light elementary particles between the test bodies, its magnitude should be less or equal to the half-width of the confidence interval

\[
|P^\text{hyp}(z)| \leq \Delta_{\text{tot}}|P^\text{th}(z) - P^\exp(z)|.
\]

This inequality permits to obtain constraints on new forces. Really, the hypothetical pressure \(P^\text{hyp}\) between the two plates at a separation \(z\) can be found as a negative integral of the interatomic Yukawa potentials over the volumes of plates with subsequent differentiation of the obtained result with respect to separation. Thus, this pressure depends on \(\alpha_G\) and \(\lambda\):

\[
P^\text{hyp}(z) = -G\alpha_G \lambda^2 e^{-z/\lambda} F(\rho_i^{(1)}, \rho_j^{(2)}, \lambda),
\]

where the explicit form for \(F\) is determined by the structure of both plates which usually consist of several layers with the densities \(\rho_i^{(1)}\) and \(\rho_j^{(2)}\). The Newtonian gravitational force acting between closely spaced plates is negligibly small.

The constraints on \((\lambda, \alpha_G)\) following from the two measurements of the Casimir pressure by means of micromechanical torsional oscillator are shown in figure 14 by lines 1a and 1b. Line 1b is obtained from the first experiment \[\text{11}\], and line 1a from the second, improved experiment \[\text{12, 53}\]. For comparison, in the same figure the constraints following from old Casimir force measurements between dielectrics \[\text{7, 9}\] are shown by line 2, from torsion pendulum experiment \[\text{12, 54}\] by line 3 (this is a continuation to smaller \(\lambda\) of the line 7 in figure 13), and from the experiment by means of an atomic force microscope \[\text{18, 57}\] by line 4. As is seen from figure 14, the constraints obtained from the measurements of the Casimir force by means of micromechanical torsional oscillator completely fill in the gap between the modern constraints obtained using an atomic force microscope, and those obtained using a torsion pendulum.

Further work is needed to significantly strengthen the constraints on the predictions of extra-dimensional physics and other extensions of the standard model in a wider interaction range. For this purpose, experiments over a wider separation range which use smoother and thicker metal coatings on the surfaces of the test bodies are planned.

VI. CONCLUSIONS

As is evident from the foregoing, the Casimir effect is the subject of diverse theoretical and experimental studies, and applications in both fundamental physics and nanotechnology. This is due to the fact that the concept of zero-point vacuum fluctuations, central to the Casimir effect, is one of the most general, fundamental, and even puzzling concepts of modern physics. There are many other important applications of the Casimir effect in elementary particle physics, gravitation and cosmology, atomic and condensed matter physics which can not be discussed here due to the lack of space. They are connected with the bag model of hadrons, mechanisms of spontaneous compactification of extra dimensions, inflationary cosmology, problem of dark matter, theory of Rydberb atoms, atomic friction, wetting phenomena etc. \[\text{10, 11}\].

In our opinion, the most striking development of the last few years on the subject is the precision measurements of the Casimir force between metal surfaces. The novel experimental approaches have opened up new fields of application of dispersion forces and called for further theoretical investigations in the case of real boundaries. The puzzle of the thermal Casimir force between real metals which was discovered at this point calls for a few additional remarks. In fact, the most of theoretical output in the Casimir effect before the year 1997, when the intensive experimental work had begun, was produced by field theorists. The standard pattern of research in this area is the use of exact
mathematical tools in the framework of physical theory applied to a rather simple model of a system under study. It was desirable that the model used provide full correspondence to all relevant properties of the physical system. This ideal pattern is, unfortunately, unattainable in condensed matter physics. The physical systems under consideration there (real metals, for instance) are so complicated that one must consider a set of different approximate models to theoretically describe different properties of a system. It is self-evident that in doing so all basic principles of quantum mechanics, quantum electrodynamics, thermodynamics and statistical physics must be preserved. This preservation, however, is not automatic. If the model used leads to violation of some basic principle (as occurs in the Drude model approach to the thermal Casimir force) this should be considered as a signal that the model under consideration does not reflect some feature of the physical system important for the phenomenon of our interest. At the same time, this model can be entirely adequate in certain physical situations (Drude model, for instance, provides good description for many electrical properties of metals and for optical processes with real photons). Another model (the Leontovich impedance in our case) may be adequate in the area where the Drude model fails. At present, several experiments to measure the small thermal effect in the Casimir force have been proposed [56, 57, 58, 59]. The performance of these experiments will bring final resolution to the puzzle discussed in this paper.

We expect that the role of the van der Waals and Casimir forces in nanotechnology will increase. Miniaturization is the main tendency in modern technology and, thus, below some transition size-scale, these forces will become dominant. The application of the Casimir force as a test for fundamental physics also seems to be very promising. The relatively cheap, table-based laboratory experiments on the Casimir effect may be considered as an alternative for huge accelerators. All this suggests that we are only at the threshold of an important scientific line of inquiry devoted to the Casimir effect.

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Figures
FIG. 1: Configuration of two parallel plates of area $S$ made of ideal metal spaced $z$ apart.
FIG. 2: Zero-point vacuum oscillations; left: in empty space; right: in between two ideal metal plates.
FIG. 3: Two semispaces made of some real material with the dielectric permittivity $\varepsilon(\omega)$ and reflections of electromagnetic oscillations on their boundary planes.
FIG. 4: Schematic diagram of the experimental setup for measuring the Casimir force between a plate and a sphere using an atomic force microscope.
FIG. 5: Measured average Casimir force as a function of plate-sphere separation (open squares) compared with theory taking into account corrections due to surface roughness and finite conductivity (solid line) and without any correction (dashed line).
FIG. 6: Schematic diagram of the experimental setup for measuring the lateral Casimir force between the corrugated plate and the sphere using an atomic force microscope.
FIG. 7: Measured average lateral Casimir force as a function of the lateral displacement between corrugations on a sphere and a plate (solid squares) compared with theory (solid line).
FIG. 8: Casimir entropy per unit area as a function of temperature at a separation 1 µm between plates computed using the plasma model (solid line) and the Drude model (dashed line).
FIG. 9: Schematic diagram of the experimental setup for the determination of the Casimir pressure between two parallel gold coated plates using a micromechanical torsional oscillator.
FIG. 10: Differences between theoretical and experimental Casimir pressures between gold coated plates (dots) and the absolute errors of these differences at 95% confidence (solid lines) as functions of separation; left: theory uses the impedance approach; right: theory uses the Drude model approach.
FIG. 11: Schematic diagram for the model of oscillator actuated by the Casimir force.
$\Delta F(z) = F^{\text{th}} - F^{\text{exp}} (\text{pN})$

FIG. 12: Differences of theoretical and mean measured Casimir force acting between silicon plate and gold coated sphere (dots) and the absolute errors of these differences at 95% confidence (solid lines) as functions of separations.
FIG. 13: Constraints on the strength of the Yukawa interaction as a function of interaction range obtained from the experiments of Eötvös-type (lines 1, 2), Cavendish-type (lines 3–6), and from the measurement of the Casimir force by means of torsion pendulum (line 7).
FIG. 14: Constraints on the strength of the Yukawa interaction obtained from the measurements of the Casimir force by means of micromechanical torsional oscillator (lines 1a and 1b) and earlier experiments (lines 2–4). See text for a more detailed characterization of the lines.