LOCAL $SO(8) \times SU(8)$ INVARIANCE IN $N = 8$ SUPERGRAVITY
AND ITS IMPLICATION FOR SUPERUNIFICATION

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ABSTRACT

We review the construction of $N = 8$ supergravity with local $SO(8) \times SU(8)$ invariance and discuss some possible consequences for superunification.

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INTRODUCTION

The unification of elementary particles and their fundamental forces is probably one of the most daring endeavours in theoretical physics. It necessarily involves the extrapolation in energy over many orders of magnitude where no direct experimental information is, or is expected to become available. Thus one has to rely mainly on theoretical prejudices. Supergravity offers a unique and fascinating starting point for such a unification, which includes the gravitational forces ab initio, and which treats fermions and bosons in a symmetric fashion. The fact that gravity is an intrinsic part of these theories may seem to have a deep theoretical significance, but in practice it is one of the major obstacles for their application since the mass scale at which supergravity is defined is thus set by the Planck mass: 10^{19} proton masses. Nevertheless, there have been courageous attempts to investigate the implications of supergravity for phenomenologically relevant energies which have met with some success¹.

Extended supergravity is characterized by the number of independent supersymmetries under which the theory is invariant. These N supersymmetries are naturally fused with a rigid $SO(N)$ symmetry group. The largest supergravity theory is defined for $N = 8$ ²⁻⁴. In their pioneering paper⁴, Cremmer and Julia demonstrated that extended supergravity theories have more symmetries than the aforementioned rigid $SO(N)$, which are hidden in the conventional approach. First, there is an (off-shell) local group H without a kinetic term for the H gauge connections. Secondly there is a rigid (on-shell) non-compact symmetry group G which is an invariance

*) Talk given by H. Nicolai

of the equations of motion. H is then isomorphic to the maximal compact subgroup of G , and the scalar fields of the theory are described as the coset space G/H . For $N = 8$, $G = E_{7(+7)}$ and $H = SU(8)$, and it is this case which we will consider here. Lower N supergravities may be obtained by consistent truncation. It is possible to impose a gauge condition for the local H symmetry, which corresponds to choosing a specific parametrization of the coset space. In that case, the equations of motion are invariant under a rigid H group; its $SO(N)$ subgroup is a manifest symmetry of the Lagrangian in this formulation, which corresponds precisely to the $SO(N)$ group mentioned above.

Since N extended supergravity contains $(2)^N$ Abelian gauge fields in the adjoint representation of $SO(N)$ one may attempt to promote the rigid $SO(N)$ to a local gauge group. This programme has been carried out for $N \leq 4$ ⁵ and, more recently, for $N = 5$ ⁶. The latter paper also contains lowest order results for $N = 8$. Thus these theories contain a gauge coupling constant g in addition to the gravitational coupling constant κ (which we put equal to one in what follows).

In this contribution we wish to report on a successful attempt to formulate $N = 8$ supergravity with local $SO(8)$ gauge invariance⁷. An important aspect of this work is that the local $SU(8)$ invariance group can be preserved in the process of gauging $SO(8)$. To clarify the significance of this fact we recall that previously (i.e., for $N \leq 5$) the gauging was achieved in the manifestly $SO(N)$ invariant formulation, which is obtained in the local H gauge. The $SO(N)$ group that was gauged was thus embedded in the rigid $SU(N) \times SU(N)$ of $G \times H$. Here we proceed differently by leaving the local $SU(8)$ intact and by gauging the $SO(8)$ subgroup of E_7 instead. The conventional form with local $SO(8)$ and without $E_7 \times SU(8)$ may be recovered by fixing the local $SU(8)$ gauge afterwards. At present it is not known whether this theory follows from a non-trivial reduction from 11 dimensional supergravity⁸.

There are essentially two reasons for keeping the local $SU(8)$ throughout the calculation, the first one being technical: the $E_7/SU(8)$ coset structure turns out to be just as indispensable for the construction and the consistency proof of gauged $N = 8$ supergravity as in the ungauged case, even though E_7 is no longer a symmetry of the theory since it is broken by the $SO(8)$ gauging. The second reason is even more important. It has been known for quite some time⁹ that an $SO(8)$ gauge group is too small to comprise the observed particle states, and on the basis of this observation it has been argued that an $SO(8)$ gauging is undesirable. Our results show that such an objection is not valid since both $SU(8)$ and $SO(8)$ may be relevant for the particle spectrum. In fact, we believe that the existence of $N = 8$ supergravity with local $SO(8) \times SU(8)$ has important implications for possible superunification scenarios;

some of these will be discussed in Section 4. We should add that some interesting features of this theory have been known for some time. For instance, the β function of the $SO(8)$ gauge coupling constant g vanishes in the one-loop approximation¹⁰. Furthermore, the gauging generates a potential for the scalar fields which is not bounded from below. Although this poses serious difficulties, it is conceivable that such a potential may ultimately trigger spontaneous breaking of supersymmetry, $SU(8)$, or both. We will comment further on this in Section 4. Whether such a mechanism is compatible with previously proposed ones to obtain spontaneously broken $N = 8$ supergravity without gauging¹¹ remains an interesting problem.

RESULTS

In our conventions we will closely follow Refs. 3 and 6. The field multiplet of the $N = 8$ theory contains one graviton e_μ^a which is a singlet with respect to $SO(8) \times SU(8)$, eight gravitinos ψ_μ^i and 56 spin 1/2 fields χ^{ijk} which are assigned to representations of chiral $SU(8)$ and singlets under $SO(8)$, and 28 vector fields A_μ^{IJ} which transform as a 28 under $SO(8)$. The scalar fields are represented by a 56-bein ("Sechsfundfünzigbein")

$$\mathcal{V} = \begin{bmatrix} u_{ij}^{IJ} & v_{ijkl} \\ v^{klIJ} & u_{KL}^{kl} \end{bmatrix} \quad (2.1)$$

which is an element of E_7 in the fundamental representation. $\mathcal{V}$ transforms under local $SU(8)$ from the left, and under E_7 from the right; its inverse can be written in terms of the same submatrices u and v

$$\mathcal{V}^{-1} = \begin{bmatrix} u^{ij}_{IJ} & -v_{klIJ} \\ -v^{ijKL} & u_{KL}^{kl} \end{bmatrix} \quad (2.2)$$

All assignments have been collected in the Table below. Note that these are off-shell assignments in contradistinction to the ungauged

	e_μ^a	ψ_μ^i	A_μ^{IJ}	χ^{ijk}	u_{ij}^{IJ}	v^{ijIJ}
$SO(8)$	1	1	28	1	28	28
$SU(8)$	1	8	1	56	$\overline{28}$	28

case with $E_7(\text{rigid}) \times SU(8)(\text{local})$ where only the field strengths and their duals, but not the vector fields themselves, could be fitted into on-shell representations of E_7 . We write the $N = 9$ Lagrangian as follows:

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2} e R(e, \omega) - \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} \bar{\psi}_\mu^i \gamma_\nu \overleftrightarrow{D}_\rho \psi_{\sigma i} - \\ & -\frac{1}{8} e [F_{\mu\nu IJ}^+ F_{\mu\nu KL}^+ (2S^{IJ, KL} - \delta^{IK} \delta^{JL}) + \text{h.c.}] - \\ & -\frac{1}{2} e [F_{\mu\nu IJ}^+ O_{\mu\nu}^{+ KL} S^{IJ, KL} + \text{h.c.}] - \\ & -\frac{1}{2} e \bar{\chi}^{ijk} \overleftrightarrow{D} \chi_{ijk} - \frac{1}{96} e |A_\mu^{ijkl}|^2 - \\ & - \frac{1}{12} e [\bar{\chi}_{ijk} \gamma^\nu \gamma^\mu \psi_{\nu e} A_\mu^{ijkl} + \text{h.c.}] \end{aligned} \quad (2.3)$$

where $F_{\mu\nu IJ}^+$ ($F_{\mu\nu IJ}^-$) denotes the self-dual (antiself-dual) Abelian fields strengths of the 28 vectors A_μ^{IJ}

$$F_{\mu\nu}^{IJ} = F_{\mu\nu IJ}^+ + F_{\mu\nu IJ}^- = 2\partial_{[\mu} A_{\nu]}^{IJ} \quad (2.4)$$

$S^{IJ, KL}$ is defined in terms of the submatrices of the 56-bein by the condition

$$(u_{ij}^{IJ} + v_{ij IJ}) S^{IJ, KL} = u_{ij}^{KL} \quad (2.5)$$

and $O_{\mu\nu}^{+ IJ}$ is defined by

$$\begin{aligned} u_{IJ \mu\nu}^{ij} O_{\mu\nu}^{+ IJ} = & \frac{\sqrt{2}}{144} \eta \epsilon^{ijklmnpq} \bar{\chi}_{klm} \sigma_{\mu\nu} \chi_{npq} - \\ & - \frac{1}{2} \bar{\psi}_{\lambda k} \sigma_{\mu\nu} \gamma^\lambda \chi^{ijk} + \\ & + \frac{\sqrt{2}}{2} \bar{\psi}_\rho^i \gamma [\rho_{\sigma \mu\nu} \gamma^\sigma] \psi_\sigma^j \end{aligned} \quad (2.6)$$

The derivatives in (2.3) are covariant with respect to local Lorentz and local $SU(8)$ transformations; hence, besides the standard spin connection ω_μ^{ab} , we have $SU(8)$ gauge fields $\mathcal{B}_\mu^i{}_j$ which satisfy

$$(\mathcal{B}_\mu^i{}_j)^* = -\mathcal{B}_\mu^j{}_i; \quad \mathcal{B}_\mu^i{}_i = 0 \quad (2.7)$$

These gauge fields occur in D_μ according to

$$D_\mu \phi^i = \partial_\mu \phi^i + \frac{1}{2} \mathcal{B}_\mu^i{}_j \phi^j \quad (2.8)$$

where ϕ^i is an $SU(8)$ vector in the fundamental representation.

The SU(8) gauge fields $\mathcal{B}_{\mu j}^i$ do not correspond to dynamic degrees of freedom (at least classically); they can be expressed in terms of the physical fields of N = 8 supergravity. This dependence can be viewed as the result of an algebraic equation of motion (first order form) or of a conventional constraint (second order form). For our purposes it is most convenient to choose the second option; this brings our results in direct correspondence with those of Ref. 3. Also, the quantity $\mathcal{A}_{\mu}^{ijkl}$, which characterizes the scalar kinetic terms in (2.3), is dependent. The dependence of $\mathcal{A}_{\mu}$ and $\mathcal{B}_{\mu}$ on the 56-bein is determined by the requirement of E_7 invariance; the only quantity of that kind which contains one derivative is given by $D_{\mu} \mathcal{V} \cdot \mathcal{V}^{-1}$. This 56×56 matrix transforms covariantly under local SU(8) and takes its values in the Lie algebra of E_7 . The dependence of $\mathcal{A}_{\mu}$ and $\mathcal{B}_{\mu}$ is now defined by

$$D_{\mu} \mathcal{V} \cdot \mathcal{V}^{-1} = \begin{bmatrix} 0 & -\frac{\sqrt{2}}{4} \mathcal{A}_{\mu ijkl} \\ -\frac{\sqrt{2}}{4} \mathcal{A}_{\mu}^{mnpq} & 0 \end{bmatrix} \quad (2.9)$$

The diagonal blocks of (2.9) characterize the SU(8) subalgebra and are used to express $\mathcal{B}_{\mu}$ in terms of the submatrices of $\mathcal{V}$ and their covariant derivatives. The part of the algebra orthogonal to SU(8) defines $\mathcal{A}_{\mu}$ in a similar fashion. Note that $\mathcal{A}_{\mu}$ does not explicitly depend on $\mathcal{B}_{\mu}$ in this way. Its classification as a component of the E_7 Lie algebra implies

$$\mathcal{A}_{\mu}^{ijkl} = \frac{1}{24} \eta \epsilon^{ijklmnpq} \mathcal{A}_{\mu mnpq} \quad (2.10)$$

This is a typical example of the kind of argument that is of crucial importance throughout this paper. It is not possible to show the validity of (2.10) directly from the explicit dependence of $\mathcal{A}_{\mu}$ on the 56-bein; instead, we have to rely on group-theoretic arguments based on E_7 .

It is possible to view the matrix $\begin{bmatrix} \mathcal{B} & \mathcal{A}^* \\ \mathcal{A} & \mathcal{B}^* \end{bmatrix}$ as a connection of a local E_7 group. In that context, (2.9) specifies that the connection is pure gauge and has vanishing E_7 fields strengths. Indeed, application of a second SU(8) covariant derivative D_{ν} on (9) and antisymmetrization in μ and ν leads to

$$\begin{aligned} ([D_{\mu}, D_{\nu}] \mathcal{V}) \mathcal{V}^{-1} &= -\frac{1}{8} \begin{bmatrix} 0 & \mathcal{A}^* \\ \mathcal{A} & 0 \end{bmatrix}_{\mu} \begin{bmatrix} 0 & \mathcal{A}^* \\ \mathcal{A} & 0 \end{bmatrix}_{\nu} - \\ &\quad - \frac{\sqrt{2}}{4} \begin{bmatrix} 0 & D_{\mu} \mathcal{A}^* \\ D_{\nu} \mathcal{A} & 0 \end{bmatrix} - (\mu \leftrightarrow \nu) \end{aligned} \quad (2.11)$$

On the other hand, the commutator of two covariant derivatives is equal to the field strength; as $SU(8)$ acts on V from the left, the left-hand side of (2.11) takes the simple form

$$([D_\mu, D_\nu]V) \cdot V^{-1} = \begin{bmatrix} \delta_{[m}^{[i} \mathcal{F}(\mathcal{B})_{\mu\nu n]}^{j]} & 0 \\ 0 & \delta_{[p}^{[k} \mathcal{F}(\mathcal{B})_{\mu\nu q]}^{\ell]} \end{bmatrix} \quad (2.12)$$

where $\mathcal{F}(\mathcal{B})_{\mu\nu}^i$ denotes the $SU(8)$ field strengths. Equations (2.11) and (2.12) play a crucial role in establishing the invariance of the supergravity action; this requires a number of partial integrations which lead to $SU(8)$ field strengths by means of the Ricci identity (2.12) or to derivatives on $\mathcal{A}_\mu$. By using (2.11) and (2.12) we find that the antisymmetric derivative of $\mathcal{A}_\mu$ vanishes and that the $SU(8)$ field strength can be expressed in terms which cancel against other variations. Indeed, (2.11) and (2.12) were previously found by requiring supersymmetry invariance of the action.

The gauging of $SO(8)$ is now affected by further extending the covariant derivative with respect to local $SO(8)$ embedded in the E_7 group. For instance, we define

$$D_\mu u_{ij}^{IJ} \equiv \partial_\mu u_{ij}^{IJ} + \mathcal{B}_\mu^k [i u_{j]}^k{}^{IJ} - 2g A_\mu^K [I u_{j]}^{J] K} \quad (2.13)$$

where A_μ^{IJ} are the 28 vectors of $N = 8$ supergravity. At the same time we replace the field strengths $F_{\mu\nu}^{IJ}$ by their fully $SO(8)$ covariant counterparts

$$F_{\mu\nu}^{IJ} \equiv 2\partial_{[\mu} A_{\nu]}^{IJ} - 2g A_{[\mu}^{IK} A_{\nu]}^{KJ} \quad (2.14)$$

The presence of the order g terms in the Lagrangian and transformation rules violates the supersymmetry invariance of the original action. To re-establish the invariance one has to introduce new terms in the Lagrangian and transformations. These can be parametrized in terms of three tensorial functions A_{1-3} which depend on the scalars contained in the 56-bein. The parametrization takes the following form⁶

$$\begin{aligned} \delta_g \bar{\psi}_\mu^i &= -\sqrt{2} g \bar{\epsilon}_j \gamma_\mu A_1^{ji} \\ \delta_g \bar{\chi}^{ijk} &= -2g \bar{\epsilon}^\ell A_{2\ell}^{ijk} \end{aligned} \quad (2.15)$$

$$\begin{aligned}
\mathcal{L}_g &= \sqrt{2}g e A_{1ij} \bar{\psi}_\mu^i \sigma^{\mu\nu} \psi_\nu^i + \text{h.c.} \\
&+ \frac{1}{6} g e A_{2jkl}^i \psi_i^\mu \gamma_\mu \chi^{jkl} + \text{h.c.} \\
&+ g e A_3^{ijk,lmn} \bar{\chi}_{ijk} \chi_{lmn} + \text{h.c.} \\
L_{g^2} &= g^2 e \left(\frac{3}{4} |A_1^{ij}|^2 - \frac{1}{24} |A_{2jkl}^i|^2 \right)
\end{aligned} \tag{2.15}$$

Note that the SU(8) tensors A_{1-3} must satisfy certain symmetry properties as a consequence of the way in which they appear in (2.15). The solution for A_{1-3} can be found by requiring that all $gF\bar{E}\psi$ and $gF\bar{E}\chi$ variations of the Lagrangian cancel:

$$\begin{aligned}
A_1^{ij} &= \frac{4}{21} T_m^{ijm} \\
A_{2m}^{ijk} &= -\frac{4}{3} T_m^{[ijk]} \\
A_3^{ijk,lmn} &= \frac{\sqrt{2}}{108} \eta \epsilon^{ijkpqr} [T_m^{lmn}]_{pqr}
\end{aligned} \tag{2.16}$$

where the SU(8) tensor T is defined by

$$T_\ell^{kij} \equiv (u_{IJ}^{ij} + v^{ijIJ})(u_{\ell m}^{JK} u_{KI}^{km} - v_{\ell mJK} v^{kmKI}) \tag{2.17}$$

which is manifestly antisymmetric in i and j. Note that T is not invariant with respect to the full E_7 group, but it is invariant under local SO(8) and covariant under SU(8).

In order for the solution (2.16) to be consistent, the tensor T has to obey a number of non-trivial identities; these follow from the E_7 structure of the 56-bein. For instance, one can prove that T admits the following decomposition

$$T_\ell^{kij} = T_\ell^{[kij]} + \frac{2}{7} \delta_\ell^{[i} T_m^{j]mk} \tag{2.18}$$

where T_m^{jmk} is symmetric in j,k and $T_\ell^{[kij]}$ is completely antisymmetric and traceless, i.e.,

$$T_k^{[kij]} = 0 \tag{2.19}$$

For the proof of invariance of the Lagrangian in order g and g^2 , one also needs the following identities in terms of A_1 and A_2

$$\begin{aligned}
D_\mu A_1^{ij} &= -\frac{1}{24}\sqrt{2} \mathcal{A}_\mu^{ik\ell m} A_{2k\ell m}^j - (i \leftrightarrow j) \\
D_\mu A_{2i}^{jkl} &= -\frac{\sqrt{2}}{2} A_{1im} \mathcal{A}_\mu^{mjkl} - \frac{3}{4}\sqrt{2} \mathcal{A}_\mu^{mn} [jk A_{2imn}^\ell] - \\
&\quad - \frac{\sqrt{2}}{4} \mathcal{A}_\mu^{mnp} [j \delta_i^k A_{2mnp}^\ell] \\
A_{2i}^{klm} A_{2klm}^j - 18 A_{1ik} A_1^{kj} &= \\
&= -\frac{1}{8} \delta_i^j (A_{2p}^{klm} A_{2klm}^p - 18 A_{1kl} A_1^{kl}) \\
- A_{1it} A_{2jkl}^t + \frac{\eta}{24} \epsilon_{abcpqr} [jk A_{2\ell}^a]^{abc} A_{2i}^{pqr} &= \\
&= \text{antisymmetric and self-dual in } [ijkl]
\end{aligned} \tag{2.20}$$

where the defining equation (2.9) for $\mathcal{A}_\mu$ now includes the covariantization with respect to local $SO(8)$. The proof of (2.20) is again based on the E_7 properties of V .

SPECIAL GAUGE CHOICE

It is instructive to impose an $SU(8)$ gauge choice which corresponds to an explicit parametrization of the $E_7/SU(8)$ coset space. In the symmetric gauge⁴, the 56-bein becomes

$$\mathcal{V} = \exp \begin{bmatrix} 0 & -\frac{\sqrt{2}}{4} \phi_{ijkl} \\ -\frac{\sqrt{2}}{4} \bar{\phi}^{mnpq} & 0 \end{bmatrix} \tag{3.1}$$

and introducing the variable $y^{[4]}$

$$y_{ij,kl} \equiv \phi_{ijmn} \left(\frac{\tanh \sqrt{(1/8) \bar{\phi} \phi}}{\sqrt{\bar{\phi} \phi}} \right)^{mn}_{kl} \tag{3.2}$$

(Matrix-multiplication always over pairs of indices). We obtain an explicit representation for $^{[4]}$

$$U = \begin{bmatrix} P^{-\frac{1}{2}} & -P^{-\frac{1}{2}} y \\ -\bar{P}^{-\frac{1}{2}} \bar{y} & \bar{P}^{-\frac{1}{2}} \end{bmatrix} \quad (3.3)$$

with

$$P_{ij}^{kl} \equiv \delta_{ij}^{kl} - y_{ijmn} \bar{y}^{mnkl} \quad (3.4)$$

[the notation is the same as in Ref. 3]. The inverse 56-bein is given by formula (2.2). Inserting this parametrization into T_{ijkl} , we obtain the functions A_1 as infinite series of the self-dual scalar fields ϕ^{ijkl} . For A_1 and A_2 these expressions read as follows up to cubic order

$$\begin{aligned} A_1^{ij} &= (1 - \frac{1}{96} |\phi|^2)^{-\frac{1}{2}} \delta^{ij} + \\ &+ \frac{\sqrt{2}}{96} \phi^{ikmn} \phi_{mnpq} \phi^{pqkj} + O(\phi^4) \\ A_2^{ijkl} &= -\frac{\sqrt{2}}{2} (1 - \frac{1}{144} |\phi|^2) \phi^{ijkl} - \frac{3}{8} \phi_{mnl} [i \phi^{jk}]^{mn} \\ &+ \frac{\sqrt{2}}{16} \phi_{lpqr} \phi^{pqrs} [i \phi^{jk}]^{rs} + O(\phi^4) \\ |\phi|^2 &\equiv \phi^{ijkl} \phi_{ijkl} \end{aligned} \quad (3.5)$$

A_3 is related to A_2 by (2.16). In the reduction to $N = 5$, these results can be compared with those of Ref. 6. Both $SO(8)$ and $SU(8)$ are affected by the gauge choice (3.1); however, the non-trivial $SO(8)$ subgroup acting on both $SO(8)$ and $SU(8)$ indices is preserved. Determining $\mathcal{B}_{\mu j}^i$ from the $SO(8)$ covariantized version of (2.9), we find

$$\begin{aligned} \mathcal{B}_{\mu j}^i &= \frac{2}{3} \left\{ (\partial_{\mu} P^{-\frac{1}{2}} \cdot P^{-\frac{1}{2}})_{jm}^{im} - (\partial_{\mu} (P^{-\frac{1}{2}} y) \cdot \bar{P}^{-\frac{1}{2}} \bar{y})_{jm}^{im} - \right. \\ &\left. - 2g A_{\mu}^{mp} (P^{-\frac{1}{2}})_{jr}^{qm} P^{-\frac{1}{2}}{}_{pq}^{ir} - (P^{-\frac{1}{2}} y)_{jr}^{qm} (\bar{P}^{-\frac{1}{2}} \bar{y})^{pqir} \right\} \end{aligned} \quad (3.6)$$

In order to rewrite (3.6) in a more suggestive form we introduce the fully $SO(8)$ covariant derivative $\mathcal{D}_{\mu}$

$$\begin{aligned} \mathcal{D}_{\mu} P^{-\frac{1}{2}}{}_{jr}^{pq} &\equiv \partial_{\mu} P^{-\frac{1}{2}}{}_{jr}^{pq} - 2g A_{\mu t} [j P^{-\frac{1}{2}}{}_{r}]^t{}^{pq} - \\ &- 2g A_{\mu t} [P^{-\frac{1}{2}}{}_{jr}]^t{}^{pq} \end{aligned} \quad (3.7)$$

which now acts on all indices. In terms of $\mathcal{D}_\mu$, (3.6) becomes

$$\mathcal{B}_{\mu j}^i = \frac{2}{3} \left\{ \mathcal{D}_\mu P^{-\frac{1}{2}} \cdot P^{-\frac{1}{2}} \right\}_{jm}^{im} - (\mathcal{D}_\mu (P^{-\frac{1}{2}} y) \cdot \bar{P}^{-\frac{1}{2}} \bar{y})_{jm}^{im} \left\} + \right. \\ \left. + \frac{4}{3} g A_{\mu t} [j \delta_r^{ir}] t \right. \quad (3.8)$$

where we have also used the fact that $P^{-1} - (P^{-\frac{1}{2}} y)(\bar{P}^{-\frac{1}{2}} \bar{y}) = \mathbb{1}$. Hence,

$$\mathcal{B}_{\mu j}^i = -2g A_\mu^{ij} + \text{non-linear SO(8) covariant terms} \quad (3.9)$$

In the special gauge (3.1) there is no longer a distinction between SO(8) and SU(8) indices, and Eq. (3.9) shows that A_μ^{IJ} will now couple minimally to all fields that carry SO(8) and/or SU(8) indices. This corresponds to the standard formulation of supergravity with local SO(N) that has been obtained for $N \leq 5$.

We now turn to the discussion of the cosmological term induced by the gauging. It is important that for $N \geq 4$ this is not a cosmological constant but rather a scalar field potential which is furthermore rather badly behaved being unbounded from below. From (2.15) it is evident that the potential

$$g^{-2} \mathcal{P}(V) = \frac{1}{24} |A_{2i}^{jkl}|^2 - \frac{3}{4} |A_1^{ij}|^2 \quad (3.10)$$

is the difference between two positive definite terms and as such can take values between $+\infty$ and $-\infty$. However, one should keep in mind that A_1 and A_2 are not independent but depend on the 56-bein in the way specified in the preceding section. To analyze this aspect a little further it is advantageous to go to the special gauge, and we consider here the cases $N = 4$ and $N = 5$.

i) For $N = 4$ ⁵, there is one complex scalar ϕ and the potential

$$\mathcal{P}(\phi) = -2 - \frac{4}{1-|\phi|^2}, \quad |\phi|^2 \leq 1 \quad (3.11)$$

is unbounded from below but bounded from above. The singularity at finite $|\phi|^2$ is an artifact of the coset parametrization since the variable ϕ corresponds to the variable y introduced in (3.2).

ii) For $N = 5$ ⁶ there are five complex scalars ϕ^i and the potential is ($\phi^1 \equiv \phi_i^*$)

$$\mathcal{P}(\phi_i) = -2 - \frac{4}{1-\phi_i \phi^i} + \\ + \frac{1}{2(1-\phi_i \phi^i)^2} [(\phi_i \phi^i)^2 - |\phi_i \phi_i|^2], \quad \phi_i \phi^i \leq 1 \quad (3.12)$$

which, of course, is still unbounded from below. The Cauchy-Schwarz inequality tells us that

$$|\phi_i \phi^i|^2 - |\phi_i \phi_i|^2 \geq 0 \quad (3.13)$$

and is saturated if and only if

$$\phi_i = e^{i\alpha} \phi^i \quad (3.14)$$

for some $\alpha \in [0, 2\pi)$. In all those directions the potential tends to $-\infty$ as $|\phi|^2 \rightarrow 1$; this set in particular includes all possible $N = 4$ truncations. In any other direction, (3.13) is strictly positive and as its coefficient in (3.12) diverges faster than the second term in (3.12) for $|\phi|^2 \rightarrow 1$, the potential tends to $+\infty$ there. It is amusing that the set (3.14) with "bad behaviour" constitutes a set of measure zero on the sphere $|\phi|^2 = 1$ in $\mathbb{C}^5$.

For $N = 8$ one may also insert the special gauge choice into (3.1) thereby obtaining the $N = 8$ potential explicitly in terms of the self-dual scalar fields. Unfortunately, the resulting expression is rather unwieldy since it cannot be reduced to simply contracted structures such as $\phi^{ijkl}\phi_{ijkl}$, and therefore we shall not discuss it here. The simplicity of (3.10) in terms of A_1 and A_2 is entirely lost in any special gauge.

CONCLUDING REMARKS

The result that $N = 8$ supergravity exists with local $SO(8) \times SU(8)$ widens the range of possible scenarios for superunification. One possibility is that $SO(8) \times SU(8)$ is actually relevant for the particle spectrum at the Planck mass scale. The problem of embedding the $SU(3) \times SU(2) \times U(1)$ invariance of strong, weak and electromagnetic interactions into $SO(8) \times SU(8)$ may then be resolved in a variety of ways. At any rate, the $SO(8)$ gauge interaction breaks the E_7 invariance of the theory. As is well known, unitary representations of the non-compact E_7 group are infinite dimensional and therefore superunification based on $E_7 \times SU(8)$ would lead to a proliferation of particle states at the Planck mass¹². One has argued that an infinite number of particle states is a welcome feature which leads to a necessary re-arrangement of helicity states. However, the fact that another version of $N = 8$ supergravity is now available may require a fundamental re-orientation of the superunification strategy.

Since the local $SO(8)$ and $SU(8)$ are realized in a different fashion in the Lagrangian they will presumably play different roles. One possibility which we find rather intriguing is the following "preconfinement" hypothesis: one may assume that the $SO(8)$ Yang-Mills interactions provide the force which confines the preons at the Planck mass scale. In that case, all "observable" states would have

to be $SO(8)$ singlets. Since the only supergravity fields that carry $SO(8)$ indices are the spin 0 and spin 1 fields, we are led to the conclusion that only the graviton, the gravitinos and the spin 1/2 fields correspond to observable particle states. An important aspect of this mechanism is that it gives us a well-defined confinement criterion [namely " $SO(8)$ -neutrality"] which would explain why the graviton is not confined. Such a criterion appears to be lacking in the conventional approach to superunification. The $SO(8)$ preconfinement hypothesis also leads to supersymmetry breaking: because only part of the basic supermultiplet is confined, $N = 8$ supersymmetry can no longer be realized. Some dynamical restoration of symmetry may take place because the bound states may fill the gap and lead to a supersymmetric spectrum again. However, this would presumably lead to a lower N version of rigid supersymmetry which may be relevant at much lower mass scales than local $N = 8$ supersymmetry. If the latter is broken at the Planck scale one is no longer forced to put the bound states into $N = 8$ supermultiplets which could possibly circumvent some problems of the standard scheme. In view of the fact that the β function is known to vanish at one loop and is conjectured to vanish to all orders, the $SO(8)$ preconfinement would have to be non-perturbative. Further studies of the dynamical aspects are badly needed and we wish to emphasize that these should now be undertaken within a broader context than before.

We have already mentioned the fact that the potential induced by the gauging has the annoying feature of being unbounded from below. At present one can envisage (at least) two possible resolutions to this problem. The first is that the potential may not be all that significant any more if it lies in the confined sector, and we remind the reader that unbounded potentials have been encountered and dealt with in physics before. Another possibility which may be related to the first is that additional terms in the potential are generated dynamically by the same mechanism that produces the kinetic terms for the $SU(8)$ connections. Such additional terms could remedy the defect of unboundedness and would correspond to a quantum mechanical stabilization of the potential. The effective potential would then be a sum of two terms such that

$$\mathcal{I}_{\text{eff}}(V) = \mathcal{I}(V) + \mathcal{I}_{\text{dyn}}(V) \geq \text{const.} \quad (4.1)$$

Indeed, the form of $\mathcal{I}(V)$ in (3.10) bears a striking resemblance to a Higgs-like potential with two mass terms for the $SU(8)$ composite tensors A_1 and A_2 but with higher order (ϕ^4 -like) interactions missing. If those were generated dynamically with the correct sign one would obtain a spontaneous breaking of $SU(8)$ via a non-vanishing vacuum expectation value

$$\langle A_1^{ij} \rangle \neq 0 \quad (4.2)$$

for example. We stress that in the ungauged theory there is no such possibility for breaking $SU(8)$ due to the absence of a scalar field potential: if one insists on E_7 invariance, there is no way of constructing non-trivial $SU(8)$ objects from the 56-bein U . Non-vanishing vacuum expectation values for A_1 and A_2 would give masses to the spinors via (2.15) and this might entail some interesting phenomenological consequences. Thus, the gauging of $SO(8)$ might well turn out to be indispensable for bridging the gap between $N = 8$ supergravity and present-day phenomenology.

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