UPDATED CONSTRAINTS ON AXIONS FROM SN1987A

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Abstract

We recently determined a lower bound on the axion decay constant (upper bound on the axion mass) by demanding that the axion emission rate in SN1987A to be small enough to be consistent with the observed neutrino emission. We estimate here the magnitude of corrections to our previous results due to high density effects, $\rho$-exchange and the restoration of chiral symmetry. Assuming a non-degenerate core, we now find that for naive quark model couplings $f_a \geq 0.2 \times 10^{-12}$ GeV ($m_a \leq 3.6 \times 10^{-4}$ eV) which is a factor of 4 different from our previous limits assuming a degenerate core. Limits using couplings derived from EMC measurements are a factor of 2-4 weaker. We show that there is no window in $f_a$ for non-freely-streaming axions. These limits still imply that the remaining window for axions is the one which is the most interesting for cosmology.
Astrophysical considerations\textsuperscript{1-4)} provide the most significant constraints on the properties of the invisible axion\textsuperscript{5)}, the pseudo-Nambu-Goldstone boson which provides the most attractive solution to the strong CP problem\textsuperscript{6)}. The detection of neutrinos from SN1987A\textsuperscript{7)} has offered the possibility of significantly extending the previous constraints\textsuperscript{1-10)}. Indeed, since the lower bound on the axion mass decay constant inferred from SN1987A\textsuperscript{8-10)} approaches the upper bound permitted by cosmological considerations\textsuperscript{11)}, the axion "window" is in danger of closing. However, in calculating the effect of axion emission on the predicted flux of neutrinos from SN1987A, various and different simplifying assumptions and approximations have been made, leading to quantitatively different bounds on $f_a$\textsuperscript{1-10)}. For example, our previous bound\textsuperscript{8)} on $f_a$ is a factor 3-8 (depending on $\cos \beta$) more stringent than that derived by Turner\textsuperscript{10)}. In view of the potential importance of the supernova bound to the axion mass, we and others\textsuperscript{12)} have re-examined our previous calculations with an eye to checking (some of) the approximations and to correcting any omissions and/or errors.

It is the purpose of this Letter to re-evaluate our earlier bound on $f_a$ from SN1987A\textsuperscript{1)} and to compare our new constraint with those derived by others\textsuperscript{10,12)}. In computing axion emission from nucleon-nucleon bremsstrahlung in the hot, dense core of a supernova, it is important to know if the nucleons are degenerate or non-degenerate. Models for SN1987A\textsuperscript{12)} suggest semi-degenerate conditions are likely to have obtained. In our earlier calculation\textsuperscript{8)}, we have used a degenerate nucleon approximation while, in contrast, Turner\textsuperscript{10)} employed a non-degenerate approximation. Recently, Brinkmann and Turner\textsuperscript{12)} have evaluated numerically axion emission from nucleon-nucleon-axion
bremsstrahlung for arbitrary nucleon degeneracy. They conclude that even under the semi-degenerate conditions likely to be relevant, the non-degenerate rate is a very good approximation (to better than a factor of 2) while the degenerate rate overestimates the axion emission by a significant factor. As a result, our earlier bound $f_a \geq 8 \times 10^{11}$ GeV (corresponding to a bound $m_a \leq 9 \times 10^{-5}$ eV)$^8$ is likely to have been too high. The new results presented below have been calculated using the numerical rates of Brinkmann and Turner$^{12}$ for a non-degenerate core. In what follows, we will present our limits on the axion parameters assuming a non-degenerate core and discuss other uncertainties in the calculation and estimate the magnitude and direction of their effects.

As before$^8$, we work with conventions such that the axion mass and coupling constant are defined by

$$m_a = 7.2 \times 10^{-5} \text{eV} \left(10^{12} \frac{\text{GeV}}{f_a}\right) (N/6) \quad (1)$$

where $N$ is the total number of quark flavors taken to be 6

$$g_{an} = \frac{C_{AN} m_a}{f_a} \quad (2)$$

where $m_N$ is the nucleon mass.

A comment on the axion-nucleon couplings given by Eq. (2) is in order here. We showed in ref. 8 that

$$C_{ap} = 2[-2.76\Delta u - 1.13\Delta d + 0.89\Delta s - \cos2\theta(\Delta u - \Delta d - \Delta s)]$$

(3)
$$C_{an} = 2[-2.76\Delta d - 1.13\Delta u + 0.89\Delta s - \cos 2\beta(\Delta d - \Delta u - \Delta s)]$$

where the $\Delta q_i$ are defined by $\langle p | q_i \gamma_\mu s q_i | p \rangle = s_\mu \Delta q_i$ where $s_\mu$ is the proton spin. Previously, we presented two estimates of the $\Delta q_i$ and hence of the coefficients $C_{an}$, one based on the Naive Quark Model (NQM):

$$\Delta u = 0.97, \Delta d = -0.28, \Delta s = 0$$

(4)

$$C_{ap} = -4.7 - 2.5 \cos 2\beta, C_{an} = -0.61 + 2.5 \cos 2\beta$$

and one based on a recent EMC measurement of deep inelastic polarized $\mu$-p scattering:

$$\Delta u = 0.71, \Delta d = -0.54, \Delta s = -0.26$$

(5)

$$C_{ap} = -3.1 - 3.0 \cos 2\beta, C_{an} = 0.93 + 2.0 \cos 2\beta$$

As in ref. 8, we will quote here both bounds based on the NQM (4) and on the EMC (5). However we would like to mention here two developments which may be taken as favouring the EMC values. (1) Analyses of elastic $^{15,16}$ of elastic $^{(\nu p) \rightarrow (\nu n)}$ scattering indicate independently that $\Delta s$ is non-zero: $\Delta s = -0.15 \pm 0.09$. (2) It has been pointed out $^{17,16}$ that the EMC result is explained naturally by the Skyrme model of the nucleon which should become exact in the chiral limit $m_u, m_d, m_s \rightarrow 0$ and the number of colours $N_c \rightarrow \infty$. These two observations make the EMC values (5) of the $\Delta q_i$ less outrageous than they may have appeared at first sight.
Our limits will be derived assuming that axions stream freely out of the core, for the following reason. Previously, we found that the mean free path for the three body process $a + N + N \rightarrow N + N$ is given by

$$\lambda^{-1} = \frac{i\pi^2}{(2.7)T^4}$$  \hspace{1cm} (6)$$

where $i$ is the total axion emission rate. For the degenerate case we find that $\lambda = 4.6 \times 10^{-15}(f_a/1\text{ GeV})^2$ cm so that axions could escape a $10^6$ cm core for $f_a \geq 10^{10}$ GeV. For $f_a \leq 10^{10}$ GeV axions continue to diffuse at a rate in excess of allowed supernova luminosity unless $f_a \leq 10^{8}$ GeV. For the non-degenerate case under consideration we find

$$\lambda = 1.3 \times 10^{-13}(f_a/1\text{ GeV})^2$$  \hspace{1cm} (7)$

indicating that axions freely escape the core for $f_a \geq 3 \times 10^9$ GeV and continue to saturate the luminosity unless $f_a \leq 6 \times 10^7$ GeV. Such a low value of $f_a$ could only be tolerated by the red giant limits if $\cos\beta < 0.065$, where $\cos^2\beta = v^2/(v^2 + v_a^2)$ relates the two Higgs vacuum expectation values. (We recall that the red giant limit is based on the process $e^+ + e^- \rightarrow e^+ + a$.) However even if electron-axion interactions are suppressed there is also an additional red giant limit due to the Primakov process which gives $f_a \geq 7.5 \times 10^8$ GeV. In addition there is the limit due to neutron star cooling, $f_a \geq 6 \times 10^8 - 3 \times 10^9$ GeV. Both of these latter limits are independent of $\beta$ and close any window for non-freely-streaming axions.

The validity of the limits to be derived below also depends on the approximations made in arriving at the matrix elements and energy loss
rates. We now look at possible corrections to these rates due to \( \rho \)-exchange and finite density and the restoration of chiral symmetry. The squared matrix element for axion bremsstrahlung from \( nn \) or \( pp \) pairs is approximately\(^3\):

\[
|M|^2 = \frac{256}{3} \frac{g_{\pi NN}^4}{m_N^2} \frac{2}{g_{aN}} \left[ \frac{k^4}{(m_N^2 + k^2)^2} + \frac{l^4}{(l^2 + m_N^2)^2} + \frac{g_{\pi NN}^2 k^2}{(k^2 + m_N^2)(l^2 + m_N^2)} \right] 
\]

(8)

in the one-pion exchange approximation. Initial studies of axion bounds from the supernova 1987A ignored the possible effects of density and temperature on the physical parameters \( m_N, g_{\pi NN}, g_{aN} \), and \( m_\pi \) appearing in (8), and neglected possible other meson exchange diagrams. A first step towards considering these effects is given in ref. 19: here we discuss these effects with a view to assessing their impact on our previous bound on \( f_a \).

A review of the nuclear equation of state as relevant to supernovae and neutron stars is given in ref. 20. Non-relativistic\(^21\) and relativistic\(^22\) calculations of the effective nucleon mass \( m_N^* \) seem to agree when the density is close to the nuclear density

\[
\frac{m_N^*}{m_N} = 0.80 \text{ to } 0.85 \text{ for } \rho = \rho_o 
\]

(9)

The relativistic calculations\(^22\) suggest that

\[
\frac{m_N^*}{m_N} = 0.5 \text{ for } \rho = 3\rho_o 
\]

(10)
which we will take as a plausible estimate in the core of the neutron star. To discuss the density-dependence of $g_{\pi NN}$ we use the Goldberger-Treiman relation

$$g_{\pi NN} = \frac{m_N g_A}{f_{\pi}}$$  \hspace{1cm} (11)

and rely on estimates of the density-dependence of $g_A$ and $f_{\pi}$ given in ref 23. According to one estimate

$$\frac{g_A^*}{g_A} = [1 + (2/3)^2 (72/25) \cdot \frac{2\rho}{2.1 m^3_{\pi}} g'_o]^{-1}$$  \hspace{1cm} (12)

and

$$\frac{f_{\pi}^*}{f_{\pi}} = 1 - (2/3)^2 (72/25) \frac{2\rho}{2.1 m^3_{\pi}} (g_A^*/g_A)$$  \hspace{1cm} (13)

where $g'_o = 0.6$. Taking $\rho = \rho_o$ with $\rho_o = 0.46 \ m^3_{\pi}$, equation (12) gives $g_A^*/g_A = 0.75$ in accordance with the known quenching of $g_A$ in nuclear matter, and (13) gives $\frac{f_{\pi}^*}{f_{\pi}} = 0.50$. For the case $\rho = 3\rho_o$ of interest to us, (12) and (13) give

$$\frac{g_A^*}{g_A} = 0.50, \quad \frac{f_{\pi}^*}{f_{\pi}} = 0.16$$  \hspace{1cm} (14)

Inserting these values into the Goldberger-Treiman relation (11), we arrive at the first estimate
\[
\frac{g^{*}_{\pi NN}}{g_{\pi NN}} = \left(\frac{m^*_{N}}{m_{N}}\right) \left(\frac{g_A^*}{g_A}\right) \left(\frac{f_{\pi}^*}{f_{\pi}}\right) \approx 1.6
\]

(15)

The second estimate of ref. 20 gives a density-dependence of \(g_A^*\), \(f_{\pi}^*\) and hence \(g_{\pi NN}^*\) which are qualitatively similar but quantitatively different. They are

\[
g_A^*/g_A \approx 0.8, \quad \frac{f_{\pi}^*}{f_{\pi}} \leq 0.5
\]

(16)

when \(m^*/m_N \approx 0.5\) as we advocated above. Inserting (16) into the Goldberger-Treiman relation (11) we find

\[
g_{\pi NN}^*/g_{\pi NN} \geq 0.8
\]

(17)

The same estimates can be used for the density-dependence of \(g_{\alpha NN}^*\). Analogously to (15,17) we find

\[
\frac{g_{\alpha NN}^*}{g_{\alpha NN}} = \left(\frac{m^*_{N}}{m_{N}}\right) \left(\frac{g_A^*}{g_A}\right) = \left(\frac{0.25}{0.4}\right)
\]

(18)

using the two methods of ref. 23. Combining the estimates (15) or (17) with (18), we find the following possible density-dependence of the overall prefactor in \(|M|^2\) (8):

\[
\frac{\frac{4}{m_{N}} g_{\pi NN}^2}{\frac{m_{N}}{m_{N}}^2} \left(\frac{g_{\pi NN}}{g_{\alpha NN}}\right)^* \left(\frac{g_{\pi NN}}{g_{\alpha NN}}\right) = (0.26 \text{ to } 1.5)
\]

(19)
It therefore seems that our previous neglect of the possible density-dependences of these prefactors did not lead us far astray\*. In view of the uncertainties in (19), we will in fact retain our previous low-density values for the subsequent numerical analysis.

The density-dependence of $m_\pi$ is more uncertain, but fortunately less significant. We think it likely that pion condensation\textsuperscript{24) } takes place at some sufficiently high density $\rho_c$ where $m_\pi(\rho_c) = 0$. We expect on general physical grounds that $m_\pi$ decreases monotonically as $\rho$ increases from zero to $\rho_c$. The simple Ansatz for pion condensation discussed in refs. 17, 22 would yield a linear dependence of $m_\pi^2$ on $\rho$:

$$\left(\frac{m_\pi^*}{m_\pi}\right)^2 = 1 - \rho/\rho_c$$

(20)

but other models give more complicated density-dependences of $m_\pi$. As discussed in ref. 20, 25, $\rho_c$ may be as low as $2.2\rho_o$, in which case there would be pion condensation at our nominal density $\rho = 3\rho_o$\textsuperscript{**}. The effect of setting $m_\pi$ to zero when integrating $|M|^2$ over phase space in the non-degenerate limit is to increase the energy loss rate by a factor $\sim 1.5^{19}$). We will neglect this possible decrease in $m_\pi$ and increase in energy loss rate in our subsequent numerical analysis.

The effect\textsuperscript{21) } of including $\rho$ exchange in the NN axion bremsstrahlung calculation is to replace

\* This differs from the conclusions reached in ref. 19.

\** If pion condensation were indeed to occur inside the neutron star, one might expect that other axion emission processes such as $\pi\pi \rightarrow \pi\alpha$ could compete with NN bremsstrahlung. We have checked that in fact NN bremsstrahlung still dominates over $\pi\pi \rightarrow \pi\alpha$. 

\[ (m_\pi^2 + k^2)^{-2} \rightarrow \left( (m_\pi^2 + k^2)^{-1} - \frac{C_\rho}{(m_\rho^2 + k^2)^{\frac{1}{2}}} \right)^2 \] 

(21)

in \( |M|^2 \) (8), where \( C_\rho \) is estimated to be between 1.25 and 1.67. Taking the latter estimate to be more conservative, we find that the non-degenerate energy loss rate for the dominant \( n + p \rightarrow n + p + a \) process is reduced by a factor \( = 3 \). Polarization effects on the pion-exchange mechanism can further reduce the energy loss rate by a factor \( = 2^{19} \), resulting in overall reduction by a factor of \( = 6 \).

Previously, we had assumed a degenerate core and used the energy loss rate calculated by Iwamoto \(^3\) due to neutron-neutron scattering\(^*\).

\[ \dot{\varepsilon} = \frac{31}{3780\pi} \frac{g^2_{\pi N}}{4} (kT)^6 \rho_{an}^2 m_N^{-2} P_F(n)F(x_n) \]

\[ = 8.2 \times 10^{43} \left( \frac{1 \text{ GeV/f}_n}{\text{erg/cm}^2\text{s}} \right)^2 \rho_{an}^2 \left( \frac{m}{2\pi F(x_n)} \right)^{1/3} \frac{6}{\text{MeV}} F(x_n) \text{ erg/cm}^2\text{s} \] 

(22)

where \( P_F(n) \) is the neutron Fermi momentum, \( X_n \) the abundance of neutrons with mass density \( \rho_{12} = \rho/10^{12} \text{ g/cm}^3 \), \( x_n = m_{\pi}/2P_F(n) \) and \( F(x) = 1 - \frac{3}{2}x \tan^{-1}(1/x) + x^2/2(x^2 + 1) \). By isospin invariance the rate due to proton-proton scattering is the same with the substitution \( C_{an} \rightarrow C_{ap} \), \( X_n \rightarrow X_p \) and \( x_n \rightarrow x_p \). To get the rate due to neutron-proton scattering, we must multiply this rate by 10 and make the substitutions \( C_{an} \rightarrow (C_{an} + C_{ap})/2 \), \( X_n^{1/3} \rightarrow (X_n^{1/3} + X_p^{1/3})/2 \) and \( x_n \rightarrow x_{np} = m_{\pi}/(P_F(n) + P_F(p)) \). Previously we had incorrectly multiplied

\(^*\)With the substitution \( g_{\pi N}^2 = 4\pi f^2 (4m_N^2/m_{\pi}^2) \), with \( 4\pi f^2 = 1 \), one recovers the previously used expressions.
the n-p rate by an additional factor of 2. Since the n-p rate is the most
important we would naively expect that the limit on $f_a$ would be reduced by
about $(1/2)$.

We have now performed a similar analyses using the non-degenerate rates
calculated by Brinkmann and Turner $^{12}$)

$$
\dot{\epsilon} = 9.7 \times 10^{46} \left(\text{1GeV}/f_a\right)^2 C_\text{an}(\rho_{14} X_n)^2 T_{\text{MeV}}^{3.5} \text{erg/cm}^3 \text{s} \quad (23)
$$

Similarly, the rate due to p-p scattering is found by $C_{\text{an}} \to C_{\text{ap}}$, and due to
n-p scattering by multiplying by 10 and $C_{\text{an}} \to (C_{\text{an}} + C_{\text{ap}})/2$, $X_n^2 \to X_n X_p$.

We find that the total axion emission rates are lowered only by a
factor of 10 when the non degenerate rates are used instead of degenerate
rates in a numerical calculation. In addition the total rate is lowered by
the factor of $-2$ in the n-p rate mentioned above so that our bound on $f_a$ is
lowered by an overall factor of 4 relative to our previous result $^{**}$. Doing
a perturbation calculation similar to that described in our previous paper $^8$
we now find (using the non-degenerate axion emission rate)

$$
f_a \geq 0.17 \times 10^{12} \text{ GeV} \left(1 - 0.051 \cos 2\beta + 0.31 \cos^2 2\beta\right)^{1/2} \quad \text{NQM} \quad (24)
$$

$$
f_a \geq 0.082 \times 10^{12} \text{ GeV}(1 + 1.67 \cos 2\beta + 1.09 \cos^2 2\beta)^{1/2} \quad \text{EMC} \quad (25)
$$

Figure 1 displays the above curves (as equalities) with $f_a$ normalized to
unity for $\beta = 0$ taking the NQM values. (This puts $f_a(\beta = 0)|_{\text{EMC}} = .84$.)

$^{**}$ There may be a significant correction to this factor due to the $\rho$-exchange
and polarization effects mentioned earlier.
Four evolutionary calculations were also done for the $\beta = 0$ NQM value of $G_\text{an}$ and $C_\text{ap}$ using $f_a = 0.8 \times 10^{12}$ GeV, $0.3 \times 10^{12}$, $0.15 \times 10^{12}$ GeV, and $0.08 \times 10^{12}$ GeV. These results are shown in Figures 2 and 3 (along with the $f_a = \infty$ result). It can be seen that the previous limiting case $f_a = 0.8 \times 10^{12}$ GeV is now completely acceptable with very little axion emission. It is also difficult to exclude $f_a = 0.3 \times 10^{12}$ whereas $f_a = 0.15 \times 10^{12}$ GeV is probably not acceptable. We feel $f_a \geq 0.2 \times 10^{12}$ GeV can be inferred conservatively from the perturbation calculation and the fact that the axion emission had not leveled off in the evolutionary calculation when the calculation was terminated for $f_a = 0.15 \times 10^{12}$ GeV.

The major difference with our earlier results is traceable to the difference between the degenerate axion emission rate and the non-degenerate rate. As mentioned above, for the same conditions of temperature and density, we find the non-degenerate rate lower by a factor of ~20; this translates into a factor of 4 difference to our bounds on $f_a$ and $m_a$. Although our earlier work overestimated the axion emission rate, others have underestimated this rate. For example, neutron-proton axion bremsstrahlung dominates over proton-proton and neutron-neutron axion bremsstrahlung. Incorrect accounting by Turner led to a rate which was too small by a factor of ~2. Note however, that Brinkmann and Turner have corrected this in their recent paper. Another source of discrepancy is in the criterion adopted for an "acceptable" model with axion emission. In our previous work, we incorporated the effect of axion emission on the exploding supernova core; Raffelt and Seckel only approximated this effect and Turner simply required that the axion luminosity be $\leq 10^{53}$ ergs$^{-1}$ which, for the ~10 sec duration of the observed neutrino emission,
would correspond to $\leq 10^{54}$ ergs emitted in axions. In contrast, we required that the neutrino emission extend over $\geq 7$ seconds; this corresponds to a total axion emission of $\leq 3 \times 10^{53}$ ergs.

While this work was in final preparation, we received a preprint from Burrows et al. who also have integrated axion emission into a full stellar collapse calculation as we did previously and have done in the present work. The Burrows collapse model has a lower central temperature than achieved in the Mayle-Wilson models. Nevertheless, the conclusion of Burrows et al. is quite similar to ours with minor differences depending on what criteria one selects to obtain a constraint.

In summary: we have reanalyzed the limits on the axion decay constant $f_a$ using improved non-degenerate axion emission rates. We find that our results are lower by a factor of 4 so that a conservative bound is $f_a \geq 0.2 \times 10^{12}$ GeV ($m_a \leq 3.6 \times 10^{-4}$ eV) for NQM couplings and $f_a \geq 0.05 \times 10^{12}$ GeV ($m_a \leq 1.4 \times 10^{-3}$ eV) for EMC couplings. In addition, the limits on $f_a$ may be lowered in both cases by about $\sqrt{6}$ due to $\rho$-exchange and polarization effects. Furthermore, we find no window in $f_a$ for non-freely-streaming axions. This leaves a cosmologically interesting window for the axion with $10^{11}$ GeV $\leq f_a \leq 10^{12}$ GeV.
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References


Figure Captions

Figure 1. The relative values of $\hat{f}_a$, normalized so that $\hat{f}_a = 1$ for $\beta = 0$ for the NQM model ($\hat{f}_a = f_a / 0.19 \times 10^{12}$ GeV), as obtained from equations (24), (25) requiring equal energy loss to axions and neutrinos, plotted versus angle $\beta$ for NQM and EMC models.

Figure 2. Time evolution of the energy emitted in neutrinos ($\nu$) and axions (a) for $f_a = 0.8 \times 10^{12}$ GeV, $f_a = 0.3 \times 10^{12}$ GeV, $f_a = 0.15 \times 10^{12}$ GeV, $f_a = 0.08 \times 10^{12}$ GeV, and $f_a = \infty$, assuming $\beta = 0$ in the NQM model.

Figure 3. Antineutrino luminosities versus time for the five models of Figure 2. For SN1987a, neutrinos were emitted for $\sim 12$ seconds.
Figure 1.
Figure 3.