Wightman-Function Approach to the Relativistic Complex-Ghost Field Theory

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The relativistic complex-ghost field theory is covariantly formulated in terms of Wightman functions. The Fourier transform of the 2-point Wightman function of a complex-ghost pair is explicitly calculated, and its spontaneous breakdown of Lorentz invariance is compared with that of the corresponding Feynman integral.

1 Introduction

Abe and the present author have proposed and established the general method for solving quantum field theory, formulated as the canonical operator formalism, in the Heisenberg picture. The set of the full-dimensional (anti)commutators for field operators is obtained as the solution to the q-number Cauchy problem constructed by field equations and equal-time (anti)commutation relations, and then the representation of this operator solution is constructed by giving the set of Wightman functions. In contrast to the axiomatic quantum field theory, however, the Wightman functions thus obtained do not, in general, satisfy the norm-positivity condition of the state-vector space. That is, the natural framework of the Lagrangian quantum field theory is the indefinite-metric theory.

One of the striking properties of the indefinite-metric theory is that the eigenvalues of a hermitian operator are not necessarily real, that is, we generally encounter complex-energy eigenvalues for the Hamiltonian. This fact contradicts the usual spectral condition postulated in the axiomatic quantum field theory. The "energy-positivity condition" has also been made use of the above-mentioned method for solving quantum field theory in determining Wightman functions. That is, we have required every Wightman function \( W(x_1, ..., x_n) \) to be a boundary value of an analytic function from the lower half-planes of the time differences \( x_1^0 - x_2^0, ..., x_{n-1}^0 - x_n^0 \); this property is what follows from the positivity of energy. Hence, if there are complex-energy states, we must examine whether or not the "energy-positivity condition" is still applicable to our method.

The complex-energy states should not appear in the physical world, that is, they should be unphysical states. Hence they are called the "complex ghosts". A single complex ghost has zero norm, and it cannot appear in the final state if the initial state is a real-energy

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state because of the energy conservation. However, if a complex ghost exists, there is always a complex-conjugate ghost, and the state consisting of a pair of a complex ghost and its conjugate has real energy and can have negative norm. Hence, the energy conservation cannot forbid the appearance of a complex-ghost pair and it is expected that the unitarity of the physical S-matrix is violated. This fact was indeed confirmed in nonrelativistic models.

In 1969-1970, however, Lee and Wick\(^3\),\(^4\) (and Lee\(^5\) alone) pointed out that the violation of the unitarity of the physical S-matrix does not occur in the relativistic complex-ghost field theory. This is because as the expression for the relativistic energy is a square root of a sum of a mass squared and a spatial momentum squared, the complex conjugation of a complex mass does not generically imply the complex conjugation of the corresponding energy; therefore, the unitarity cut due to the complex-ghost pair is absent. This wonderful result, however, is achieved at the cost of the violation of Lorentz invariance, as was proved by the present author\(^6\) and as was explicitly confirmed by Gleeson, Moore, Rechenberg, and Sudarshan\(^7\) (GMRS) subsequently. Lee and Wick\(^8\) attributed the violation of Lorentz invariance to the fact that spatial momentum is real while energy is complex, and wished to admit the use of complex spatial momenta by adopting the S-matrix-theoretical rule proposed by Cutkosky, Landshoff, Olive, and Polkinghorn\(^9\) previously. The violation of Lorentz invariance, however, is not a direct consequence of the reality of spatial momenta, because the complex energy is encountered only on the mass shell and quantum field theory is formulated on the basis of the off-the-mass-shell quantities. Indeed, the present author\(^10\) showed that the relativistic complex-ghost field theory can be formulated manifestly covariantly without using complex masses in the action and that it is actually Lorentz invariant at the operator level.

The purpose of the present paper is to reformulate the relativistic complex-ghost field theory in terms of Wightman functions. Its manifestly covariant formulation done previously was based on the momentum-space consideration:\(^10\) After introducing creation and annihilation operators for real-mass fields explicitly, the complex-ghost fields were constructed by means of the Bogoliubov transformation. In the present paper, we directly solve the Cauchy problem for the 4-dimensional commutators and then construct the Wightman functions without going to the momentum space. We show that the “energy-positivity condition” works well in spite of the presence of complex ghosts. The 2-point Wightman functions of the fundamental fields are shown to be Lorentz invariant. The violation of Lorentz invariance, however, takes place for the 2-point Wightman functions of composite fields. We explicitly calculate the Fourier transform of the 2-point Wightman function of a complex-ghost pair, and compare it with the corresponding Feynman integral calculated by GMRS. Although no
spectral representation holds in the present case, the expression for the Wightman function is found to be something like the spectral function for the Feynman amplitude.

The present paper is organized as follows. In §2, we apply the Wightman-function approach to the complex-ghost theory and find the Wightman functions explicitly. In §3, we review the spontaneous breakdown of Lorentz invariance in the Feynman integral involving a complex-ghost-pair intermediate state. In §4, we explicitly calculate the Fourier transforms of the 2-point Wightman functions of a complex-ghost pair and of two complex ghosts. The final section is devoted to discussion.

2 Formulation of the theory

We start with the Lagrangian density

$$\mathcal{L} = \frac{1}{2} \sum_{j=1}^{2} (-1)^{j-1} (\partial^{\mu} \phi_j \cdot \partial_{\mu} \phi_j - m^2 \phi_j^2) - \gamma m \phi_1 \phi_2, \quad (2.1)$$

where $\phi_1$ and $\phi_2$ are hermitian scalar fields and $m$ and $\gamma$ are positive constants.

Field equations are

$$(\Box + m^2) \phi_1 + \gamma m \phi_2 = 0,$$

$$(\Box + m^2) \phi_2 - \gamma m \phi_1 = 0, \quad (2.2)$$

and non-vanishing equal-time commutation relations are

$$[\partial_0 \phi_j(x), \phi_j(y)]_{x^0 = y^0} = -(-1)^{j-1} i \delta(x - y). \quad (2.3)$$

We set

$$[\phi_j(x), \phi_k(y)] \equiv i \Delta_{jk}(x - y) \quad (2.4)$$

and

$$\Delta \equiv \text{matrix}(\Delta_{jk}). \quad (2.5)$$

We then have the following Cauchy problem:

$$(\Box + m^2 + i \gamma m \sigma_3) \Delta(x - y) = 0 \quad (2.6)$$

together with

$$\Delta(x - y)_{x^0 = y^0} = 0,$$

$$\partial^a \Delta(x - y)_{x^0 = y^0} = -\sigma_3 \delta(x - y). \quad (2.7)$$

where $\sigma_i$ denotes the Pauli matrix.
We diagonalize (2.6) by using a unitary matrix \( U \equiv (i + \sigma_1)/\sqrt{2} \). Because \( U \sigma_3 U^{-1} = \sigma_3 \) and \( U \sigma_2 U^{-1} = -\sigma_2 \), we obtain
\[
(\Box^r + m^2 + i\gamma m \sigma_3) \hat{\Delta}(x - y) = 0
\] (2.8)
together with
\[
\hat{\Delta}(x - y)|_{x^0 = y^0} = 0,
\]
\[
\partial_{x^0} \hat{\Delta}(x - y)|_{x^0 = y^0} = \sigma_2 \delta(x - y),
\]
where \( \hat{\Delta} \equiv U \Delta U^{-1} \).

We define the complex-mass \( \Delta \)-function by the Cauchy problem
\[
(\Box^r + M^2) \Delta(x - y; M^2) = 0,
\] (2.10)
\[
\Delta(x - y; M^2)|_{x^0 = y^0} = 0,
\]
\[
\partial_{x^0} \Delta(x - y; M^2)|_{x^0 = y^0} = -\delta(x - y),
\] (2.11)
where \( M^2 \) is a complex number such that \( \Re M^2 > 0 \). The explicit expression for the complex-mass \( \Delta \)-function is given by
\[
\Delta(\xi; M^2) = \frac{1}{(2\pi)^3} \int d\mathbf{p} \frac{\sin(\mathbf{p} \cdot \xi - E_p \xi^0)}{E_p} \\
= -\frac{1}{2\pi \xi^0} \left[ \delta(\xi^2) - \frac{M^2}{2} \theta(\xi^2) \frac{J_1(M \sqrt{\xi^2})}{M \sqrt{\xi^2}} \right],
\] (2.12)
where \( E_p = \sqrt{M^2 + \mathbf{p}^2} \) and \( J_1 \) denotes a Bessel function. The complex-mass \( \Delta \)-function is, of course, Lorentz invariant, though it behaves exponentially as \( \xi^2 \to \infty \).

The solution to the Cauchy problem (2.8) with (2.9) is given by
\[
\hat{\Delta}(x - y) = i\sigma_+ \Delta(x - y; M^2) - i\sigma_- \Delta(x - y; M^*),
\] (2.13)
where \( \sigma_\pm \equiv (\sigma_1 \pm i\sigma_2)/2 \) and \( M^2 \equiv m^2 + i\gamma m \). Hence we have
\[
\Delta(x - y) = U^{-1} \hat{\Delta}(x - y) U = \sigma_3 \Re \Delta(x - y; M^2) - \sigma_1 \Im \Delta(x - y; M^2).
\] (2.14)

We proceed to considering the representation of the above operator solution. We introduce the vacuum \(|0\rangle \) and set \( \langle 0|0 \rangle = 1 \). The 1-point Wightman functions \( \langle 0|\phi_j(x)|0 \rangle \) are set equal to zero so as to be consistent with translational invariance and field equations. Then the 2-point truncated Wightman functions are the same as the untruncated ones.
The 2-point Wightman functions \( \langle 0 | \phi_j(x) \phi_k(y) | 0 \rangle \) must be constructed so as to be consistent with the commutator functions given by (2.14) and with the energy-positivity condition. We define the “positive-energy” complex-mass \( \Delta \)-function by

\[
\Delta^+(\xi; M^2) \equiv \frac{1}{(2\pi)^3} \int dp \frac{\exp ip \xi - Ep \xi^0}{2Ep}.
\]

Note that the momentum integral is convergent because \( \Im E_p \) tends to zero as \( |p| \to \infty \).

Evidently, \( \Delta^+(\xi; M^2) \) is Lorentz invariant and has the properties

\[
\Delta^+(\xi; M^2) + \Delta^+(-\xi; M^2) = i\Delta(\xi; M^2),
\]

\[
\Delta^+(-\xi; M^2) = [\Delta^+(\xi; M^2)]^*.
\]

Furthermore, because \( \Re E_p > 0 \), \( \Delta^+(\xi; M^2) \) is a boundary value of an analytic function from the lower half-plane of \( \xi_0 \) in conformity with the requirement of the energy-positivity condition.

From (2.14), we find that the 2-point (truncated) Wightman functions are given by

\[
\langle 0 | \phi_j(x) \phi_j(y) | 0 \rangle = (-1)^j \frac{1}{2}[\Delta^+(x - y; M^2) + \Delta^+(x - y; M^{*2})],
\]

\[
\langle 0 | \phi_1(x) \phi_2(y) | 0 \rangle = \langle 0 | \phi_2(x) \phi_1(y) | 0 \rangle = -\frac{1}{2i} [\Delta^+(x - y; M^2) - \Delta^+(x - y; M^{*2})].
\]

Of course, all higher-point truncated Wightman functions vanish. Thus, all Wightman functions of the fundamental fields are Lorentz invariant.

3 Violation of Lorentz invariance

As we have shown in §2, the theory is strictly Lorentz invariant as far as the Wightman functions of the fundamental fields are concerned. Quite interestingly, however, Lorentz invariance is no longer valid for composite fields. In this section, we briefly review the results obtained previously for the 1-loop Feynman integral involving a complex-ghost-pair intermediate state.

In the Fourier representation of the Feynman propagator \( \Delta_F(\xi; M^2) \) (but not \( \Delta_F(\xi; M^{*2}) \)), we need to introduce a complex contour \( C \), which runs above the pole located at \( p_0 = \sqrt{M^2 + p^2} \) in spite of the fact that it lies on the upper half-plane. Therefore, in order to have the momentum-space Feynman integral, we encounter the position-space integrals of the following type:

\[
f(k_0) \equiv \frac{1}{2\pi} \int_{-\infty}^{\infty} dx^0 e^{\pm i(k_0 - \lambda)x^0},
\]

(3.1)
where $\lambda$ is a complex number. Naively, this integral is divergent and mathematically meaningless. By adopting a Gaussian adiabatic hypothesis in defining the Dyson S-matrix, however, we can show that (3.1) should be defined in the sense of the “complex $\delta$-function”,\(^{10}\) $f(k_0) = \delta_c(k_0 - \lambda)$.\(^2\) This notion is defined by extending the definition of the Schwartz distribution in the following way: Let $\varphi(k_0)$ be a test function, which is an arbitrary function holomorphic in an appropriate strip domain around the real axis; then

$$
\int_{-\infty}^{\infty} dk_0 \varphi(k_0) \delta_c(k_0 - \lambda) \equiv \frac{-1}{2\pi i} \oint dk_0 \frac{\varphi(k_0)}{k_0 - \lambda}, \tag{3.2}
$$

where the contour goes around $k_0 = \lambda$ anticlockwise. Of course, if $\lambda$ is real, the complex $\delta$-function reduces to the ordinary $\delta$-function.

Now, the Feynman integral involving a complex-ghost-pair intermediate state is given by

$$
F_{MM^*}(p) \equiv \frac{1}{(2\pi)^4} \int dq \int_{C} dq_0 \frac{1}{(q^2 - M^2)((p - q)^2 - M^*{}^2)}, \tag{3.3}
$$

where we understand that its ultraviolet divergence is appropriately subtracted. The integrand of (3.3) has four poles in the $q_0$ plane. The contour $C$ runs from $-\infty$ to $+\infty$, passing through below the two poles $-E_q$ and $p_0 - E_{p-q}^*$ and above the other two poles $E_q$ and $p_0 + E_{p-q}^*$. Carrying out the contour integration of (3.3), we obtain

$$
F_{MM^*}(p) = \frac{1}{2(2\pi)^3} \int dq \left( \frac{1}{E_q} + \frac{1}{E_{p-q}^*} \right) \frac{1}{p_0^2 - (E_q + E_{p-q}^*)^2}. \tag{3.4}
$$

When $q$ runs over the whole three-dimensional space, the locus of

$$
p_0 = E_q + E_{p-q}^*, \tag{3.5}
$$

which corresponds to the unitarity cut in the real-mass case, sweeps a 2-dimensional fish-shaped domain $D$ on the $p_0$ plane. Its boundary curve $\Gamma$ intersects the real axis at only one point $p_0 = 2 \Re E_{p/2}$. Evidently, the quantity $s_{\min}$ is not Lorentz invariant, where

$$
s_{\min} \equiv 4(\Re E_{p/2})^2 - p^2 \quad (s_{\min} < (M + M^*)^2 \text{ for } p \neq 0). \tag{3.6}
$$

GMRS explicitly carried out the momentum integration of (3.4). Their result is \(^3\)

$$
F_{MM^*}(p) = \frac{1}{16\pi^2} \Re \int_{\Gamma_s} \frac{ds'}{s - s'} \left[ \frac{\sqrt{p^2(M^2 - M^*{}^2)}}{s'\sqrt{s' + p^2}} + \frac{\sqrt{(s' - (M + M^*)^2)(s' - (M - M^*)^2)}}{s'} \right], \tag{3.7}
$$

where $s \equiv p_0^2 - p^2$ and the contour $\Gamma_s$ is the lower half part of the image of $\Gamma$ by the mapping $s' = p_0^2 - p^2$; it runs from $s_{\min}$ to $\infty$.

\(^2\)This notion was introduced first by the present author\(^{11}\) in 1958.

\(^3\)In (3.7), an overall factor 1/2 has been corrected. This error was committed in (A3) of their paper.\(^7\)
4 Wightman function of a complex-ghost pair

In this section, we explicitly calculate the Fourier transform of the 2-point Wightman function of a complex-ghost pair. Its formal expression is given by

$$W_{MM^*}(p) \equiv \int d^4 \xi \Delta^{(+)}(\xi; M^2) \Delta^{(+)}(\xi; M^{*2}) e^{ip\xi}. \quad (4.1)$$

Of course, naively (4.1) is meaningless because $\Delta^{(+)}(\xi; M^2)$ is exponentially increasing in its absolute value. Substituting (2.15) into (4.1) and carrying out one of spatial momentum integrations, we obtain

$$W_{MM^*}(p) = \frac{1}{(2\pi)^3} \int dq \frac{1}{4E_q E_{p-q}} \int_{-\infty}^{\infty} d\xi^0 \exp[i(p_0 - E_q - E_{p-q})\xi^0]. \quad (4.2)$$

Now, the energy factor inside the exponential of (4.2) is complex for $p \neq 0$, so that the integration over $\xi^0$ is meaningless naively as stated above. We define the integral over $\xi^0$ by means of the complex $\delta$-function as in §3. That is, according to (3.2), we consider

$$\int_{-\infty}^{\infty} dp_0 \varphi(p_0)W_{MM^*}(p) = \oint dp_0 \varphi(p_0)I(p), \quad (4.3)$$

where

$$I(p) \equiv \frac{i}{4(2\pi)^3} \int dq \frac{1}{E_q E_{p-q}(p_0 - E_q - E_{p-q})}. \quad (4.4)$$

Without loss of generality, we can take the coordinate system defined by $p = (0, 0, p_3 > 0)$. Employing the cylindrical coordinates for $q$ and setting $\rho^2 = q_1^2 + q_2^2$, we obtain

$$I(p) = \frac{i}{32\pi^2} \int dq_3 \int_{-\infty}^{\infty} d\rho^2 \frac{d\rho^2}{E(\rho^2, q_3)E^*(\rho^2, -q_3)[p_0 - E(\rho^2, q_3) - E^*(\rho^2, -q_3)]}, \quad (4.5)$$

where

$$E(\rho^2, q_3) \equiv \sqrt{M^2 + \rho^2 + \left(\frac{p_3}{2} + q_3\right)^2}. \quad (4.6)$$

By transforming the integration variable $\rho^2$ into a complex variable

$$q_0 = E(\rho^2, q_3) + E^*(\rho^2, -q_3), \quad (4.7)$$

the integral is remarkably simplified into

$$I(p) = \frac{i}{16\pi^2} \int dq_3 \int_{\alpha(q_3)}^{\infty} d\rho_0 \frac{d\rho_0}{q_0(p_0 - q_0)}, \quad (4.8)$$

where

$$\alpha(q_3) \equiv E(0, q_3) + E^*(0, -q_3). \quad (4.9)$$

The integration over $q_0$ is easily carried out; we obtain

$$I(p) = \frac{i}{16\pi^2 p_0} \int_{-\infty}^{\infty} dq_3 \log \frac{\alpha(q_3) - p_0}{\alpha(q_3)}. \quad (4.10)$$
Transforming the integration variable \( q \) into \( \alpha = \alpha(q_3) \) for \( q_3 \geq 0 \) and \( \alpha = \alpha(-q_3) = [\alpha(q_3)]^* \) for \( q_3 \leq 0 \), we have

\[
I(p) = \frac{-i}{16\pi^2 p_0} \left( \int_{\Gamma^+} + \int_{\Gamma'^+}^* \right) d\alpha \frac{dq_3(\alpha)}{d\alpha} \left[ \log(\alpha - p_0) - \log \alpha \right] - \frac{1}{\alpha - p_0} + 1 \alpha + c. \tag{4.11}
\]

Here, \( q_3(\alpha) \) is the inverse function of \( \alpha = \alpha(q_3) \); explicitly,

\[
q_3(\alpha) = p_3(M^2 - M^*2) + \alpha \sqrt{(\alpha^2 - p_3^2 - (M + M^*)^2)(\alpha^2 - p_3^2 - (M - M^*)^2)} / 2(\alpha^2 - p_3^2), \tag{4.12}
\]

where the sign of the square root has been chosen in such a way that \( q_3 = \alpha/2 \) when \( M = 0 \) and \( q_3 > p_3/2 \) as seen from (4.9); \( c = i/16\pi^2 \); the contour \( \Gamma^+ \) is the image of the positive real axis by the mapping \( \alpha = \alpha(q_3) \) for \( q_3 \geq 0 \), that is, it is the upper boundary curve of \( D \) introduced in §3. Both \( \Gamma^+ \) and \( \Gamma'^+ \) can be deformed into a real interval; that is,

\[
I(p) = -\frac{i}{8\pi^2 p_0} \int_{2\Re p/2}^{\infty} d\alpha q_3(\alpha) \left( \frac{1}{\alpha - p_0} - \frac{1}{\alpha} \right) + c. \tag{4.13}
\]

Now, we substitute (4.13) into (4.3):

\[
\int_{-\infty}^{\infty} dp_0 \varphi(p_0) W_{MM^*}(p) = \int_{-\infty}^{\infty} dp_0 \varphi(p_0) \left[ -\frac{i}{8\pi^2} \int_{2\Re p/2}^{\infty} d\alpha \frac{q_3(\alpha)}{\alpha(\alpha - p_0)} + c \right] = \int_{2\Re p/2}^{\infty} d\alpha \varphi(\alpha) \frac{1}{4\pi} \frac{q_3(\alpha)}{\alpha}. \tag{4.14}
\]

Rewriting the right-hand side of (4.14) into

\[
\int_{-\infty}^{\infty} dp_0 \varphi(p_0) \frac{1}{\pi} \frac{q_3(p_0)}{p_0} \theta(p_0 - 2\Re p/2), \tag{4.15}
\]

we find that

\[
W_{MM^*}(p) = \frac{1}{4\pi} \frac{q_3(p_0)}{p_0} \theta(p_0 - 2\Re p/2). \tag{4.16}
\]

From (4.16) with (4.12), therefore, our final result is

\[
W_{MM^*}(p) = \frac{1}{8\pi} \left[ \frac{\sqrt{p^2(M^2 - M^*2)}}{s^2 + s + p^2} + \frac{\sqrt{(s - (M + M^*2))(s - (M - M^*2))}}{s} \right] \theta(s - s_{\min}) \tag{4.17}
\]

for the general value of \( p \).

Thus, \( W_{MM^*}(p) \) is not only Lorentz non-invariant but also complex-valued. In contrast to the real-mass case, in which the 2-point Wightman function is the discontinuity function of the Feynman amplitude according to the Cutkosky rule, \( W_{MM^*}(p) \) has no direct connection with \( F_{MM^*}(p) \). Nevertheless, comparing (4.17) with (3.7), we find that the relationship between both expressions are quite analogous to that in the real-mass case.
For completeness, we calculate the Wightman function of two complex ghosts,

\[ W_{MM}(p) = \int d^4\xi \left[ \Delta^{(+)}(\xi; M^2) \right]^2 e^{ip\xi} = \frac{1}{(2\pi)^3} \int dq \frac{1}{4EqE_{p-q}} \int_{-\infty}^{\infty} d\xi_0 \exp[i(p_0 - E_q - E_{p-q})\xi_0]. \]  

(4.18)

The calculation is carried out almost in the same way except for the fact that the locus of \( p_0 = E_q + E_{p-q} \) does not intersect the real axis. Therefore, the final result cannot be expressed in terms of the ordinary function or distribution. Introducing the notion of the "complex \( \theta \)-function" by

\[ \theta_c(k_0 - \lambda) \equiv \int_{-\infty}^{k_0} dk_0' \delta_c(k_0' - \lambda), \]  

(4.19)

we can write the final result in the following way:

\[ W_{MM}(p) = \frac{1}{8\pi} \sqrt{\frac{s - 4M^2}{s}} \theta_c(s - 4M^2). \]  

(4.20)

Of course, as \( \Im M \to 0 \), both (4.17) and (4.20) tend to the well-known expression for the Wightman function (which equals the discontinuity function of the Feynman amplitude along the real axis) in the equal-real-mass case.

5 Discussion

In the present paper, we have successfully applied the method for solving quantum field theory in the Heisenberg picture to the covariant formulation of the complex-ghost theory. In spite of the fact that there is complex energy spectrum, we see that the energy-positivity condition can be used without trouble.

We have discussed the spontaneous breakdown of Lorentz invariance encountered in the relativistic complex-ghost theory from the viewpoint of the Wightman function. We have explicitly calculated the Fourier transform of the 2-point Wightman function of a complex-ghost pair and that of two complex ghosts. The former indeed exhibits the violation of Lorentz invariance. In spite of the fact that the spectral representation does not hold for the Feynman amplitude, the relation between the Feynman amplitude and the corresponding Wightman function is quite analogous to that in the real-mass case.

We have seen that the introduction of the complex \( \delta \)-function is quite essential in those considerations. From (2.18), we have

\[ \langle 0|\phi_j(x)|^2\phi_j(y)|^2|0\rangle = \frac{1}{2} [\Delta^{(+)}(x - y; M^2) + \Delta^{(+)}(x - y; M^*2)]^2. \]  

(5.1)
According to (4.17) and (4.20), therefore, its Fourier transform is neither Lorentz invariant nor expressible in terms of the ordinary distributions. This result is rather surprising in view of the fact that the Lagrangian density (2.1) is a very simple one and explicitly contains no complex numbers. That means that it is very difficult to avoid the appearance of such possibilities a priori in the generic framework of the Lagrangian quantum field theory.

References

1) For a review, see N. Nakanishi, Prog. Theor. Phys. 111 (2004), 301. Further references are contained therein.