SCALING RELATIONS OF DWARF GALAXIES WITHOUT SUPERNOVA-DRIVEN WINDS

KONSTANTINOS TASSIS$^{1,2}$, ANDREY V. KRAVTSOV$^{1,2,3}$ & NICKOLAY Y. GNEadin$^{1,4}$

Draft version September 29, 2006

ABSTRACT

Nearby dwarf galaxies exhibit tight correlations between their global stellar and dynamical properties, such as circular velocity, mass-to-light ratio, stellar mass, surface brightness, and metallicity. Such correlations have often been attributed to gas or metal-rich outflows driven by supernova energy feedback to the interstellar medium. We use high-resolution cosmological simulations of high-redshift galaxies with and without energy feedback, as well as analytic chemical evolution modeling, to investigate whether the observed correlations can arise without supernova-driven outflows. We find that the simulated dwarf galaxies exhibit correlations similar to those observed as early as $z \approx 10$ and the addition of realistic levels of supernova energy feedback has no appreciable effect on these correlations. We also show that the correlations can be well reproduced by our analytic chemical evolution model that accounts for gas inflow but without outflows, and star formation rate obeying the Kennicutt-Schmidt law with a critical density threshold. We argue that correlations in simulated galaxies arise due to the increasingly inefficient conversion of gas into stars in low-mass dwarf galaxies rather than supernova-driven outflows. We also show that the decrease of the observed effective yield in low-mass objects, often used as an indicator of gas and metal outflows, can be reasonably reproduced in our simulations without outflows. We show that this trend can arise if a significant fraction of metals in small galaxies is spread to the outer regions of the gas disk outside the stellar extent via mixing. In this case the effective yield can be significantly underestimated if only metals within the stellar radius are taken into account. Measurements of gas metallicity in the outskirts of gaseous disks of dwarfs would thus provide a crucial test of this explanation.

Subject headings: cosmology: theory – galaxies: evolution – galaxies: formation – galaxies: abundances – galaxies: fundamental parameters – galaxies: dwarf

1. INTRODUCTION

Observational studies of galaxies have revealed the existence of tight correlations between their global stellar and dynamical properties, such as circular velocity, mass-to-light ratio, stellar mass, surface brightness, and metallicity. For example, scalings of the mean metallicity of a galaxy with other global properties such as its luminosity, stellar mass, or total mass, have been established for galaxies of a large range of masses and morphologies (e.g., Lequeux et al. 1979; Garnett & Shields 1987; Zaritsky et al. 1994; Garnett 2002; Prada & Burkert 2002; Dekel & Woosley 2003; Tremonti et al. 2004; Simon et al. 2006). Theoretical modelling of these correlations can help us to identify key physical processes shaping global properties of galaxies.

Supernova energy feedback to the interstellar medium (ISM) with putative associated gas outflows from galaxies has long been a favored mechanism to explain these correlations and other properties of low-mass galaxies (e.g., Larson 1974; Dekel & Silk 1986; Arimoto & Yoshii 1987). The strong effect of the SN feedback on properties of low-mass galaxies is also a standard assumption of the semi-analytic models of galaxy formation (e.g., Lacey et al. 1993; Kauffmann et al. 1993; Cole et al. 1994; Somerville & Primack 1999; Benson et al. 2003; Croton et al. 2006; Dekel & Woosley 2003). Some authors argue, for example, that the correlations between circular velocity, stellar mass, metallicity, and stellar surface densities they find for the nearby dwarf galaxies can be reproduced with a simple semi-analytic model incorporating the effect of supernova feedback on the gas component of dwarf galaxies.

The existence of galactic winds has indeed been observationally established in star-forming galaxies at low and high redshifts (e.g., Heckman et al. 2000; Pettini et al. 2001; Martin et al. 2002; Strickland et al. 2004; Martin 2005; Ott et al. 2005; see Veilleux et al. 2005 for a recent review). An indirect evidence for metal loss in winds is the high-metallicity of the diffuse intergalactic medium in groups and clusters (e.g., Renzini et al. 1993). However, the extent to which the outflows affect the global properties of galaxies and whether the outflow gas escapes the gravitational potential of its host halo or rains back down to the disk remains uncertain.

Mac Low & Ferrarris (1999) and D’Ercole & Brighenti (1999) used numerical simulations to show that global gas blowout is inefficient in all but the smallest mass halos ($M \lesssim 10^7 M_\odot$; see also Marcolini et al. 2006), although metal-enriched SN ejecta may be removed efficiently. It is also uncertain whether correlations of metallicity with galaxy stellar mass and mass-to-light ratios can be attributed solely to such winds.

In the context of cosmological simulations, the SN feedback is usually ineffective, unless ad hoc phenomenological recipes enhancing SN feedback are employed (e.g., Navarro & White 1993; Navarro & Steinmetz 1997; Thacker & Couchman 2001; Marri & White 2003; Springel & Hernquist 2003; Scannapieco et al. 2006). It is likely that this inefficiency is at least partially due to the inability of current cosmological simulations to resolve the scales and processes relevant to stellar feedback. However, it is also possible that the actual effects of feedback on the global properties of galaxies are fairly small in reality. Cosmological simulations at this point cannot make ab initio predictions about the importance or inefficiency of stellar feed-
back. Observations, on the other hand, although showing
evidence for the presence of large-scale winds in actively
starforming galaxies (e.g., see Veilleux et al. 2005 for re-
view), are rather uncertain in their estimates of wind mass
loss to provide reliable direct observationally-motivated feed-
back recipes for implementation in simulations. Nevertheless,
we can gauge the importance of supernova energy injection
by comparing galaxies formed in simulations performed with
different assumptions about feedback to observations. In this
respect, the smallest dwarf galaxies present the ideal test case
because they can be expected to be the most susceptible to the
effects of SN feedback due to their shallow potential wells
(Larson 1974; Dekel & Silk 1986).

The role of feedback in the formation of dwarf galaxies has
been investigated in several recent studies. Tassis et al. (2003)
used Eulerian AMR simulations of galaxy formation with star
formation and SN feedback and found that correlations be-
 tween mass-to-light ratio and metallicity and stellar mass and
metallicity, similar to those observed for the nearby dwarfs,
are exhibited by dwarf galaxies in their simulations at $z \gtrsim 3$.
The simulations of Tassis et al. (2003) used a rather extreme
amount of energy per supernovae to maximize the effects of
feedback. They did not test, however, whether feedback or
some other mechanism is in fact the dominant factor in shap-
ing the correlations. More recently, Kobayashi et al. (2006)
found a tight relation between stellar metallicity and stellar mass
at all redshifts in their Smooth Particle Hydrodynamics
cosmological galaxy formation simulations, which they
attributed to the mass-dependent galactic winds. The winds in
the model of Kobayashi et al. (2006) do affect the gas and
metal content of galaxies significantly, with the effect increas-
ing towards lower mass systems (see their Fig. 16), which
plays an important role in shaping the resulting correlation of
metallicity and stellar mass. De Rossi et al. (2005) have
recently studied the origin of luminosity- and stellar mass-
metallicity relations in cosmological simulations. They find
that correlations similar to those observed arise already at high
redshifts, although supernova feedback in their simulations is
not efficient. This is consistent with our results presented
below. Most recently, Brooks et al. (2006) presented a study
of the origin and evolution of the mass-metallicity relation in
cosmological simulations. They also concluded that the re-
lation arises not due to the mass loss in winds, but due to in-
creasing inefficiency of star formation in smaller mass galax-
ies. However, they argue that low effective yields of dwarf
galaxies can only be explained by the loss of metals in winds.
We argue below that there is an alternative explanation for the
low effective yields of dwarfs.

In this paper we investigate the origin of the observed mass-
metallicity and other correlations of dwarf galaxies with the
specific goal of testing the role of SN feedback. In particu-
lar, we discuss results of several high-resolution cosmological
simulations of galaxy formation started from the same initial
conditions and run with the same prescriptions for star for-
formation and metal enrichment of the ISM, but with different
models for gas cooling, UV heating (optically thin approxi-
mation vs. self-consistent radiative transfer of the UV radia-
tion), and supernova feedback (see §4 for details). We follow
the simulations until $z \sim 3$.

Although our models simulate galaxies at high redshifts, we
study the evolution of the correlations with redshift from $z = 9$
to $z \sim 3$ and show that the trends with metallicity and stellar mass
are approximately preserved as the galaxy population evolves allowing extrapolation of the results to the present
epoch. Therefore, our results and conclusions give us insight
to the processes responsible for the properties of observed
low-redshift dwarfs. There is indeed observational indication
that the stellar mass-metallicity correlation, for example, is
already established by $z \sim 1–2$ (Kobulnicky & Kewley 2004;
Savaglio et al. 2003; Erb et al. 2004), although the metallic-
ities of galaxies of a given stellar mass appear to be lower
by $\approx 0.3$ dex at $z = 2$ compared to $z = 0$ (Savaglio et al.
2005; Erb et al. 2006). In addition, for some of the Local Group
dwarf galaxies that exhibit these correlations, a sig-
nificant fraction of their stellar mass was built at $z \gtrsim 3$ (e.g.,
Dolphin et al. 2005). The correlations for these dwarfs should
thus be largely set at high redshifts, which we probe with our
simulations.

Our main result is that correlations similar to those ob-
served can be reproduced in all runs, which suggests that su-
pernova energy feedback is not required to explain the ob-
served properties of dwarfs. To interpret the simulation re-
sults we use a simple, open box chemical evolution model.
We show that a model with gas accretion but no outflows and
the Kennicutt law for star formation with the critical density
threshold for star formation reproduces the results of our sim-
ulations.

This paper is organized as follows. In §2 we summarize the
observationally established correlations between global prop-
erties of dwarf galaxies that we wish to interpret. Our analytic
chemical evolution calculations are described in §3. In §5 we
describe in detail our cosmological simulations, and in §5 we
discuss how global properties of the simulated galaxies are
correlated with each other, and we compare these correlations
to the observed ones, and to the analytic chemical evolution
model of §3. A discussion of observational results on the ef-
ective yield and corresponding constraints on the existence of
outflows is given in §5. We discuss our findings in §7 and
we summarize our conclusions in §8.

2. OBSERVED CORRELATIONS

One of the tightest relations of the Local Group dwarf
galaxies is the correlation between the dynamical mass-to-
light ratio and average metallicity (Prada & Burkert 2002):

$$\log(M_{\text{dyn}}/L_V) \propto -[\text{Fe}/\text{H}].$$

(1)

An additional set of correlations relates the stellar mass, $M_*$,
with the metallicity $Z$, the surface brightness $\mu_*$, and the
maximum circular velocity $V_m$ of dwarf galaxies. These can be
parameterized as the power law relations:

$$Z \propto M_*^{\alpha} ; \quad \mu_* \propto M_*^{\beta} ; \quad V_m \propto M_*^{\gamma} .$$

(2)

The existence of these correlations (in several cases in the form
of correlations between absolute magnitudes, rather than stellar
masses, and $Z$, $\mu_*$ and $V_m$) has been estab-
lished through observations of galaxies with a large
range of masses and in a variety of settings, including
nearby dwarfs (e.g., Skillman et al. 1989; Dekel & Woosley 2002;
Lee et al. 2006; van Zee & Haynes 2006), dwarf galaxies in
the Sloan Digital Sky Survey (SDSS; Tremonti et al. 2004;
Kauffmann et al. 2003; Blanton et al. 2003; Bernardi et al.
2003), the Hubble Deep Field (Driver 1999), the Ursa
Major cluster (Bell & de Jong 2001) and the Virgo cluster
(Ferguson & Binggeli 1994). However, the exact values of
the slopes of these relations are affected by systematic uncer-
tainties involved in obtaining stellar masses from the observed
quantities (magnitudes). In general, the slopes depend on the
mass range in which they are determined.
For example, in the case of the $Z - M_*$ scaling, Dekel & Woo (2003) find that for the Local Group dwarfs in the mass range of $10^{10} M_\odot \lesssim M_* \lesssim 10^{10} M_\odot$, $n_2 \sim 0.4$, while the SDSS data of Tremonti et al. (2004) (see also Gallazzi et al. 2005) indicate that $n_2$ increases with decreasing $M_*$: it is quite flat for $M_* \gtrsim 3 \times 10^{10} M_\odot$, while it appreciably exceeds the Dekel & Woo (2003) value by $M_\sim \sim 10^{8} M_\odot$. Most recently, Lee et al. (2006) find $n_2 \approx 0.29 \pm 0.03$ for the dwarf galaxies in their sample.

In the case of the $\mu_* - M_*$ scaling, Dekel & Woo (2003) find for the Local Group dwarfs the slope of $\mu_* \sim 0.55$, although their data points exhibit a scatter around the best-fit line which is large compared to their adopted error bars. For the same scaling, Rauchmann et al. (2003) using SDSS data for the surface density of stellar mass within the half-light radius, find that for a given value of $M_*$ there is appreciable scatter (about an order of magnitude) in $\mu_*$, and the median value of $\mu_*$ scales roughly as $M^{0.65}_* \times 3 \times 10^{10} M_\odot$, and it becomes almost independent of $M_*$ for larger $M_*$.

In the case of the $M_* - V_m$ scaling (the Tully-Fisher relation), Bell & de Jong (2001) using galaxies with $3 \times 10^{8} M_\odot \lesssim M_* \lesssim 10^{10} M_\odot$ find $n_2 \sim 0.21 \pm 0.24$, and a shallower correlation with baryonic mass (i.e., stars plus gas), $n_2 \sim 0.27 \pm 0.30$. Several recent studies also find that the $V_m - M_*$ relation for low-mass galaxies ($V_m \lesssim 90$ km/s) is steeper and exhibits considerably more scatter than the baryonic Tully-Fisher relation of the same galaxies (Matthews et al. 1998; McGaugh et al. 2000; Bell & de Jong 2001; McGaugh et al. 2003; Ghe et al. 2006). These studies find slopes of $n_2 \approx 0.1 - 0.2$ for the $M_* - V_m$ relation of the dwarf galaxies (McGaugh et al. 2004) and $n_2 \approx 0.25 - 0.33$ for the baryonic $M_{\text{baryon}} - V_m$ relation (e.g., McGaugh et al. 2000; Verheijen & Bell & de Jong 2001; McGaugh 2003; Ghe et al. 2006). For the Local Group dwarfs, Dekel & Woo (2003) obtain $n_2 \sim 0.37$ for $10^{8} M_\odot \lesssim M_* \lesssim 10^{10} M_\odot$ and $n_2 \sim 0$ for $10^{6} M_\odot \lesssim M_* \lesssim 10^{8} M_\odot$.

3. CHEMICAL EVOLUTION MODEL

In this section, we use simple analytic galactic chemical evolution calculations to derive two fundamental correlations between global properties of dwarf galaxies. First, we seek a relation between metallicity and mass-to-light ratio, since the correlation between these two quantities is tight and prevalent in observed systems. In addition, we use the model to derive relations between stellar mass, gas mass, stellar surface density, metallicity, and circular velocity of high-z galaxies. We do so under the following main assumptions: (1) that gas outflows escaping the potential of the host galaxy are negligible, (2) that gas is distributed in an exponential disk with its scale-length scaling with the galaxy circular velocity as indicated in cosmological simulations (see eq. [1]), and (3) that star formation in the gas disks is governed by the Kennicutt (1998) star formation law and that gas at surface densities below a critical threshold value (e.g., Martin & Kennicutt 2001) does not form stars.

Our models use assumptions and approximations appropriate for the high redshifts ($z \lesssim 3$) that we follow in our cosmological simulations. In all calculations presented in this section we adopt the instantaneous recycling approximation: i.e., we assume no time delay between the birth of a generation of stars and the corresponding metal enrichment of the ISM. Hence, we implicitly assume that the metal enrichment due to type II supernovae is dominant. This is appropriate for all but the most massive dwarf galaxies.

We additionally take the luminosity of an object, $L$, to be proportional to its stellar mass. This approximation is appropriate for older stellar populations. We use it here because we assume that the stellar populations in our high-z galaxies would actually be observed today, when they have aged substantially.

Finally, we assume that at the redshifts of interest, the gas mass of the objects is proportional to their total (dynamical) mass, $M_g \propto M$. Since at the early cosmic epochs we consider $M_g \approx M_b$, this proportionality stems from a more general assumption that the baryonic mass $M_b$ is proportional to the total mass. As we will see in this section, for the range of metallicities considered here this last assumption holds for all but the smallest objects in the simulation ($M \lesssim 10^6 M_\odot$), which we exclude in our subsequent analysis of other global quantities.

In the following calculations, the metallicity $Z$ used is the metallicity of the gas, although as discussed in detail in this section for the objects of interest at high $z$ the stellar metallicity tracks the ISM gas metallicity very well (see Fig. 4).

3.1. Mass-to-light ratio vs. metallicity

We start with the interpretation of the correlation between the mass-to-light ratio and metallicity found by Prada & Burkert (2003): $M/L \propto Z$. As we discussed above, for the galaxies at high redshifts $M_g \propto M$. Assuming that their stars would be observed old at the present epoch: $L \propto M_*$. The correlation can then be recast as a relation between metallicity and star-to-gas--mass ratio:

$$Z \propto \frac{M_*}{M_g}.
\quad (3)$$

This relation can be derived from a fairly general model for the chemical evolution. Assuming no inflow (i.e., that accreting gas has zero metallicity) or outflow of heavy elements, the rate of change of the mass of metals in a galaxy is

$$\frac{d}{dt}(M_* Z) = q_* \psi - aZ \psi,
\quad (4)$$

where $q_*$ is the fraction of mass going into star formation that will be returned to the ISM in the form of newly synthesized metals, $\psi$ is the star formation rate (mass per unit time) of the galaxy, $a$ is the fraction of mass of newly formed stars that is locked up in the form of long-lived stars or stellar remnants and is thus removed from the gas phase. Equation (4) states that the rate of change of metal mass is equal to the rate of production of newly synthesized metals minus the mass of heavy elements locked in long-lived stars and stellar remnants.

Equation (4) can be further simplified if we assume that $aZ \ll q_*$. This is an excellent approximation for metallicities significantly smaller than solar. The assumption will hold as long as $Z \ll q_*/a$, where $Z$ here is the metal mass fraction of the gas (not in solar units). Since $a \lesssim 1$, a conservative limit for $Z$ for the assumption to hold is $Z \ll q_*$. An additional assumption is that the approximation adopted here can be avoided by using a stellar population synthesis model and full integration of the equations. We chose to use appropriate approximations here to keep the equations and the model as transparent as possible. The adopted approximations do not change the model correlations significantly.

Note that in principle the approximations adopted here can be avoided by using a stellar population synthesis model and full integration of the equations. We chose to use appropriate approximations here to keep the equations and the model as transparent as possible. The adopted approximations do not change the model correlations significantly.

In the case of the particular metallicity-enhancement scheme implemented in our simulations, and for SN type II only, $q_* = \int_{m_*}^{100 M_\odot} f(m) \phi(m) dm / \int_{100 M_\odot}^{1000 M_\odot} m \phi(m) dm$, where $f_* = \min(0.2, 0.01 m_* - 0.06)$, and $\phi(m)$ is the Miller-Scalo initial mass function.
approximation that can be made is that both \( a \) and \( q_z \) are approximately constant. Although the fraction of mass returned into the ISM, \( q_z \), is in general not constant and increases with time, the increase after the initial few tens of millions of years is slow and this approximation is sufficiently accurate to derive approximate relations.

With these considerations, equation (4) can be re-written as

\[
d\frac{d}{dt}(M_g Z) \approx q_z \psi. \tag{5}
\]

The rate of change of the stellar mass is

\[
d\frac{dM_*}{dt} = a \psi. \tag{6}
\]

Combining Equations (5) and (6) gives

\[
d\frac{d}{dt}(M_g Z) = d \left( \frac{q_z}{a} M_* \right), \tag{7}
\]

which, for initial conditions \( M_{g,0} = 0 \) and \( Z_0 = 0 \), gives

\[
M_g Z \propto M_* \Rightarrow Z \propto \frac{M_*}{M_g} \propto \frac{M_*}{M}, \tag{8}
\]

in agreement with observations, and, as shown in [5], the results of our simulations. Note that the exact functional form of the star formation law does not enter this calculation. This implies that stochasticity and scatter in star formation will not affect this correlation.

3.2. Stellar Mass - Gas Mass Relation

The relation between stellar mass and total gas mass in dwarf galaxies is central to understanding the origin of correlations between other observable quantities\(^7\). In this section, we derive such a relation. In subsequent sections, we will use it to explain the correlations discussed in [2] and we will directly compare it against our simulation results.

The rate of change of the gas mass, assuming that there are no significant gas outflows due to SN-driven winds, is

\[
d\frac{d}{dt}(M_g) = F - a \psi, \tag{9}
\]

where \( F \) is the gas mass accretion rate. If the dependence of \( \psi \) and \( F \) on \( M_g \), as well as any explicit dependence on redshift, are known, then equations (5) and (6) can be integrated in time as a system of two ordinary differential equations. Different initial conditions for \( M_g \) at some early time (for which we can safely assume that \( M_* = 0 \)) will produce different pairs of \((M_g, M_*)\) at any time of interest. In this way, we calculate parametrically the prediction of our chemical evolution model for the \( M_* - M_g \) relation. We consider the functional dependence of \( F \) and \( \psi \) on the gas mass in the following two sections.

3.2.1. Gas Mass Accretion Rate

In the spherical collapse model, an overdensity of mass \( M \) at a time \( t \) accretes matter at a rate \( F = \frac{dM}{dt} \propto M \) [Fillmore & Goldreich 1984, Bertschinger 1985]\(^8\). Such a function. Performing the above integrations we find \( q_z = 0.015 \), therefore the assumption holds as long as \( Z < 0.015 \) or, equivalently, as long as \( Z \) is much lower than solar \((Z_{\odot} \approx 0.02)\). All of the protogalaxies that we are considering have \( Z < 0.1Z_{\odot} \), and hence this assumption always holds in our case.

\(^7\) Note that this relation may not hold for some galaxies, if, for instance, their gas is lost due to tidal or ram pressure stripping.

\(^8\) Although analytic solutions were derived for an Einstein - de Sitter universe, the result holds under spherical collapse in any cosmological model. The relation is also consistent with the results of cosmological simulations, in which halo mass grows with time as \( M(z) \propto \exp[-Cz] \) (Wechsler et al. 2002), which gives \( dM/dt = f(z)M \). In general, the proportionality constant depends nontrivially on cosmic time (with the exception of the Einstein-de Sitter universe where the evolution is self-similar and the proportionality constant is simply \( \propto t^{-1} \)), as well as the cosmological parameters.

Since we consider early epochs in the evolution of galaxies we assume that the baryon mass is dominated by gas. For example, for the simulated halos in the redshifts of interest: \( M_g \gtrsim 10M_* \). We assume thus that \( M_g \approx M_b = \Omega_b M/\Omega_m \), so

\[
F = f(z)M_g. \tag{10}
\]

To get the proportionality constant \( f(z) \) for equation (10) we use the following simple recipe

\[
F = 4\pi [r_{vir}(z)]^2 (1 + \delta_{ext}) \rho_{m,z}(z)v_{acc}(z), \tag{11}
\]

where \( \rho_{m,z} = \rho_{m,0}(1+z)^{-3} \) is the mean matter density of the universe at redshift \( z \), \( r_{vir}(z) = \left[ \frac{3M}{4\pi \rho_{m,0}(1+\delta_{vir})} \right]^{1/3} \) is the virial radius of an object of mass \( M \) at redshift \( z \), \( \delta_{vir} \) is the virial overdensity (equal to \( 18\pi^2 \approx 178 \) for the high redshifts considered here), \( 1 + \delta_{ext} \) is the local compression factor right outside the virial radius of the object, and \( v_{acc}(z) = \sqrt{2GM/r_{vir}(z)} \) is the free-fall velocity\(^9\). Equation (11) is only approximate, and we use the value of \( 1 + \delta_{ext} \) to set its exact normalization so that it adequately reproduces out simulations results, which we find to be the case for \( 1 + \delta_{ext} \approx 20 \). Similar values of the external compression factor used in analytic cosmological accretion models yield results in general agreement with the accretion rates and energetics of objects in cosmological simulations [Pavlidou & Fields, 2006].

\(^9\) Note that \( r_{vir} \propto M^{1/3} \) and \( v_{acc} \propto M^{1/2} r_{vir}^{-1/2} \propto M^{1/3} \) so \( F \propto r_{vir}^2 v_{acc} \propto M. \)
3.2.2. Star Formation Rate

To calculate $\psi(M_*)$ we assume that gas in each halo settles into an exponential disk obeying the empirical Kennicutt star formation law\(^10\), with stars forming only above a certain surface gas density threshold. Specifically, we assume that the star formation rate per surface area, $\Sigma_*$, depends on the surface density of gas, $\Sigma_g$, as

$$\Sigma_*(r) = \Sigma_0 \exp \left( -\frac{r}{r_d(z)} \right).$$

(Kennicutt 1998). The surface gas density profile is assumed to be

$$\Sigma_g = 2.5 \times 10^{-4} \left( \frac{\Sigma_g}{M_\odot \text{pc}^{-2}} \right)^{1.4} \left( \frac{M_\odot}{10^9 \text{M}_\odot \text{pc}^{-2} \text{yr}^{-1}} \right).$$

Following Kravtsov et al. (2004), we adopt the following scaling of the disk scale radius, $r_d$, with the total halo mass:

$$r_d(z) = 2^{-1/2} \lambda v_{\text{vir}}(z) \exp \left[ \frac{10^4 K}{T_{\text{vir}}(M,z)} \right].$$

where $v_{\text{vir}}(m,z)$ is the virial temperature of a halo of mass $m$ virializing at redshift $z$, and $\lambda$ is the angular momentum parameter, which has a log-normal distribution of values with scale and shape parameters 0.045 and 0.56 respectively (e.g., Vivitska et al. 2002) with the peak of the distribution at $\lambda = 0.033$). The finite spread in possible angular momentum parameters contributes to the scatter in the observed correlations. The constant $c$ is calibrated against our simulations data at every redshift, and $c = 1$ is found to give an excellent fit.

The scaling given by equation (14) adopts the commonly used scaling ($r_d \propto M_{\text{vir}}$) for massive halos (Mo et al. 1998), while for systems with the virial temperatures $T_{\text{vir}} \lesssim 10^4$ K, it assumes that the gas cannot cool efficiently and cannot settle into a rotationally supported disk but settles into a more extended equilibrium configuration in the halo (see Kravtsov et al. 2004). As a result, the associated gas surface density decreases steeply with decreasing mass and star formation is inefficient or is completely suppressed in smaller objects. As we show below, it is this inefficiency of star formation in dwarf galaxies that is primarily responsible for the observed correlations.

Star formation is suppressed for gas surface densities $\Sigma_g = \Sigma_{\text{th}} \approx 5 M_\odot \text{pc}^{-2}$ (Martin & Kennicutt 2001), which implies the threshold radius of

$$r_{\text{th}}(z) = r_d(z) \ln \left( \frac{\Sigma_0(z)}{\Sigma_{\text{th}}(z)} \right).$$

This radius decreases with decreasing galaxy mass. In other words, star formation proceeds within smaller radii in galaxies of smaller masses.

The star formation rate $\psi$ can then be calculated as follows:

$$\psi(M_*,z) = \int_0^{r_{\text{th}}} \Sigma_g(r) 2 \pi r dr,$$

$$= M_\odot \text{yr}^{-1} \frac{2 \pi (2.5 \times 10^{-4})}{1.4^2} \left( \frac{r_d(z)}{\text{kpc}} \right)^2 \left( \frac{\Sigma_0(z)}{M_\odot \text{pc}^{-2}} \right)^{1.4},$$

$$\times \left[ 1 - \left( 1 + \frac{1.4 r_d(z)}{r_d(z)} \right) \exp \left( -\frac{1.4 r_d(z)}{r_d(z)} \right) \right].$$

This result for the star formation rate per surface area, $\Sigma_*$, controls the location of the break). We find that the best fit to our data at $z = 4$ is obtained when both these free parameters have their fiducial values, $\lambda = 0.033$ (the most probable value) and $\Sigma_{\text{th}} = 5 M_\odot \text{pc}^{-2}$ (the “canonical” observational value). Figure II shows the sensitivity of our chemical evolution model to the values of $\lambda$ and $\Sigma_{\text{th}}$. The solid line represents the $z = 4$ model results for the fiducial parameter values (which are also the best-fit parameters). The dashed lines show the range of $\lambda$ values that brackets the scatter of our simulation data points at $z = 4$. The range of $\lambda$ values represented here is $0.023 \lesssim \lambda \lesssim 0.044$ (which contains $\approx 40\%$ of the area under the $\lambda$–probability distribution, and is centered around the distribution peak). In this case, $\Sigma_0$ is kept at its fiducial value. The dotted lines show the range of $\Sigma_{\text{th}}$ values that brackets the scatter of our data. The range of $\Sigma_{\text{th}}$ values represented here is $2 M_\odot \text{pc}^{-2} \lesssim \Sigma_{\text{th}} \lesssim 10 M_\odot \text{pc}^{-2}$. In this case, $\lambda$ is kept at its fiducial value.

4. NUMERICAL SIMULATIONS

In this study we use four simulations (Table I) of the early ($z \gtrsim 3$) stages of evolution for a Lagrangian region of a MW-sized system of total (dark matter and baryons) virial mass $\approx 10^{12} h^{-1} M_\odot$ at $z = 0$. The simulations were performed using the Eulerian, gasdynamics + N-body Adaptive Refinement Tree (ART) code (Kravtsov et al. 1997, Kravtsov et al. 1999). In both the gasdynamics and gravity calculations, a large dynamic range is achieved through the use of adaptive mesh refinement (AMR). At the analyzed epochs, the galaxy has already built up a significant portion of its final total mass: $1.3 \times 10^{10} h^{-1} M_\odot$ at $z = 9$ and $2 \times 10^{11} h^{-1} M_\odot$ at $z = 4$. The evolution is started from a random realization of a gaussian density field at $z = 50$ in a periodic box of $6 h^{-1}$ Mpc with an appropriate power spectrum and is followed assuming flat
ACDM model; \( \Omega_0 = 1 - \Omega_{\Lambda} = 0.3, \Omega_b = 0.043, h = H_0/100 = 0.7, n_s = 1, \) and \( \sigma_8 = 0.9. \)

To achieve the high mass resolution, a low-resolution simulation was run first and a galactic-mass halo was selected. A lagrangian region corresponding to five virial radii of the object at \( z = 0, \) corresponding to a region of \( \sim 3h^{-1} \) comoving Mpc in diameter, was then identified at \( z = 50 \) and resampled with additional small-scale waves (Navarro & White 1994, the specific procedure described here is described in Klypin et al. 2001). The total number of DM particles in the high-resolution lagrangian region is \( 2.64 \times 10^6 \) and their mass is \( m_{\text{DM}} = 9.18 \times 10^9 \text{M}_\odot. \) In addition to the main progenitor of the MW-sized system this Lagrangian region contains several dozens of smaller galaxies spanning a wide range of masses. We will use these galaxies to analyze correlations between their properties.

The code used a uniform \( 64^3 \) grid to cover the entire computational box. The lagrangian region, however, was always unconditionally refined to the third refinement level, corresponding to the effective grid size of \( 512^3. \) As the matter distribution evolves, the code adaptively and recursively refines the mesh in high-density regions beyond the third level using two refinement criteria: gas and DM mass in each cell. A mesh cell was tagged for refinement if its gas or DM mass exceeded \( 1.2 \times 10^9 \text{M}_\odot \) and \( 3.7 \times 10^9 \text{M}_\odot, \) respectively. The maximum allowed refinement level was \( l_{\text{max}} = 9. \) The volume of high-density cold star forming disks forming in DM halos was refined to \( l_{\text{max}} = 9. \) The physical size of mesh cells in the simulations was \( \Delta x = 26.16 [10/(1+z)]^{2/3} \text{pc}, \) where \( l \) is the cell’s level of refinement. Each refinement level was integrated with its own time step \( \Delta t_l = \Delta t_0 (l/\text{pc})^{-2} \approx 2 \times 10^4 \text{yr}. \) The time steps are set by a global CFL condition.

The analyzed simulations were started from the same initial conditions, but evolved using different assumptions about cooling and heating processes accompanying galaxy formation. All four runs include star formation using the recipe described in Kravtsov 2003. Namely, the gas is converted into stars on a characteristic gas consumption time scale, \( \tau_g \); \( \rho_s = \rho_g/\tau_g. \) We use constant, density-independent \( \tau_g. \) This is different from the commonly used density-dependent efficiency, but may be more appropriate for star forming regions on \( \sim 100 \text{pc} \) scales which are resolved in the simulations where constant star formation efficiency is indicated by observations (e.g., Young et al. 1996, Wong & Blitz 2002). As shown by Kravtsov 2003, such constant efficiency assumption at the scales of molecular clouds results in the Kennicutt-like star formation law on kiloparsec scales. The star formation was allowed to take place only in the densest cold regions, \( \rho_g > \rho_{\text{SF}} \) and \( T_g < T_{\text{SF}}, \) but no other criteria (like the collapse condition \( \nabla \cdot v < 0 \)) were imposed. We used \( \tau_g = 4 \text{Gyrs}, T_{\text{SF}} = 10^4 \text{K}, \) and \( \rho_{\text{SF}} = 1.64 \text{M}_\odot \text{pc}^{-3} \) corresponding to atomic hydrogen number density of \( n_{\text{H, SF}} = 50 \text{cm}^{-3}. \)

Our fiducial simulation (FEC) includes metallicity-dependent cooling and UV heating due to cosmological ionizing background with equilibrium cooling and heating rates tabulated for the temperature range \( 10^5 < T < 10^9 \text{K} \) using the Cloudy code (ver. 96b4; Ferland et al. 1998). The cooling and heating rates take into account Compton heating/cooling of plasma, UV heating, atomic and molecular cooling. It also includes energy feedback and chemical enrichment due to supernovae assuming the Miller & Scalo (1979) stellar initial mass function (IMF) and stellar masses in the range \( 0.1-100 \text{M}_\odot. \) All stars with \( M_\star > 8 \text{M}_\odot \) deposit \( 2 \times 10^{51} \text{ergs} \) of thermal energy and a mass \( f_2 M_\star \) of heavy elements in their parent cell (no delay of cooling was introduced in these cells). The metal fraction is \( f_2 = \min(0.2, \rho_{\text{SF}}/10^{-6} \text{M}_\odot). \) which crudely approximates the results of Woosley & Weaver (1995).

The second simulation, NFEC, is identical to the FEC run in all respects, except that it did not include the energy injection due to SNe (chemical enrichment due to SNe is still included). Note that the cooling rates in the run with feedback accounted for the local metallicity of the gas, while in the run with no feedback the significantly lower zero-metallicity cooling rates were used.

The third simulation, FNEC-RT, includes star formation and SN enrichment and energy feedback in the same way as the FEC run, but uses self-consistent 3-D radiative transfer of UV radiation from individual stellar particles using the OTVET algorithm (Gnedin & Abel 2001, Iliev et al. 2006) and follows non-equilibrium chemical network of hydrogen and helium species (the details of the specific implementation of the OTVET algorithm on adaptively refined meshes will be described elsewhere). The simulation thus includes non-equilibrium cooling and heating rates which make use of the local abundance of atomic, molecular, and ionic species and UV intensity followed self-consistently during the course of the simulation. The metallicity-dependence of cooling rates is taken into account using optically thin equilibrium metal cooling functions from Sutherland & Dopita (1993) in the high metallicity regime and from Penston (1970) and Dalgarno & McCray (1972) in the low metallicity regime. The molecular hydrogen formation on dust is taken into account using Cazaux & Spaans (2004) rates, and gas cooling on dust is based on Draine (1981).

A severe limitation of our current treatment of molecular hydrogen formation is that self-shielding of molecular lines in the Lyman-Werner band is not taken into account, which results in a substantial underestimate of the molecular fractions at the high density regime \( (n > 100 \text{cm}^{-3}). \) Note, however, that since our star formation criterion is based on the total rather than molecular gas density, this does not affect our star formation rates significantly.

Finally, the fourth simulation, F2NEC-RT, is identical to the FNEC-RT but SN energy is injected with a delay of \( 10^7 \text{yr} \) after the star forming event and energy release is spread over \( 10^7 \text{yr} \) after that. This variation is intended to test the sensitivity of our results to the specific details of the feedback implementation. Energy injection with delay was argued to enhance the efficiency and impact of the SN feedback on the ISM (Slyz et al. 2005), as the stellar particles have time to leave the densest regions where they form and release their energy at lower densities where the cooling time is longer. This change of feedback recipe results in a significant enhancement of the star formation efficiency. Perhaps surprisingly, the F2NEC-RT simulation forms nearly twice as much stellar mass as the simulation FNEC-RT. The reason is simple. With a delayed energy release the star forming regions are generally left intact and continuing forming stars in the simulation F2NEC-RT, while in the FNEC-RT the temperature of gas in star forming regions is increased due to SN feedback and star formation ceases for some period of time.

We show below that the details of the specific implementation of feedback in our simulations do not affect the correlations of galaxy properties and our conclusions.

5. RESULTS

\begin{equation}
\text{ACDM model: } \Omega_0 = 1 - \Omega_{\Lambda} = 0.3, \Omega_b = 0.043, h = H_0/100 = 0.7, n_s = 1, \text{ and } \sigma_8 = 0.9.\end{equation}
Fig. 2. — Gas and stellar mass fractions of objects in our simulations plotted against the total (dark matter + total baryonic) mass of each object. Left panel: simulation FEC; middle panel: simulation NFEC; right panel: simulation FNEC-RT. Open triangles show the gas fraction ($M_g/M_{tot}$), while stars show the star fraction $M_*/M_{tot}$. Different colors correspond to different redshifts as detailed in the legend.

Fig. 3. — Gas mass vs. stellar mass for objects in each of the analyzed simulations. Left panel: simulation FEC; middle panel: simulation NFEC. Right panel: the FNEC-RT simulation. Different symbols correspond to different redshifts as detailed in the legend. The solid and dashed lines show the results of our analytic chemical evolution model for $z = 4$ and $z = 9$, respectively.

Fig. 4. — Gas metallicity vs. stellar metallicity. Left panel: simulation FEC; middle panel: simulation NFEC; right panel: FNEC-RT simulation. Different symbols correspond to different redshifts as detailed in the legend. The solid line corresponds to $Z_{gas} = Z_*$. 
In this section, we discuss relations between global properties of galaxies in our cosmological simulations, how these depend on the detailed physics included in each simulation, and how they compare to the observationally established correlations and our analytic chemical evolution models of §3.

The panels in the figures discussed in this section are arranged in the following way: left panel - FEC simulation (SN feedback, uniform UV background, tabulated heating/cooling rates); middle panel - NFEC simulation (no SN energy feedback, uniform UV background, tabulated heating/cooling rates); right panel - FNEC-RT simulation (SN feedback, 3-D radiative transfer). We do not show the results of the F2NEC-RT simulations in all of the figures for clarity. In all cases, the results of the F2NEC-RT simulations are quite similar to those of the FNEC-RT run.

Figure 2 shows the gas and stellar mass fractions within the virial radius as a function of the virial mass for all objects in the simulations. For systems with \( M \gtrsim 10^9 M_\odot \), the gas fractions are approximately independent of mass, while at smaller masses a variety of effects (e.g., gas stripping, reionization) act to suppress the gas fractions and increase the scatter. This result is qualitatively consistent with the findings of the previous studies (e.g., Gnedin 2000; Chiu et al. 2001; Tassis et al. 2003; Hoef polit al. 2005). Figure 2 also shows that the stellar fractions for the redshifts and masses under consideration is an order of magnitude smaller than the gas fractions. This is due both to the inefficiency of star formation in these small systems and the fact that we analyze them at high redshifts. The objects with smaller masses do contain a universal share of produced very similar results. This demonstrates explicitly the correlation of the chemical evolution in stars for every object in our simulations at different redshifts. The solid line corresponds to the one-to-one relation \( Z_{\text{gas}} = Z_* \). Indeed, for the objects and redshifts we consider, the metallicity of stars tracks that of the gas very closely.

One of the tightest correlations observed for the nearby dwarf galaxies is the correlation of the dynamical mass-to-light ratio and metallicity (Prada & Burkert 2002). In §3.1 we showed that such correlation should naturally arise in low-metallicity dwarf galaxies without gas outflows and regardless of their star formation rate as long as baryon mass is approximately proportional to their total mass. Figure 5 clearly shows that for galaxies in our simulations the ratio of the total (baryonic + dark matter) to stellar mass is tightly correlated with metallicity, independent of the details of star formation and cooling and inclusion of SN feedback. The form of the correlation, \( M_{\text{tot}}/M_\ast \propto Z^{-1} \) is similar to that exhibited by the observed dwarf galaxies. Note that the correlation arises for objects at the earliest epochs and that galaxies move along the correlation as they evolve. This can explain the tightness of the correlation exhibited by the local dwarf galaxies. It also implies that a similar correlation should be expected to hold for the observed galaxies at high redshifts.

Figure 6 shows the mass-weighted metallicity of stars of the simulated galaxies as a function of stellar mass. The correlation of the two quantities is very tight and can be described by a power law at small masses. At large stellar masses, \( M_\ast \gtrsim 5 \times 10^9 M_\odot \), the correlation becomes shallower. This is qualitatively consistent with the flattening of the \( Z-M_\ast \) relation found in the SDSS data (Tremonti et al. 2004), although in their observations the break occurs at much larger masses: \( m \sim 3 \times 10^{10} M_\odot \). Note, however, that this figure should not be directly compared to the present-day correlation over a wide range of stellar masses because massive galaxies will undergo substantial evolution in both their stellar mass and metallicity between \( z = 3 \) and the present, while the evolution of dwarf galaxies is expected to be much slower. For some of the dwarf galaxies the evolution can be halted at high redshifts if their gas is removed by some process (e.g., ram pressure stripping). Thus, the comparison with the correlations exhibited by the local galaxies is most meaningful for smaller mass objects.

The solid lines in Figure 7 show the results of our chemical evolution model. The model reproduces the slope of the correlation in the two mass regimes, the location of the break, and the observed redshift evolution seen in the simulated galaxies. The correlation in the model arises mainly due to the inefficiency of star formation in dwarf galaxies and the existence of star formation threshold, without gas outflows. Interestingly, although some assumptions of our analytic model are expected to break down at late times, direct extrapolation of the model to \( z = 0 \) indicates that the flattening of the \( M_\ast -Z \) relation shifts to \( M_\ast \sim \text{few} \times 10^{10} M_\odot \), in agreement with the SDSS data.

Figure 7 shows the stellar surface density, \( \mu_\ast \), against galaxy stellar mass. The surface density is calculated from the simulation data as \( \mu_\ast \propto M_\ast/R_\ast^2 \), where \( R_\ast \) is the size of...
Fig. 5.— Ratio between total mass and stellar mass (assumed to be proportional to the mass-to-light ratio of each object), plotted against stellar metallicity. Left panel: simulation FEC; middle panel: simulation NFEC; right panel: FNEC-RT simulation. Different symbols correspond to different redshifts as detailed in the legend. The solid line corresponds to the $M_{\text{tot}}/M_* \propto Z^{-1}$ scaling.

Fig. 6.— Stellar metallicity as a function of stellar mass. Left panel: simulation FEC; middle panel: simulation NFEC; right panel: FNEC-RT simulation. Different symbols correspond to different redshifts as detailed in the legend. The solid and dashed lines show the results of our analytic chemical evolution model for $z = 4$ and $z = 9$, respectively.

Fig. 7.— Stellar mass over $R_*^2$ (where $R_*$ is the half-light radius, which includes 90% of the stellar mass of each object), plotted against stellar mass. Left panel: simulation FEC; middle panel: simulation NFEC; right panel: FNEC-RT simulation. Different symbols correspond to different redshifts as detailed in the legend. The solid line corresponds to the prediction of our analytic chemical evolution model for $z = 4$.
Fig. 8.— Maximum circular velocity as a function of stellar mass (Tully-Fisher relation). Left panel: simulation FEC; middle panel: simulation NFEC; right panel: FNEC-RT simulation. Different symbols correspond to different redshifts as detailed in the legend. The solid and dashed lines show the results of our analytic chemical evolution model for $z = 4$ and $z = 9$, respectively. In the lower row plots we have shifted $V_m$ by a factor of $(1+z)^{1.5}$ to correct for the explicit redshift dependence entering $V_m$ through its dependence on the virial radius.
Figure 9.— Maximum circular velocity as a function of baryonic mass (baryonic Tully-Fisher relation). Left panel: simulation FEC; middle panel: simulation NFEC; right panel: FNEC-RT simulation. Different symbols correspond to different redshifts as detailed in the legend. The solid line corresponds to the $M_{\text{baryon}} \propto V_m^3$ scaling. In the lower row plots we have shifted $V_m$ by a factor of $(1+z)^{-0.5}$ to correct for the explicit redshift dependence entering $V_m$ through its dependence on the virial radius.

the stellar system, defined as the radius which includes 90\% of the stellar mass. The data occupy a primary locus in the $\mu_* - M_*$ plane, which is traced by the prediction of our analytic chemical evolution model, in which stellar surface density is calculated as $\frac{M_*}{r^2_{\text{th}}}$. At masses $M_* \lesssim 10^8 M_\odot$ there are several outliers, as in this regime star formation becomes more stochastic. As for the other considered correlations, we see that the results of different simulations are consistent. We should note that stellar surface densities are probably less reliable than global properties, such as stellar mass, because they depend on the internal structure of the stellar distribution in galaxies and may be more susceptible to resolution effects.

Figure 8 shows the stellar mass of the objects in our simulations as a function of their maximum rotational velocity (the Tully-Fisher relation). A break of the scaling law exists at a stellar mass of $\sim 10^8 M_\odot$. For objects with stellar masses above the break, the stellar mass scales as $V_m^3$ with the best fit slopes of $3.41 \pm 0.33, 3.34 \pm 0.33$, and $3.37 \pm 0.22$ for the FEC, NFEC, and FNEC-RT simulations. Below the break the $V_m - M_*$ relation steepens considerably. The location of objects on the $M_* - V_m$ plane exhibits a considerable shift with redshift. This is a trivial consequence of the redshift-dependence of the virial density.

From the virial theorem we have
\[
V_m \propto \sqrt{M/R_{\text{vir}}}. \tag{18}
\]
At the high redshifts under consideration, the universe is matter-dominated, and the virial overdensity $\delta_{\text{vir}} = \rho_{\text{vir}}/\rho_m - 1 \approx 18\pi^2$ is essentially independent of $z$. The virial radius is then determined from the relation
\[
\frac{4}{3} \pi R_{\text{vir}}^3 \rho_m(1+\delta_{\text{vir}}) = M. \tag{19}
\]
Given that $\rho_m(z) = \rho_{m,0}(1+z)^3$, we get
\[
R_{\text{vir}} \propto \frac{M^{1/3}}{1+z}. \tag{20}
\]
and hence
\[
V_m \propto M^{1/3} \sqrt{1+z}. \tag{21}
\]

In the lower panels of Figure 8 we have scaled the $V_m$ values of all data points of the upper panel by $\sqrt{1+z}$ to correct for this redshift evolution. Using $M \propto M_g$ and the $M_* = M_g(M_f)$ dependence calculated using our analytic model, we get the solid and dashed lines shown in Figure 8 for $z = 4$ and $z = 9$, respectively. Once again, the model describe the results of the simulation very well.

Finally, Figure 9 shows the baryonic Tully-Fisher relation, where the total baryonic mass (stars and gas) of each object is plotted against $V_m$. In this case, the $M_{\text{baryon}} \propto V_m^3$ scaling (represented by the solid line) shows no evidence of a break. This is similar to the behavior of the observed dwarfs, which exhibit a break in the $M_* - V_m$ relation, but a tight power law $M_b - V_m$ relation (see § 2).

6. THE EFFECTIVE YIELD

The effective yield, defined as
\[
y_{\text{eff}} = \frac{Z}{\ln(1/f_g)}, \tag{22}
\]
FIG. 10.— Effective yield as a function of circular velocity of galaxies in our simulations at $z = 4$. Left column: simulation FEC (with supernova energy feedback); right column: simulation NFEC (no supernova energy feedback). Upper panel: effective yield $y_{\text{eff}}$ calculated taking into account the mass of gas, stars, and metals within the virial radius of each object; lower panel: effective yield $y_{\text{eff}}^*$ calculated taking into account the mass of gas, stars, and metals within the radius enclosing 90% of galaxy stellar mass.

has been widely used as a diagnostic of the evolution of the baryonic component of galaxies, and more specifically as a “litmus test” of the validity of the closed-box approximation. Here, $f_g = M_g / (M_\ast + M_g)$ is the fraction of baryons in the gas phase. Under the closed-box assumption, the effective yield is always equal to the true yield $y_{\text{true}}$, defined as the mass in newly synthesized metals returned to the ISM by a stellar population normalized to the stellar mass of this population locked-up in stellar remnants and long-lived stars.\footnote{Edmunds (1990) showed that $y_{\text{eff}}$ cannot exceed $y_{\text{true}}$. Values of $y_{\text{eff}}$ lower than $y_{\text{true}}$ are indicative of the deviations from the closed-box model due to either inflows or outflows of gas and metals. Observationally, dwarf galaxies ($V_m \lesssim 100 \text{ km/s}$) have low values of $y_{\text{eff}}$. This has been interpreted as significant outflows of metals and/or gas, which can reduce the metallicity and/or the gas fraction and are expected to be more prevalent in smaller objects with shallower gravitational potentials. (Garnett 2002, Tremonti et al. 2004, Pilyugin et al. 2004; Dalcanton 2006).}

In the previous section we showed that global correlations of galaxy properties arise in our simulations without SN-driven outflows. We have shown that these correlations can be reproduced with an open-box evolution with cosmological mass inflow but no outflows and 1) mass-dependent gas distribution given by equation (14) and 2) star formation obeying the Kennicutt law \textit{with a surface density threshold}. In this model the correlations, similar to those of observed galaxies, arise due to the increasing fraction of gas at surface densities below the star formation threshold density (and, correspondingly, inefficiency of star formation) in smaller mass objects. This trend, in turn, is due to inefficiency of cooling and more extended gas distributions for smaller masses. Given that the observed values of the effective yield are considered to be evidence for outflows, in this section we compare the predictions of our simulations for the values of the effective yield.

Upper panels of Figure 10 show the effective yield as a function of circular velocity for the objects in simulations FEC (with feedback) and NFEC (no feedback) at $z = 4$. Here $y_{\text{eff}}$ is calculated taking into account all of the gas and metals inside the virial radius of each object. As expected, in the absence of outflows the effective yield is constant and is close to the true yield adopted in simulations. However, we should recognize that in observations the metallicity and gas fraction are generally measured only within the stellar extent of galaxies (e.g., Tremonti et al. 2004). The lower panels in Figure 10 show the values of $y_{\text{eff}}$ calculated using gas and metals only within the stellar extent, defined as the radius that includes 90% of the total stellar mass. In this case, there is a marked decrease of $y_{\text{eff}}$ with decreasing circular velocity at $V_m \lesssim 100 \text{ km/s}$. The values of the effective yield for the small-mass systems are now comparable to those measured for dwarf galaxies.

The fact that the effective yield decreases drastically when measured within the stellar extent means that the stellar re-
regions in our simulated galaxies do not evolve as closed box systems. Given that within the virial radius the evolution is close to the closed box, the metals and gas must leave the stellar region but remain within the virial radius.

Indeed, our analysis shows that substantial fraction of heavy elements is outside the stellar extent. Figure 11 shows the local metallicity of the gas as a function of the distance from the center of the object for two simulated galaxies, of total masses of 5.3 × 10^9 M⊙ and 2.3 × 10^9 M⊙. The vertical line in each panel represents the radius containing 90% of the stellar mass. In the massive object, the extent of stellar distribution is comparable to the size of the extent of the enriched gas (i.e., the size of the constant metallicity region). On the other hand, in smaller object the enriched gas has a considerably more extended distribution than stars. Hence, an appreciable fraction of the total mass of metals produced during the lifetime of the object is outside the stellar system. Observational measurements of metallicity, however, are generally limited to within the stellar distribution, which can lead to a significant under-estimate of effective yield.

The trend with galaxy mass is demonstrated in Figure 12, which shows the metal mixing length l_{mix}, the radius at which the metallicity reaches 1/e of its central value, normalized to the radius of the stellar disk, as a function of the circular velocity of the object, for simulations FEC and NFEC. For higher mass objects the mixing length is always comparable to their stellar disk radius, while in smaller objects l_{mix} is significantly larger than the stellar disk radius, and more so as the mass of the objects becomes smaller.

Given that NFEC simulation shows that the spread of metals is not due to SN-induced outflows, the most likely mechanism of metal diffusion is turbulent mixing in the galactic disks. The turbulence can be driven by gravitational instabilities or by mergers with other galaxies and dark matter subhalos, frequent at high redshifts. Another possible source of turbulence are the cold accretion flows (Birnboim & Dekel 2003; Keres et al. 2005) reaching well inside the virial radius and stirring up the gas.

Quantitatively, the impact of extensive metal mixing in small objects can be seen as follows. Let the effective yield of an object as it would be measured if metals, gas, and stars inside the virial radius were accounted for, be y_{eff}. For an object with low M_*/M_⊙, Eq. (22) can be rewritten as

\[ y_{\text{eff}} = \frac{M_Z}{M_\text{g}} \approx \frac{M_Z}{M_*} \sim L \]

If now we observe the same object only out to the stellar disk radius, then we will only be observing a fraction of its total metal mass, \( \beta_Z M_Z \), a fraction of its total gas mass, \( \beta_g M_g \), and almost all of its total star mass, \( M_* \). Then, the observationally determined effective yield, \( y_{\text{eff}}^* \), will be

\[ y_{\text{eff}}^* = \frac{(\beta_Z M_Z)/(\beta_g M_g)}{\ln (1 + M_*/(\beta_g M_g))} \approx \frac{\beta_Z M_Z}{M_*} = \beta_Z y_{\text{eff}} \]

assuming that the gas fraction within the stellar disk radius is still high.

For large objects, in which the mixing length is comparable to the extent of star formation, \( \beta_Z \approx 1 \) and \( y_{\text{eff}}^* \) is not offset appreciably from \( y_{\text{eff}} \). However, in smaller objects due to the inefficiency of star formation and the limited extent of the starforming disk, \( \beta_Z \ll 1 \) and observationally determined \( y_{\text{eff}}^* \) is significantly smaller than \( y_{\text{eff}} \). This can possibly explain a trend of lower \( y_{\text{eff}}^* \) with decreasing object mass seen in the observational data. In this case, there is no need to invoke supernova energy feedback or outflows escaping the gravitational potential of the host halo to theoretically reproduce such a trend. If this explanation is correct, the prediction of our model would be that the gas well outside of the extent of stellar disk and outside of regions of active star formation should be significantly enriched with heavy elements. Although to our knowledge there are no current observational constraints on the metallicity of such gas\(^{12}\), metallicity measurements in absorption and tests of our conjecture may be possible in the future. The current measurements for \( \sim L_\text{s} \) galaxies indicate that cold gas is significantly enriched to large radii where little or none in situ star formation should be occurring (Chen et al. 2005).

7. DISCUSSION

Before a detailed comparison with observational results is attempted, it is important to consider possible evolutionary effects acting on the low-mass galaxies between the high redshifts studied in this paper and \( z = 0 \). Dwarf galaxies that are

\(^{12}\) Some existing measurements of the metallicity of neutral HI gas for dwarf galaxies (e.g., Aloisi et al. 2003; Leclercq des Etangs et al. 2004) indicate that the metallicity of the neutral gas is comparable or somewhat below that of the HI gas for which the metallicities are typically measured. These measurements of the neutral phase metallicity, however, are still limited to regions within the stellar extent.
currently observed to be star-dominated may have undergone an almost closed-box phase at some point in their history, after star formation overtook gas accretion, or they may have had their gas stripped off in the process of merging. Since our simulations have been limited to $z \gtrsim 3$ and our analytic models lack the ingredients necessary to follow either one of the scenarios described above, we cannot yet make robust predictions about the late-time properties of such objects. However, dwarf galaxies that are currently observed to contain significant amount of gas also show indications that they have not experienced significant star formation activity at late times. Hence, high-redshift correlations between their metallicity, total mass, stellar mass and surface brightness are expected to have been preserved as “fossils” until the present epoch. When properties of Local Group dwarfs are plotted against each other to derive correlations between observables, the two types of dwarfs do not occupy different loci on these plots, and the slopes of the correlations are very close to the ones we found in our models of high-redshift objects. This is a strong indication that any evolutionary processes operating at $z \lesssim 3$ and which we have not accounted for in our models do not act to negate or significantly alter the correlations established at higher redshifts. We plan to revisit the problem of late-time evolution of dwarfs in the future, using cosmological simulations extended to late times as well as an array of more detailed chemical evolution calculations.

With these issues in mind, it is nevertheless interesting to see how our simulation results compare with observations of dwarf galaxies in the present-day universe. In Figure 13 we plot observed properties of dwarf galaxies, the $z = 4$ results from the FEC simulation, and the predictions of the analytic chemical evolution model. The upper left panel shows the metallicity as a function of stellar mass. Observational data are taken from Dekel & Wid (2003) and from Lee et al. (2006), their Table 1. In the case of the Lee et al. (2006) data, we used the conversion relation fitted by Simon et al. (2006) to convert $12 + \log(O/H)$ to $[\text{Fe}/H]$. The upper right panel shows the surface brightness as a function of stellar mass. In the lower left panel, we plot the stellar mass as a function of the maximum circular velocity (the Tully-Fisher relation), corrected against explicit evolution with redshift. Finally, in the lower right panel we show the effective yield as a function of maximum circular velocity. The overplotted observational points are taken from Lee et al. (2006), their Table 5, Garnett (2002) (their Table 4) and Pilyugin et al. (2004) (we have used data on oxygen abundances and gas fractions from their Table 7 to compute the effective yield). The effective yield in the simulated objects has been calculated using the metal mass within the stellar disk, but all of the cold ($\lesssim 10^4 K$), HI-observable gas in each halo, so as to be directly comparable with the observational data. Although the scatter is appreciable both within the same dataset but most prominently between different datasets, the correlations present in the simulation results are generally very similar with the ones found in the observational data.

We have shown in §5 that in all of our models, tight correlations, similar to those observed in present-day dwarfs, are developed already at very high redshifts between observables of low-mass galaxies, and they are preserved at least until $z \sim 3$. Energy feedback for supernovae and galactic winds was not found to be either necessary or important for establishing these correlations. The evidence for this is twofold: first, the results of our simulation without supernova energy feedback exhibit no qualitative or significant quantitative deviation from the results of our two simulations with feedback; second, our chemical evolution model, which assumes no gas outflows, reproduces the results of all three simulations remarkably well. Our interpretation of these correlations in our models and the corresponding ones in observed systems is that they occur as a result of the inefficiency of star formation in low-mass systems. More specifically, we have argued that systems of virial temperature $\lesssim 10^4 K$ cannot cool efficiently and form rotationally supported disks and the associated high surface densities - hence, star formation proceeds, if at all, only in the densest central parts of these objects (a concept consistent with observations suggesting that in low-mass galaxies the surface density of cold gas is typically low, e.g., van Zee et al. 1997, Martin & Kennicutt 2001, Auld et al. 2006). As a result, stellar mass and metal enrichment are increasingly suppressed at lower masses, resulting in the observed correlations between stellar mass and other observables.

The inefficiency of star formation below the critical gas surface density also helps to explain the observed gas fractions as a function of galaxy luminosity and surface brightness (van den Bosch 2000), the stellar truncation radii and gas extent in the observed disks (van den Bosch 2001), the shallow slope of the faint-end of the galaxy luminosity function (Verde et al. 2002, Croton et al. 2006), the paucity of faint dwarf galaxies in the Local Group (Verde et al. 2002, Kravtsov et al. 2004), and observations of extreme HI-dominated dwarfs (Begum et al. 2005, 2006, Warren et al. 2006).
Fig. 13.— Observational data overlaid with results from the FEC simulation (z=4). The solid line corresponds to the analytic chemical evolution model prediction. Upper left panel: metallicity vs stellar mass. Observational data from Dekel & Woo (2003) and Lee et al. (2006). Upper right panel: surface brightness vs stellar mass. Observational data from Dekel & Woo (2003). Lower left panel: Stellar mass vs maximum rotational velocity (Tully-Fisher relation) corrected for explicit redshift evolution. Observational data from Dekel & Woo (2003) and Lee et al. (2006). Lower right panel: effective yield vs maximum rotational velocity. Observational data from Lee et al. (2006), Garnett (2002) and Pilyugin et al. (2004), which are not deficient in baryons but are nevertheless underluminous. These results imply that the star formation law governing conversion of cold gas into stars, rather than SN-driven outflows, is the dominant factor in shaping properties of faint galaxies.

In addition, there are observational indications that feedback does not in fact drive significant amounts of gas out of low-mass galaxies. Given that lower-mass galaxies would be more susceptible to strong supernova-driven gas outflows, one could expect that gas fractions should decrease with decreasing stellar mass. However, the opposite trend is observed (McGaugh & de Blok 1997; Bell & de Jong 2000; Garnett 2002; Geha et al. 2006). In fact, many of the smallest galaxies have gas fractions in excess of 0.8-0.9 (Geha et al. 2006; Lee et al. 2006).

At the same time, there is an indirect evidence for the importance of metal loss in winds. The high metallicity of diffuse intergalactic gas in groups and clusters (e.g., Renzini et al. 1993) and in the intergalactic medium around galaxies (e.g., Adelberger et al. 2005) is generally attributed to the action of winds. The winds may also play a role in faster evolution of the stellar mass-metallicity relation (De Rossi et al. 2006).

An interesting additional constraint comes from the observed HI mass function (HIMF) of galaxies (Zwaan et al. 2005). Its small-mass end is significantly shallower than the expected mass function of dark matter halos, implying that some sort of suppression of gas content in halos is needed. At this point, there is no accepted explanation of the shallow small-mass end of the HIMF, although Mo et al. (2005) have recently argued that it can be explained by a combination of gas heating by the cosmic UV background and preheating of accreting gas by filaments and pancakes in which galaxies are embedded. It is interesting that regardless of the nature of suppression mechanism it should be almost independent of halo mass. Strong mass dependence of gas suppression would likely introduce a break in the baryonic Tully-Fisher relation, which is not observed.

From a chemical evolution point of view, outflows would constitute a departure from closed-box evolution. The quantity that has been traditionally used to quantify the metallicity evolution of a galaxy and its deviation from the closed-box model is the effective yield, $Y_{\text{eff}}$ (Garnett 2002; Pilyugin et al. 2004; Tremonti et al. 2004). Lower-mass galaxies ($V_m \approx 100$ km/s) have in fact been observed to have an effective yield lower than the closed-box model prediction for their observed gas fraction, and this has been traditionally interpreted as an indication for metal outflows.

Recently, Dalcanton (2006) showed that outflows of gas with the same average metallicity as the host galaxy can only have a minor effect on the effective yield and cannot reproduce the observed reduction of $Y_{\text{eff}}$ with respect to the closed-box value in low-mass objects, and suggested that metal-enhanced outflows may instead be responsible for this effect.

In this paper, we show that there is an alternative explanation for the trend for decreased effective yields at the lowest-
mass objects. The effective mixing in the ISM at these high redshifts spreads the metals produced through star formation to larger regions in small galaxies than are accessible through observations. When the effective yield in our simulated galaxies is calculated taking into account gas, stars and metals out to the virial radius of each object, no dependence of the effective yield with mass is observed. Conversely, when we calculate the effective yield taking into account only the gas, stars and metals out to the stellar disk radius, a trend of decreasing effective yield with mass is revealed, similar to the one seen in observational data. This trend becomes much less pronounced in higher-mass galaxies, consistent with the findings of Erb et al. (2006) who found no appreciable dependence of effective yield on stellar mass for objects with $M_\ast \gtrsim 10^8 M_\odot$ and for $z \sim 2$.

We would like to stress again that gas outflows escaping the host halo are not required to explain this trend, although they may affect the exact form of the relation (normalization and slope) and its evolution. Note however that gas flows within the host halo may be important in spreading the produced metals to regions considerably more extended than the stellar disk. The consistency between results from our simulations with and without energy feedback from supernovae suggests that some other process drives turbulence and large-scale motions of the interstellar medium, such as gravitational instabilities, galaxy mergers, or cold accretion flows. This explanation of the trend of effective yield with galaxy mass can be potentially tested by measurements of the metallicity of the gas in the outskirts of the gaseous disks of dwarf galaxies.

8. CONCLUSIONS

In this paper, we use a suite of high-resolution cosmological simulations to examine the role of different physical processes in establishing the observed scaling relations of dwarf galaxies. All simulations included a recipe for star formation and metal enrichment of the ISM. Three simulations (FEC, FNEC-RT, F2NEC-RT) additionally included energy feedback from supernovae to the ISM. Simulations FNEC-RT and F2NEC-RT followed detailed 3-D radiative transfer from individual UV sources and the chemistry network of ionic species of hydrogen, helium, and molecular hydrogen to determine the radiative heating and cooling of the baryonic gas, while the other two simulations used tabulated heating and cooling rates and a uniform ionizing background. In order to interpret our simulation results, we develop and use an analytic chemical evolution model, which follows the evolution of the dynamical mass, gas, and stellar components as well as the metallicity of galaxies. We assumed an open box model with no outflows but with accretion of mass at a rate proportional to the galaxy mass, and star formation following the Kennicutt law with a critical density threshold for star formation. Our main results and conclusions can be summarized as follows.

- In all simulations, correlations between global quantities, such as the stellar mass-metallicity relation, very similar to those observed for the nearby dwarf galaxies, arise in simulated galaxies with stellar masses $10^8 M_\odot \lesssim M_\ast \lesssim 10^9 M_\odot$ for $z \gtrsim 3.3$.
- In galaxies of higher masses these correlations exhibit a break and the dependence of global properties on stellar mass flattens off. A similar flattening of the correlations is also found in observational data, although the location of the break may have evolved to higher masses by the present epoch (e.g., Tremonti et al. 2004).

- The key finding of this study is that neither the inclusion of supernova energy feedback, nor the inclusion of 3-D radiative transfer significantly affects any of these correlations, a strong indication that thermal and radiative processes (as well as associated galactic outflows and winds) are not required for the observed correlations to arise. Using our analytic model, we show that these correlations in the simulated galaxies arise due to the increasing inefficiency of star formation with decreasing mass of the object. The inefficiency of conversion of gas into stars and its dependence on mass are due to the critical gas column density threshold of the star formation law.

- We thus argue that trends similar to those exhibited by observed dwarf galaxies can arise without loss of gas and metals in wids, although winds may affect the exact form of the relation (normalization and slope) and its evolution.

- The observed trends of a decreasing effective yield with decreasing galaxy mass can be reproduced in our simulated galaxies reasonably well, and without winds, if the effective yield is calculated taking into account only the gas, stars, and metals within the stellar extent of each galaxy, as is done in most observational studies. We show that this trend is due to efficient mixing in the ISM, driven by disk instabilities, mergers, and possibly cold accretion flows. The efficient mixing redistributes the metals throughout the gaseous disks, which extend far beyond the stellar extent in smaller mass galaxies. Thus a significant fraction of metals in small galaxies is outside the stellar extent leading to a decrease of the estimated yield if only metals within the stellar radius are taken into account. This explanation can potentially be tested by measurements of the gas metallicity in the outskirts of the gaseous disks of dwarf galaxies.

We would like to thank Marla Geha for useful discussions and for sharing results prior to publications, Andrea Macciò for providing us with corrected values of $V_{10}$ for Local Group dwarfs, and Vasiliki Pavlidou for helpful discussions. We are grateful to Julienne Dalcanton, Josh Simon, and Francisco Prada for stimulating discussions of their results and Simon White, Volker Springel, Licia Verde, Hsiao-Wen Chen, Justin Read and Greg Bryan for helpful comments on the draft of this manuscript. AVK and NYG are grateful to the Institute for Advanced Study and Aspen Center for Physics for hospitality during the completion of this paper. This work was supported in part by the DOE and the NASA grant NAG 5-10842 at Fermilab, by the HST Theory grant HST-AR-10283.01 by the NSF grants AST-0206216, AST-0239759, and AST-0507666, and by the Kavli Institute for Cosmological Physics at the University of Chicago. Supercomputer simulations were run on the IBM P690 array at the National Center for Supercomputing Applications (under grant AST-020018N) This work made extensive use of the NASA Astrophysics Data System and arXiv.org preprint server.