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RECENT IDEAS ON THE COSMOLOGICAL CONSTANT PROBLEM

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1. Outline

In this talk I shall report on recent attempts at explaining the experimental smallness of the cosmological constant (CC).

I will start by recalling what the CC problem is and why its resolution cannot be a simple one. I shall then proceed with a review of the Baum-Hawking-Coleman (BHC) wormhole-based solution, stressing general features as opposed to technical details. Next I will criticize the BHC approach, by arguing that the vanishing of the CC is closely related, in that scheme, to a wormhole-induced quantum instability of the theory.

In the second part of the talk, I shall present two alternative ways of suppressing, rather than cancelling completely, the CC. The first approach is still based on topological effects, such as wormholes, while the second one is more conventionally based on perturbative quantum gravity corrections. Unfortunately, both schemes are still affected by (serious?) difficulties. They do contain, however, some new ideas which, I believe, are well worth pursuing.

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2. The cosmological constant problem

The CC problem goes back to the very early days of General Relativity. In 1917, Einstein, searching for static solutions of his equations (the expansion of the Universe was not known at the time), introduced [1,2] an extra term committing what he later described as "the biggest blunder of my life".

Denoting by G and Λ the Newton and the cosmological constant, respectively, Einstein's modified equations take the form:

$$-R_{\mu\nu} + 1/2 g_{\mu\nu} R = \Lambda g_{\mu\nu} + 8\pi G T_{\mu\nu} \quad (2.1)$$

The above equations also follow from the action:

$$\begin{aligned} \Gamma_{eff} = (16\pi G)^{-1} \int dx \sqrt{g} (2\Lambda - R) + \\ + \text{matter} + \text{higher derivative terms} \end{aligned} \quad (2.2)$$

We have denoted the action by Γ_{eff} in order to stress that we do not assume Einstein's theory to be fundamental. All we are assuming is that the effective, large distance theory of gravity is well described by the action (2.2). Indeed, this is the most general form of *local* action consistent with the invariance principles of General Relativity.

Equation (2.1) admits homogeneous, isotropic solutions, known as Friedmann-Robertson-Walker universes whose line element (metric) reads:

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 d\Omega \right] \quad (2.3)$$

where $k = 0, 1, -1$ correspond, respectively, to a flat, closed, open Universe.

Inserted into (2.1) this ansatz yields the standard cosmological equations for the scale factor $a(t)$:

$$\begin{aligned} (\ddot{a}/a) &= \frac{\Lambda}{3} - \frac{4\pi G}{3}(\epsilon + 3p) \\ H^2 &= (\dot{a}/a)^2 = -\frac{k}{a^2} + \frac{1}{3}\Lambda + \frac{8\pi G}{3}\epsilon \end{aligned} \quad (2.4)$$

where H is the Hubble "constant" and ϵ and p are the energy and pressure density respectively (defined by $T_{00} = \epsilon, T_{11} = T_{22} = T_{33} = p$). Obviously, the cosmological term corresponds to a "vacuum energy density" $\epsilon_v = \frac{\Lambda}{8\pi G}$ with an equal and opposite pressure. From Eqs. (2.4) one can easily see:

- i) the absence of a static solution if $\Lambda = 0$ and $\epsilon, p > 0$. This is the original Einstein observation[1].

ii) the fact that, if $\Lambda > 0$ and matter contributions can be neglected, $a(t)$ grows exponentially at large t :

$$\begin{aligned} a(t) &= a(0)\exp(Ht); & H^2 &= \frac{\Lambda}{3}, k = 0 \\ a(t) &= a(0)\cosh(Ht); & H^2 &= \frac{\Lambda}{3}, k = 1 \end{aligned} \quad (2.5)$$

These represent so-called inflationary epochs of the Universe.

There is a variety of good reasons [3] for an inflationary period occurring in the early Universe. This makes the CC problem even more acute since we would like to have a large, positive CC in the early Universe which magically relaxes at later times to the present tiny value. This is constrained by [2]:

$$\begin{aligned} \Lambda(now) &\lesssim H^2(now) \simeq 10^{-85} GeV^2, i.e., \\ \epsilon_v &\lesssim 10^{-47} GeV^4, \quad \text{or, finally} \\ G\Lambda &\lesssim 10^{-122} \end{aligned} \quad (2.6)$$

To get such a tiny value of ϵ_v today, things should conspire extremely well, since any phase transition (e.g., the spontaneous breaking of gauge or global symmetries) is always accompanied by a change in the vacuum energy which is [2] at the very least 40 orders of magnitude larger than the bounds (2.6).

Needless to say, it is very hard to explain this magic cancellation from the "outside", i.e., from non-gravitational physics. Apart from anthropic-principle explanations, the best chance seems to lie [2] in some mechanism provided by gravity itself and, more particularly, by quantum gravity effects. It is to proposed mechanisms of this kind that I shall devote this lecture, starting from last year's craze, also known as "Colemania".

3. The Baum-Hawking-Coleman solution

There are basically two ingredients to the BHC solution [4,5,6] of the CC problem. The first is technical (and even controversial): it consists of a set of recipes defining Euclidean Quantum Gravity [7]. It will be momentarily assumed. The second ingredient is more physical: it has to do with the possible appearance of non-localities in Γ_{eff} as a result of non-perturbative topological effects, e.g., of wormholes.

Let us give a qualitative description of wormholes [8]. These are metrics (in some theories they are even classical solutions) which have the characteristics of connecting together two smooth (e.g., flat) manifolds. Examples are depicted in Fig. 1 where,

Fig. 1: A wormhole connecting (a) two flat Universes, (b) two distant parts of one Universe

respectively, a wormhole joins together two flat Universes or connects two distant regions of the same Universe.

Let us consider the latter case in more detail. The effect of wormholes of characteristic size (width) ρ_W , is to induce, at scales much larger than ρ_W , a non-local correction to the usual local action (2.2) in the form:

$$\delta\Gamma_W = -1/2 \int dx \sqrt{g(x)} \int dy \sqrt{g(y)} C(x, y) \sim -1/2 C V^2 + O(V)$$

$$V \equiv \int dx \sqrt{g(x)} \quad (3.1)$$

where one uses the important property that, for $|x - y| \gg \rho_W$, the wormhole con-

tribution approaches a constant C . The order of magnitude of $C(\rho_W)$ is controlled by the wormhole action in the typical form:

$$C \simeq \exp\left(-\frac{S_W}{\hbar}\right) \simeq \exp\left(-3\pi\left(\frac{\rho_W}{\ell_P}\right)^2\right), \quad \ell_P^2 = G\hbar \quad (3.2)$$

For wormhole sizes an order of magnitude larger than the Planck length ℓ_P this is a very small number. We shall see, however, the way its effects can be as important as to kill, or drastically reduce, the CC.

The "proof", following Ref. [9], goes as follows. Consider the so-called partition function Z_{NL} associated with the non-local action:

$$\Gamma_{NL} = \Gamma_L + \delta\Gamma_W; \quad \Gamma_L(\Lambda) = (16\pi G)^{-1} \int dx \sqrt{g} (2\Lambda - R) \quad (3.3)$$

where $\delta\Gamma_W$ has the form (3.1). Z_{NL} is defined by the (Euclidean) functional integral:

$$Z_{NL} = \int D[g_{\mu\nu}] \exp(-\Gamma_{NL}) \quad (3.4)$$

One can get rid of the non-locality of the action by the simple trick of rewriting:

$$\exp(-\delta\Gamma_W) = \int_{-\infty}^{+\infty} d\alpha \quad \exp\left(-\frac{\alpha^2}{2C} - \alpha V\right) \quad (3.5)$$

where we have momentarily assumed $C > 0$.

After exchange of the order of integration, Z_{NL} becomes a superposition of local-action partition functions with different values for Λ

$$Z_{NL} = \int d\alpha \quad \exp\left(-\frac{\alpha^2}{2C}\right) Z_L(\Lambda + 8\pi G\alpha);$$

$$Z_L(\Lambda) \equiv \int D[g] \quad \exp(-\Gamma_L(\Lambda)) \quad (3.6)$$

The question is: which values of $\Lambda_{eff} = \Lambda + 8\pi G\alpha$ dominate the integral? One can get easily an answer by assuming that the integral over the metrics in (3.6) is

dominated by the saddle point corresponding to the classical solution:

$$R_{\mu\nu} = \Lambda_{eff} g_{\mu\nu} \quad (3.7)$$

On this classical solution the local action takes the value

$$\Gamma_L = -\Lambda_{eff}(8\pi G)^{-1} \int dx \sqrt{g(x)} \quad (3.8)$$

It is clear that negative values of Λ_{eff} are disfavoured. For positive Λ_{eff} the minimal action solution is known to be the 4-sphere of radius $r = (\frac{3}{\Lambda})^{1/2}$ which gives:

$$\Gamma_L^{min} = -(8\pi G)^{-1} \Lambda_{eff} \frac{8}{3} \pi^2 \cdot r^4 = -\frac{3\pi}{G\Lambda_{eff}} \quad (3.9)$$

Thus finally:

$$Z_{NL} \simeq \int_{-\infty}^{+\infty} d\alpha \exp \left(\frac{-\alpha^2}{2C} + \frac{3\pi}{G(\Lambda + 8\pi G\alpha)} \right) \quad (3.10)$$

is completely dominated by the value of α corresponding to $\Lambda_{eff} = 0^+$. One can similarly prove [9] that any expectation value computed with the NL action by the usual path integral coincides with the one computed with a local action and $\Lambda = 0$. This looks almost too good to be true!

4. Criticism

There have been several criticisms of the above procedure and conclusions, e.g., on the way of computing expectation values of observables in Quantum Gravity [10]. Here I shall concentrate on a different class of objections [11,12,13] related to the physical interpretation of the singularity of the integrand in Eq. (3.10).

The reason why one can be suspicious about the naive BHC procedure is that one is dealing with ill-defined functional integrals. Besides the well known difficulties with integrating over the conformal factor [7], there is an additional problem brought in by the wormholes. If the wormhole constant C is positive, as assumed in Eq.(3.5), then the non-local action is unbounded from below for very large spherical Universes. The usually invoked Wick rotation of the conformal factor [7] can be easily argued [13] to be ineffective for this new divergence of the functional integral.

It is precisely the infinity of the non-local action which induces the singularity of the integrand in Eq. (3.10). Trusting the physics of that peak is making sense of the runaway vacuum of a theory with unbounded action, typically a dangerous game.

In order to clarify this point let us consider a simpler example of a theory with unbounded action at large distances: Einstein's action with a negative CC. Using formal manipulations similar to the ones employed with wormholes, we write:

$$Z(\Lambda) = \int_{\Lambda}^{\infty} d\alpha \int D[g_{\mu\nu}] \left(\frac{V}{8\pi G} \right) \exp \left((16\pi G)^{-1} \int dx \sqrt{g} (R - 2\alpha) \right) \\ \approx \int_{\Lambda}^{\infty} d\alpha \exp \left(\frac{3}{G\alpha} \right) \left\langle \frac{V}{8\pi G} \right\rangle_{\alpha} \quad (4.1)$$

where we have again used a saddle point to integrate over the metrics. We see that, if $\Lambda > 0$, Z is dominated by $\alpha = \Lambda = \Lambda_{eff}$, while, if $\Lambda < 0$, the integral is dominated by $\alpha = \Lambda_{eff} = 0^+$. We have thus apparently shown that a negative Λ implies a vanishing Λ_{eff} ! The conclusion is again too nice to be true and (obviously?) false. The error can be attributed to using a saddle point for integrating over the metrics and *not* for integrating over α . But this is precisely what we have done in the wormhole case!

In order to get further insight into the problem let us go back to Eq.(3.10), but looking now for saddle points in α . The saddle-point condition is simply[11,13]:

$$\alpha = -\frac{24\pi^2 C}{(\Lambda + 8\pi G\alpha)^2} \quad (4.2)$$

which is a cubic equation in α and thus has either one or three real solutions. The discussion is simplified by the assumption that the wormhole "coupling" C is small because of (3.2). We shall take C small to mean:

$$C \ll \Lambda^3 (192\pi^3 G)^{-1} \quad (4.3)$$

In this case there are two kinds of solutions:

$$A) \quad \alpha \simeq -\frac{24\pi^2 C}{\Lambda^2} \ll \frac{\Lambda}{8\pi G} \implies \Lambda \simeq \Lambda_{eff} \\ B) \quad \Lambda_{eff} = \Lambda + 8\pi G\alpha = \pm 8\sqrt{3}\pi^{3/2} \left(\frac{GC}{\Lambda} \right)^{1/2} \ll \Lambda \quad (4.4)$$

The B-type saddles are only real for $\Lambda > 0$ (recall that, for the moment, $C > 0$). Obviously, these are the saddles which can be interesting for quenching Λ . What do these saddles mean physically? The answer is quite obvious if we go back to

the non-local action we started from. Varying directly Γ_{NL} we obtain a non-local modification of Einstein's equations reading:

$$R_{\mu\nu} - 1/2 g_{\mu\nu} R = (-\Lambda + 8\pi G C V) g_{\mu\nu} \quad (4.5)$$

which has a positive CC solution:

$$R_{\mu\nu} = \Lambda_{eff} g_{\mu\nu} \quad (4.6)$$

with Λ_{eff} given by:

$$\frac{(\Lambda_{eff} - \Lambda)}{8\pi G} = -\frac{24\pi^2 C}{\Lambda_{eff}^2} \quad (4.7)$$

after expressing the volume itself through Λ_{eff} . We note that the equation for Λ_{eff} is again cubic: indeed, it coincides with (4.2) after we identify Λ_{eff} with $\Lambda + 8\pi G\alpha$.

The nice thing about having gone back to Γ_{NL} is that the saddle points now have a clear, classical interpretation. By plotting Γ_{NL} as a function of V we can also see (Fig. 2a) that saddle B corresponds to a local *maximum* of Γ_{NL} . We can also conclude that, at least for the saddle points, the Euclidean Quantum Gravity machinery was not really necessary.

What about the $\Lambda_{eff} = 0$ result of BHC? That result *cannot* be obtained by a classical argument. Instead, as is quite obvious from Fig. 2a, it corresponds to the fact that Γ_{NL} is equal to $-\infty$ at $V = \infty$ if $C > 0$! The BHC result thus comes from the assumption that, quantum mechanically, the classical (de Sitter or anti de Sitter) vacuum becomes unstable and rolls away to $V = \infty$ or $\Lambda_{eff} = 0$.

We also understand now the case of $\Lambda < 0$ and no wormholes presented earlier. There too we could envisage the possibility that quantum corrections drive us away from the classical vacuum into the configuration of infinite 4-volume and vanishing CC.

Two attitudes are possible:

- i) either one accepts theories with unbounded actions and exploits the resulting instabilities; or
- ii) one works with cases in which the above instabilities are either absent or are avoided by some suitable definition of the quantum theory (i.e. of the path integral). The latter is the attitude generally adopted, for instance, in dealing with the conformal factor instability [7].

In the rest of this lecture I shall stick to this second, more conservative attitude without claiming, however, that the former is necessarily wrong. I only wish to add that other results obtained from Coleman's approach on natural constants other than Λ , crucially depend on taking seriously the infinite action instability and have no analog in the scenario that we shall now follow.

Fig. 2: Non-local Euclidean action as a function of the 4-volume V for three choices of the signs of C and Λ

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5. Quenching Λ by wormholes of the “wrong” coupling

Let us consider again the non-local action (3.3) but this time with a negative value for C . The situation can be analyzed as before either by introducing the auxiliary wormhole variable α , or, more directly, by looking for the stationary points of the non-local action. If we proceed in this second fashion we see immediately (Figs. 2b,c) that:

- i) the non-local action is now bounded from below (at least in the direction of large, smooth manifolds) and
- ii) the nature of the saddle points depends upon the sign of Λ . If $\Lambda > 0$ (Fig. 2b), there is only a real saddle corresponding to the usual de Sitter Universe with $\Lambda_{eff} = \Lambda$. Thus, in this case, the wormhole effect is negligible and inflation in the early Universe is maintained. If instead $\Lambda < 0$ (Fig. 2c), a new saddle appears (with lower Euclidean action) corresponding to:

$$\Lambda_{eff} = 8\sqrt{3}\pi^{3/2}(GC/\Lambda)^{1/2} \quad (5.1)$$

This is precisely the saddle point described before, corresponding to a huge (for very small $|C|$) quenching of the CC. We shall discuss in a moment the conditions under which this quenching can be phenomenologically sufficient.

What happens in the wormhole-variable language for $C < 0$? In particular, how is the BHC effect avoided? Referring to [11] for details, I shall just mention that, for $C < 0$, the α integration should be done on the imaginary, rather than on the real axis (i.e., via a Fourier transform). The BHC singularity is now off the integration contour and does not cause any infinity for the partition function Z_{NL} . The latter is instead dominated by a saddle point which leads exactly to the predictions described above.

The situation is nicely summarized in Hawking’s own words [13] as:

If $C > 0$, the integral over the metrics diverges; if $C < 0$, the integral over the α ’s diverges.

In the former case, the BHC instability takes place and, if one can make sense of it, the cosmological constant will vanish (and other constants of nature will be similarly determined too).

In the second case, the integral over the α ’s is simply to be rotated and everything is well defined. The BHC instability is avoided and so is, unfortunately, the prediction

that $\Lambda_{eff} = 0$. It is still possible, however, to depress in this case a *negative* CC, through the saddle point (4.4).

In order to see under which conditions such a suppression is sufficient, let us rewrite (4.4) in the form:

$$G\Lambda_{eff} = (G\Lambda)^{1/2}(CG^2/\Lambda^2)^{1/2}(8\pi\sqrt{3}\pi^{3/2}) \quad (5.2)$$

From the experimental bound (2.6) we get immediately:

$$(CG^2/\Lambda^2) < 10^{-248}/G\Lambda \quad (5.3)$$

where we can expect $G\Lambda$ to lie anywhere in the range $10^{-80} - 1$. As explained at the beginning, the smallness of C is controlled by the minimal wormhole action, so that we may assume, as an order of magnitude:

$$(CG^2/\Lambda^2) = \exp(-\hbar^{-1}S_W^{min}) = \exp(-3\pi(\rho_W^{min}/\ell_P)^2) \quad (5.4)$$

It is easy to see that the experimental bound is satisfied for a minimal wormhole size 6 to 8 times larger than ℓ_P . We thus see that the success of the scheme depends on the short distance behaviour of quantum gravity or, if we prefer, on the nature of its short distance cutoff.

Probably quantum gravity does not make sense without an ultra-violet cut-off anyway. An example of a cut-off gravity theory is provided, as we shall see in the second lecture, by string theory, which possesses a fundamental scale of its own:

$$\lambda_s = (\alpha\hbar)^{1/2} \quad (5.5)$$

If, as suggested from several points of view [14,15,16], λ_s plays the role of a minimal observable scale in string theory, then it will also provide a minimal scale for topological fixtures such as wormholes. In this case, after use of the string-unification formula [17]

$$\lambda_s^2/\ell_P^2 = 16/\alpha_{GUT} \quad (5.6)$$

one obtains the amusing result:

$$\exp(-\hbar^{-1}S_W^{min}) \simeq \exp(-48\pi/\alpha_{GUT}) \quad (5.7)$$

which shows on one hand that the suppression of Λ_{eff} is exponential in the inverse fine structure constant (cf., instanton effects) and, on the other hand, that it is numerically sufficient for an α_{GUT} lying approximately in the experimentally favoured range.

Before claiming victory, however, we still have quite a few problems to address:

1. The sign of C that we need does not seem to be the one that follows from either intuitive arguments [18] or from model calculations[13].
2. Adding more wormhole-connected universes "dilutes" the quenching effect by the average number of mutually connected Universes [11]. In the simplest scenario this number is so huge that the final suppression is only logarithmic in C and this renders the hope of suppressing sufficiently Λ almost vain.
3. Finally, the whole scheme is based on the poorly understood physics of non perturbative effects in euclidean quantum gravity.

In view of the above, I shall turn, in the last part of this talk, to an alternative mechanism which, while employing similar ideas to the ones used until now, is based on the more conventional and better understood physics of perturbative radiative corrections as a way to induce a change in the classical vacuum of the theory, the well-known Coleman-Weinberg (the "other" Weinberg and, I should add, the "other Coleman's") mechanism.

6. Quenching Λ a-la-Coleman-Weinberg

Let us briefly recall the standard Coleman-Weinberg (CW) mechanism [19] for dynamical symmetry breaking. There one considers, for instance, a massless $\lambda(\phi^*\phi)^2$ theory coupled to electromagnetism. At tree level the scalar potential has a stable, absolute minimum at $\phi = 0$ and the gauge symmetry is unbroken. Under certain restrictions on the coupling λ and of the gauge coupling, CW argued that the one-loop corrected effective potential reliably describes quantum corrections to the tree-level classical picture and that the new scalar potential acquires a term:

$$V_1 = \beta(\phi^*\phi)^2 \log(\phi^*\phi/\mu^2) \quad (6.1)$$

where $\beta > 0$ is a known function of the renormalized couplings and μ is the subtraction point. Obviously, V_1 becomes negative near $\phi = 0$ making the tree level vacuum unstable. ϕ gets a non-zero expectation value and the gauge symmetry is broken through a dynamical Higgs mechanism (the so-called CW mechanism).

How can something like this work in the case of gravity and of the CC? Let us take the tree level action to be the usual:

$$\Gamma_{tree} = (16\pi G)^{-1} \int dx \sqrt{g(x)} (2\Lambda - R) \quad (6.2)$$

and let us try to guess a one-loop correction that might do the job. Later we shall see whether or not such a correction does come out of a bona fide calculation.

The toy-model one-loop correction to (6.2) is taken [20] to be of the form:

$$\Gamma_1 = -\gamma\Lambda^2 \int \sqrt{g(x)} \log\left(\frac{\lambda_{UV}^2}{\lambda_{IR}^2}\right) dx \quad \gamma > 0 \quad (6.3)$$

where λ_{UV} is the UV cut-off needed to make sense out of quantum gravity calculations (e.g., λ_s in the case of string theory) while the much more important quantity λ_{IR} is a physical, $g_{\mu\nu}$ dependent IR cut-off that the theory itself should determine in terms of large distance physics. Our ansatz (6.3) is quite similar to the CW expression with λ_{IR} and λ_{UV} replacing $\phi^*\phi$ and μ^2 respectively. The fact that the one-loop correction to (6.2) has a logarithmic UV divergence proportional to the cosmological term with a coefficient proportional to Λ^2 is already known [21]. Thus our ansatz is not so wild... until we make an assumption on what one should write for λ_{IR} . The toy model of Ref. [20] is defined by the identification:

$$\lambda_{IR} = V^{1/4} \quad (6.4)$$

and thus has a total (non-local) action:

$$\Gamma_{eff} = \Gamma_{tree} - \gamma\Lambda^2 V \log(\lambda_{UV}^2 V^{-1/2}), \quad \gamma > 0 \quad (6.5)$$

The stationary points of (6.5) are given by:

$$R_{\mu\nu} = \Lambda_{eff} g_{\mu\nu} \quad (6.6)$$

where:

$$\frac{\Lambda_{eff} - \Lambda}{8\pi G} = -\gamma G \Lambda^2 \log(\lambda_{UV}^2 \Lambda_{eff}) \quad (6.7)$$

(6.7) is a transcendental equation for Λ_{eff} whose qualitative solutions are easily found graphically: in analogy with the wormhole case with $C < 0$, one finds that, for $\Lambda > 0$, $\Lambda_{eff} \simeq \Lambda$ while, for $\Lambda < 0$, Λ_{eff} is positive and suppressed, this time exponentially:

$$\Lambda_{eff} = \lambda_{UV}^{-2} \exp(-1/G|\Lambda|) \quad (6.8)$$

This damping is quite sufficient for $\lambda_{UV} \geq \ell_P$ and $G|\Lambda| < 0(10^{-3})$, i.e., for an easy-to-achieve set of parameters.

How about the true one-loop calculation? This is not so trivial but can be done [22] (under standard assumptions). The one-loop effective action takes, as usual, the form of the *trlog* of the operator Ω of quadratic fluctuations around the chosen metric at which the effective action is computed. There are complications due to the necessity of introducing:

- i) gauge fixing terms,
- ii) the corresponding ghost fields and, finally,
- iii) the De Witt-Vilkoviski [23] improvement needed to preserve the general co-ordinate invariance of the result.

The $tr log \Omega$ is then computed via the standard heat-kernel method as:

$$tr log \Omega = Tr \int_{\tau_{min}}^{\infty} d\tau \exp(-\tau \Omega) = \int dx \sqrt{g(x)} \int_{\tau_{min}}^{\infty} d\tau \langle x | \exp(-\tau \Omega) | x \rangle \quad (6.9)$$

In this formulation the UV-cut-off dependence comes from the small τ region of integration after identification of τ_{min} with λ_{UV}^2 . The small τ expansion of $\langle x | \exp(-\tau \Omega) | x \rangle$ is well known [21] and leads to a bunch of local operators multiplying different powers (or logarithms) of λ_{UV} .

As already argued, the IR cut-off should be intrinsic, i.e., related to the manifold on which we are performing the calculation. Obviously, the large τ behaviour is related to the spectrum of small eigenvalues of Ω , a rather "classical" mathematical problem. Somehow this spectrum depends on global properties of the manifold such as its size and shape. In the words of Mark Kc [24] one is "hearing the shape of a drum!" This is how non-locality creeps into the evaluation of Γ_1 . Note that the non-locality of quantum correction was already pointed out by Bryce De Witt as early as in 1967 [25].

The exact result is not known for general manifolds. It is, however, calculable for spheres and arguments can be given to extend the estimate to arbitrary, large and smooth enough manifolds for which boundary effects should not play a major role. The bottom-line result obtained in Ref.[22] reads:

$$\Gamma_{1 \text{ loop}} \simeq \frac{3\Lambda^2}{8\pi^2} V \log(\lambda_{UV}^2 M^2);$$

$$M^2 \equiv \max\{|\Lambda|, V^{-1/2}\} \quad (6.10)$$

which is *almost* what we had in the toy model (6.5). However, the difference between what we need and what we got is crucial. Even leaving out the fact that we got the "wrong" sign in front of the logarithm – something that could change, for instance, in supergravity – the disturbing fact remains that the argument of the log is not just V but contains the bare CC too. It is easy to see that, as a result, the logarithm can never grow large enough to appreciably change the tree level classical solution.

At a closer look, we see that the bare CC plays the role of IR cut-off much like a mass in ordinary theories. We thus lose the CW effect very much in the same way as CW would if they introduced a mass for the scalar field. This fact tells us, on one hand, that our perturbative approach may be doomed and, on the other, it may suggest a new possibility.

A positive (negative) CC looks indeed like a positive (negative) squared mass, not only in the way it regulates the IR but also in the way it controls the asymptotic behaviour of the action at large, slowly varying fields. For positive (negative) Λ the Euclidean action is bounded (unbounded) from below for large spheres (so large that the curvature term can be neglected). This has no implication classically, since there is no stationary point of the action out there. However, at the quantum level, the situation, for negative Λ , is very much the same as that of a false vacuum [26] which, non-perturbatively, can tunnel and decay.

Does then quantum gravity make sense at all for negative CC? Does one get into a runaway situation of instability which would resemble the one we have encountered with wormholes, or do quantum effects recover the sick theory through a process of quantum resuscitation [27]? And, in this latter case, is one led to a new vacuum with a drastically different (and hopefully smaller) effective CC?

Whatever the answer, it looks to me that the abrupt, qualitative change of the theory as Λ goes through 0 – a sort of phase transition – should have something to do with the eventual resolution of one of the most out (and long)-standing challenges that Nature has left to us.

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