Demonstration of a Tunable-Bandwidth White Light Interferometer using Anomalous Dispersion in Atomic Vapor

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Abstract

Recently, the design of a white-light-cavity has been proposed using negative dispersion in an intra-cavity medium to make the cavity resonate over a large range of frequencies and still maintain a high cavity build-up. This paper presents the first demonstration of this effect in a free-space cavity. The negative dispersion of the intra-cavity medium is caused by bi-frequency Raman gain in an atomic vapor cell. A significantly broad cavity response over a bandwidth greater than 20 MHz has been observed. A key application of this device would be in enhancing the sensitivity-bandwidth product of the next generation gravitational wave detectors that make use of the so-called signal-recycling mirror.

PACS numbers: 42.5, 42.50.Ct, 42.50.Gy, 42.60.Da
Optical cavities with high reflectivity mirrors are now used for signal recycling in advanced gravitational wave interferometers (such as GEO 600 and the Advanced LIGO) [1-3]. This resonantly enhances the extremely small side-band (SB) amplitudes of gravity waves. The recycling improves the shot-noise limited sensitivity at SB frequencies due to the signal build-up inside the cavity. While the GEO 600 does not use arm-end cavities, under the Advanced LIGO program the signal recycling mirror is going to be added to a system that has arm-end cavities, yielding the same degree of improvement [4,5]. However, this improvement comes at the cost of reduced detection bandwidth.

In standard optical cavities like the ones described above, there is always a trade-off between the cavity build-up and the bandwidth. Frequency responses of the gravitational interferometers [6] containing recycling cavities have to be optimized for different types of sources such as bursts, chirps from coalescing binaries, stochastic backgrounds and continuous signals. While detecting gravity waves, one would ideally like to maximize the build-up without compromising the bandwidth, for stellar events that emit gravity waves spread over a range of frequencies (up to a few KHz). It is possible to design optical cavities with a high build-up factor but without the inverse loss of bandwidth by tailoring the path length of the cavity as a function of frequency such that the round-trip phase shift remains an integer multiple of $2\pi$. This kind of frequency-independent path length change can be achieved either by using a negative dispersion intra-cavity medium with dispersion slope corresponding to a group index value close to zero or by using a pair of diffraction gratings inside the cavity [7-11]. Such a cavity can simultaneously resonate over a range of frequencies without sacrificing the cavity build-up, and is called a White-Light Cavity (WLC).

Proposals to construct such cavities using atomic phase coherence include strongly driven double-$\Lambda$ or double-gain systems that can provide controllable negative dispersion between gain or transparent line centers without absorption [7-9]. A gain based intra-cavity dispersive medium is usually preferred since the residual gain can compensate for other losses in various optical elements inside the cavity. This technique also has a fundamental advantage over any gratings based approach, as the high resolution grating elements could be significantly lossy [10]. A recent study also showed that it is impossible to construct a WLC using a pair of gratings since diffraction from gratings also gives rise to wavelength-dependent phase shifts that nearly cancels the phase from the additional path length generated by the gratings [11]. In another development, recently a small high-Q ($>10^9$) white-light resonator with an octave spanning bandwidth has been realized by using a dense and continuous (both longitudinal and transverse) mode spectrum of a whispering gallery mode resonator [12]. Although these have a great potential to be used in ring-down spectroscopy, sensing and modulation applications, the waveguide resonators are incompatible with free-space gravitational wave laser interferometers due to poor light coupling.
and throughput. They also have a non-tunable resonance condition fixed after fabrication. In contrast, if the double-gain technique is employed, the cavity bandwidth can be tuned as needed by changing the gain separation and adjusting the gain parameter. Another advantage of the double-gain technique is that the gain in resonant excitations can be observed at different wavelengths in a wide variety of materials, including inorganic and semi-conducting photorefractive materials. Therefore, the technique can be extended to different operating wavelengths, for example, $\lambda = 1.064 \mu m$ in LIGO, with the proper choice of the material medium. A potential disadvantage with the double-gain technique is that the presence of gain in the cavity can lead to undesirable self-oscillation and unstable behavior. However, these can be easily avoided by operating under a small gain regime, as exemplified by our demonstration of the WLC reported here.

In this paper, we present the details of the experimental realization of a WLC using bifrequency Raman gain in an intra-cavity atomic medium in our laboratory. Our experiment demonstrates the feasibility of controlling the dispersion slope such that the dispersion can exactly compensate for the frequency dependence of vacuum wavelength and produce a WLC. The effect of dispersion induced by gain lines on the resonator response close to the WLC condition is also simulated and a close agreement with the experimental result is obtained. In order to interpret the experimental results, we first present a simple model summarizing the essential aspects of the WLC.

Consider an optical cavity of path length $L$, containing a medium of physical length $\ell$. The effect of medium dispersion on cavity response can be easily predicted by considering the expression for the round-trip phase shift at a frequency $\omega = \omega_o + \Delta \omega$ away from the cavity resonance frequency $\omega_o$ given by:

$$\phi(\omega_o + \Delta \omega) = (\omega_o + \Delta \omega) \frac{(L - \ell)}{c} + (\omega_o + \Delta \omega) n(\omega_o + \Delta \omega) \frac{\ell}{c}$$

$$\approx N (2\pi) + \left[ \frac{(n_g - 1)}{L} + 1 \right] \frac{L}{c} \Delta \omega + O\left(\Delta \omega^2\right) + \ldots, \quad n_g = 1 + n_1, \omega_o \quad (1)$$

where $n_1$ is the first-order dispersion slope in the Taylor expansion of the medium refractive index $n(\omega)$ at $\omega = \omega_o$, $N$ is a large integer number, and $n_g$ is the group index. As can be seen from eqn.1, it is possible to choose $n_1$ (by controlling the dispersion slope) such that $\phi(\omega)$ becomes independent of frequency with respect to a first-order change, $\Delta \omega$, in the frequency. This happens for a value of $n_1 = -(1/\omega_o)(L/\ell)$, which corresponds to $n_g = 1 - L/\ell$, a small negative dispersion slope dictated by the ratio $(L/\ell)$. In a situation where the medium completely fills the entire cavity length, this will correspond to $n_1 = -1/\omega_o$ or a zero (null) value of $n_g$, known as the ideal white-light condition. Under this condition, many frequencies around $\omega_o$ will resonate simultaneously in the cavity, as the round-trip phase shift becomes independent of a first-order
change in frequency. Thus, the cavity response gets broadened without sacrificing the cavity build-up and a WLC can be realized. Fig. 1 shows an example of white-light cavity response generated by considering a cavity with an internal cavity build-up of O(10^3) and containing a gain medium with ratio (L/\ell) = 10. Negative dispersion with a slope corresponding to n_2 \approx -9 is produced by considering narrow resonant gain lines (linewidth \sim 1 MHz) with a frequency separation of 7.95 MHz. A broad cavity resonance under the white-light condition is observed without sacrificing the build-up factor shown along the vertical axis.

As discussed earlier, although there is no dephasing in the cavity response away from \omega_o to first-order in \Delta\omega, dephasing due to higher-order terms causes the cavity response to drop. This eventually limits the linewidth of the WLC to a finite value. One can evaluate the WLC linewidth (\gamma') in comparison to the empty cavity from the following relation:

\[
\frac{\gamma'}{\gamma} = \beta \frac{1}{1 + \left(\left(n_g - 1\right) + n_3 \omega_o (\gamma')^2\right) \frac{\ell}{L}}, \quad \beta = \frac{\sin^{-1}\left(\frac{1 - R}{2\sqrt{R\rho}}\right)}{\sqrt{1 - \frac{1 - R}{2R}}}, \quad n_3 = \frac{1}{6} \left. \frac{\partial^3 n}{\partial \omega^3} \right|_{\omega_0 = \omega_o}
\]

where \gamma is the empty cavity linewidth, and \rho = \exp(-\alpha\ell), where \alpha is the overall loss coefficient that causes additional linewidth broadening in the presence of the medium. In deriving the expression, the magnitude of dephasing \left| n_1, (\Delta\omega)^2, (\ell/L) \right| due to (\Delta\omega)^2 is considered small in comparison to the third-order dephasing term \left| n_1, \omega_o, (\Delta\omega)^3, (\ell/L) \right|. Note also that the dephasing contribution due to the second-order dispersion coefficient n_2 is ignored, since the characteristic dispersion profile associated with any kind of atomic resonances is anti-symmetric around the resonant frequency \omega_o. The linewidth of the WLC under ideal white-light condition (n_g = 1-L/\ell) can be estimated from eqn. 2 as \gamma' = \left(2(\beta\gamma L)/(n_3 \omega_o \ell)\right)^{1/3}.

If the intra-cavity negative dispersion is generated using Raman gain doublets in an atomic medium, the third-order slope (n_3) at the center frequency \omega_o between the gain lines can be approximated as \nu_3 = -n_1/\Gamma^2 = (1/\omega_o \Gamma^2), (L/\ell), where \Gamma corresponds to the frequency separation between the gain lines. In this case, the linewidth of WLC can be simplified to \gamma' = \left[2\beta\gamma \Gamma^2\right]^{1/3}.

Thus, a small medium length \ell (or a large ratio (L/\ell)) implies that a large, negative slope is needed for the white-light condition, while the bandwidth of the WLC remains unchanged. These analytic conclusions have also been verified via computer simulations, as described later on. The expression also implies that in order for the white light effect to be evident, the frequency separation \Gamma between the gain peaks should be much larger than the empty cavity linewidth \gamma. For the example considered in fig.1, if we substitute the values of \gamma \sim 1 MHz, \Gamma = 7.95 MHz (>> \gamma) and assume \beta = 1, the linewidth of the WLC is estimated to be 3.16 MHz. Of
course, for applications to the LIGO system, we can simply design the values necessary for the desired bandwidth.

Transparent spectral gain regions and the negative dispersion associated with gain doublets have been demonstrated in earlier experiments [13-16]. On the other hand, our experiment requires extremely small value of linear negative dispersion slope, typically $O(10^{-15})$ rad$^{-1}$ sec (equivalently, a small value of $n_e$) in order to satisfy the white-light condition. This condition has been achieved experimentally under low gain. In order to observe an appreciable white-light effect over a significant bandwidth, the frequency separation between the gain lines has been set larger than our actual cavity linewidth.

The experimental setup for the white-light cavity is shown in fig. 2a. A 10 cm long Rubidium vapor cell is placed inside a cavity as an intra-cavity gain medium. A 100 cm long ring cavity has been used for our study. The cavity consists of four mirrors with two partially transmitting plane mirrors that generate pairs of input and output ports, and two concave mirrors that allow cavity mode-matching. The empty cavity finesse is measured to be 100 and the cavity linewidth is measured to be about 3 MHz. One of the concave mirrors in the cavity is attached to a piezoelectric transducer (PZT). This is used in frequency-locking the cavity. One pair of cavity input and output ports was used to lock the cavity length at a desired resonant frequency.

The probe and pump beams used in the experiment are obtained from a CW Ti:Sapphire laser (line width ~ 1 MHz). Figure 2b shows the $5S_{1/2}, F=2$ and $F=3$ ground states and $5P_{3/2}$ excited state manifolds on the D2 line in Rb$^{85}$ that constitute a Λ-type configuration for Raman excitation. An incoherent optical pump from a diode laser is used to create a population inversion between the ground states. The optical pump frequency is locked to the $5S_{1/2}, F = 2$ to $5P_{3/2}, F' = 3$ transition in Rb$^{85}$. All the beams to the system are delivered using optical fibers. The probe beam, which is p-polarized, resonates inside the cavity.

Two pump beams of different frequencies are injected into the cavity to generate closely spaced gain lines. These pump beams are derived using two different acousto-optic modulators (AOMs). The negative dispersion is observed in the intermediate region between the gain lines. The Raman pumps and the optical pump beams, polarized orthogonally to the probe, are all combined with the probe beam inside the cavity in the co- and counter-propagating directions, respectively, using two intra-cavity polarizing beam splitters (PBS’s) before and after the cell (fig. 2a). The frequency difference between the probe and the center of the pump lines is initially matched to the ground state splitting in Rb$^{85}$ (3.0357 GHz). The gain, peaking at two different probe frequencies, is observed by scanning the probe frequency around the center frequency of the pump lines using an AOM set in a double-pass configuration.

The optical pump beam does not completely overlap (stays nearly 3 to 4 mm away) on the probe beam while observing the Raman gain. This is to ensure that the moving atoms get optically pumped before entering the probe beam. Raman gain with a large gain coefficient and sub-natural linewidth (~ 1 MHz) has been observed using a medium with modest atomic density [$N \sim O(10^{15})$] and small ground-state dephasing by introducing a flipper mirror in the cavity.
The magnetically shielded vapor cell is heated to nearly a steady temperature of 60°C using bifilarly wound coils that produce a negligible axial magnetic field.

For optimum gain, the average frequency of the probe and the pump fields is detuned below Doppler resonance where the probe does not get significantly absorbed in the cell even in the absence of the pump beams. The cavity is made resonant at the selected probe frequency $\omega_0$ by adjusting its length $L$ simply by tuning the drive voltage to the PZT mirror. The cavity length is then actively locked using a servo signal generated from the cavity output produced by a resonating locking beam sent through the other cavity port (fig. 2). The frequency of the lock beam is set at many multiples of cavity free spectral range (FSR) away from the probe frequency $\omega_0$.

We first measured directly the dispersion as seen by the probe field under the double-gain condition, using a heterodyne technique. During this measurement, the cavity resonance was interrupted by placing a flipper mirror in the probe beam path inside the cavity. For heterodyne measurements, an auxiliary reference wave was produced by frequency shifting a fraction of the probe beam outside the cavity using a 40 MHz AOM. A low noise rf mixer and a low-pass frequency filter were used to demodulate the rf signal from the detectors that allowed us to measure accurately the dispersion in the atomic medium under the gain resonances. Very small negative dispersion slopes ($n_1 \sim 3 \times 10^{-16} \text{ rad}^1 \text{ sec}$) have been achieved with gain as low as 3 dB (or smaller). When such a low gain condition prevails inside the cavity, it also avoids other undesirable effects such as self-oscillation and bistable behavior that are normally observed in the cavity response in the presence of a stronger gain medium.

The effect of anomalous dispersion on the cavity resonance is observed by scanning the probe frequency around the cavity resonance ($\omega_0$), while the cavity length is actively locked. Fig. 3 shows a sequence of cavity resonances for different frequency separations between the gain lines. Each time with an increasing frequency separation, the gain was adjusted by controlling the pump intensity to observe a white-light effect in probe transmission. Although broadened cavity response can be clearly seen from the data, the uniformity of the response is considerably degraded with increasing gain separation. This is due to our limitation in optimizing the gain required to reach the white light condition ($n_g = 1 - L/\ell$). However, in future experiments, this constraint can be circumvented using different experimental parameters such as the atomic density, the cell length, and the bandwidths of the AOMs used for generating the pumps. The cavity line-width (Fig. 3a) in the absence of gain (pumps turned off) is found to be approximately 8 MHz, which is nearly three times wider than the empty cavity line-width. This is due to extra loss introduced by intra-cavity elements and residual medium absorption. We also see that probe transmissions near white-light conditions are somewhat lower than the peak cavity transmission in the absence of gain. This could possibly be taken to imply that the cavity build-up factor is reduced as the bandwidth is increased. However, this is not the case, since the cavity transmission remains nearly the same when the white-light effect is observed with increasing
gain separations, and therefore increasing bandwidths. We attribute this reduction in peak transmission to possible additional absorption loss in probe transmission caused by optical pumping effects resulting from detuned Raman pumps. These constraints can be eliminated in future experiment by using different pumping schemes so that the probe will not see any additional absorption between the gain lines. Non-uniform response in the cavity transmission similar to the experimental results in Fig. 3 has also been observed in simulating our model when the dispersion slope differs from the exact white-light condition. Fig. 4 shows a comparison of the measured linewidth for white-light response from our experimental data with the estimated linewidth from our model. They agree reasonably well within the measurement error bars.

In a separate study, we have also demonstrated experimentally the effect of positive intracavity dispersion corresponding to slow light. For this case, we have also shown how the system becomes less sensitive to perturbations [17]. Conversely, we have shown how the WLC is more sensitive to perturbations, and how this property may be used to enhance the sensitivity of absolute rotation measurement using a ring laser gyroscope [18].

The WLC described above was demonstrated using light at 780 nm, and rubidium atoms. The idea can be easily extended to Zeeman sublevels within these transitions to realize an effective three-level system, which in turn can be used to demonstrate the type of WLC described above. One example is the Zeeman sublevel based EIT we had demonstrated earlier [19]. Another example is the electromagnetically induced transparency observed in Neon gas [20]. Thus, it may easily be possible to find vapor based transitions that would yield a WLC at 1064 nm, as required for the LIGO system. Alternatively, it is possible to realize such a WLC using non-linear wave mixing in photorefractive media [21].

In conclusion, we have experimentally demonstrated the effect of anomalous dispersion induced by an intra-cavity gain medium for realizing a white-light cavity. The origin of this effect is understood by including the medium dispersion properties in the cavity transfer function. The technique can be perfected to construct white-light cavities with performance criteria meeting the potential requirements for high-sensitivity, broadband detection of gravity waves. This work was supported in part by the Hewlett-Packard Co. through DARPA and the Air Force Office of Scientific Research under AFOSR contract no. FA9550-05-C-0017, and by AFOSR Grant Number FA9550-04-1-0189.
References


Fig. 1 Cavity response shows white-light effect for $n_g = -9$ obtained using gain-doublets corresponding to $\Gamma = 7.95$ MHz. The cavity buildup (x 2000) with WLC is maintained.
Fig. 2  (a) Schematic of the experimental set-up for the white-light cavity (b) Energy diagram showing detuned Raman excitation in D2 line $^{85}$Rb to produce bi-frequency gain at probe frequencies.
Fig. 3 Experimental results showing broadened cavity response in presence of gain doublets with varying gain separation.
Fig. 4  Comparison between the estimated white-light cavity linewidth and the actual linewidth measured from experimental data