Demonstration of a Tunable-Bandwidth White Light Interferometer using Anomalous Dispersion in Atomic Vapor

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Abstract

The concept of the “white-light cavity” has recently generated considerable research interest in the context of gravitational wave detection. Cavity designs are proposed using negative (or anomalous) dispersion in an intracavity medium to make the cavity resonate over a large range of frequencies and still maintain a high cavity build-up. This paper presents the first experimental attempt and demonstration of white light effect in a meter long ring cavity using an intracavity atomic medium. The medium’s negative dispersion is caused by bi-frequency Raman gain in an atomic vapor cell. Although the white light condition was not perfectly achieved and improvements in experimental control are still desirable, significantly broad cavity response over bandwidth greater than 20 MHz has been observed. These devices will have potential applications in new generation laser interferometer gravitational wave detectors.

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Optical cavities with high reflectivity mirrors are now used for signal recycling in advanced gravitational wave interferometers (such as GEO 600) [1-3]. This resonantly enhances the extremely small (nearly 20 orders of magnitude smaller than carrier amplitude) side-band (SB) amplitudes of gravity waves. The recycling improves the shot-noise limited sensitivity at SB frequencies due to the signal buildup inside the cavity. This is an improvement on the current LIGO interferometers, which use narrowband arm-end cavities that yield larger signal due to strain induced by gravity waves on longer effective arm lengths [4,5]. While the GEO 600 does not use arm-end cavities, the signal recycling mirror can also be added to a LIGO system that has arm-end cavities, as shown in figure 1 above, yielding the same degree of improvement. However, this improvement comes at the cost of reduced detection bandwidth.

In standard optical cavities like the ones described above, there is always a trade-off between cavity buildup and bandwidth. Frequency responses of the gravitational interferometers [6] containing recycling or end cavities have to be optimized for different type sources such as bursts, chirps from coalescing binaries, stochastic backgrounds and continuous signals in detection. While detecting gravity waves, one would ideally like to maximize the buildup without compromising the bandwidth, for these stellar events that emit gravity waves spread over a range of frequencies (up to a few KHz). It is possible to design optical cavities with high buildup without inverse loss of bandwidth by tailoring the path length of the cavity as a function of frequency such that the round-trip phase shift remains integer multiple of $2\pi$. This kind of frequency-independent path length change can be achieved either by using a negative dispersion intra-cavity medium with dispersion slope corresponding to a group index value close to zero or by using a pair of diffraction gratings inside the cavity [7-11]. Such a cavity can simultaneously resonate over a range of frequencies without sacrificing the cavity buildup, and is called a White-Light Cavity, as mentioned above.

Proposals to construct such cavities using atomic phase coherence include strongly driven double-$\Lambda$ or double-gain systems that can provide controllable negative dispersion between gain or transparent line centers without absorption [7-9]. A gain based intra-cavity dispersive medium is usually preferred as the residual gain can compensate for other losses (mainly frequency-independent) in various optical elements inside the cavity. This technique also has a fundamental advantage over any gratings based approach, as the high resolution grating elements could be significantly lossy [10]. A recent study also showed that it is impossible to construct a WLC using a pair of gratings since diffraction
from gratings also gives rise to wavelength-dependent phase shifts that nearly cancels the phase from the additional path length generated by the gratings [11]. In another interesting development, recently, a small high-Q (> 10^9) white-light resonator with octave spanning bandwidth has been realized by using dense and continuous (both longitudinal and transverse) mode spectrum of a whispering gallery mode resonator [12]. Although they have great potential to be used in ring-down spectroscopy, sensing and modulation applications, the waveguide resonators are absolutely incompatible with free-space gravitational wave laser interferometers due to extremely poor light coupling and throughput. They also have a non-tunable resonance condition fixed after fabrication. In contrast, if the double-gain technique is employed, the cavity bandwidth can be tuned as needed by changing the gain separation and adjusting the gain parameter. Another advantage of the double-gain technique is that the gain in resonant excitations can be observed at different wavelengths in a wide variety of materials, including inorganic and semiconducting photorefractive materials. Therefore, the technique can be extended to different operating wavelengths, for example, λ=1.064 µm in LIGO, with the proper choice of the material medium. A potential disadvantage with the double-gain technique is that the presence of gain in the cavity can lead to undesirable self-oscillation and unstable behavior. However, these can be easily avoided by operating under small gain regime, as exemplified by our recent demonstration of the WLC.

We now describe in detail the experimental realization of a WLC using bi-frequency Raman gain in an intra-cavity atomic medium in our laboratory. Our experiment demonstrates the feasibility of controlling the dispersion slope such that the dispersion can exactly compensate for the frequency dependence of vacuum wavelength and produce a WLC. The effect of dispersion induced by gain lines on the resonator response close to the WLC condition is also simulated and a close agreement with the experimental result is obtained. The following discussions present a simple mathematical background to understand the white-light condition necessary to realize a WLC.

Consider an optical cavity of path length L, containing a medium of physical length ℓ. The effect of medium dispersion on cavity response can be easily predicted by considering the expression for round-trip phase shift at a frequency \( \omega = \omega_o + \Delta \omega \) away from the cavity resonance frequency \( \omega_o \) given by:
\[
\phi(\omega_0 + \Delta \omega) = (\omega_0 + \Delta \omega) \left( \frac{L - \ell}{c} + \frac{\ell}{c} \right) + n(\omega_0 + \Delta \omega) \cdot \frac{\ell}{c}
\]

\[
\approx N(2\pi) + \left[ n' \omega_0 + \frac{L}{\ell} \right] \cdot \Delta \omega + O(\Delta \omega^2) + \ldots,
\]

\[
n_1 = \frac{\partial n}{\partial \omega} |_{\omega = \omega_0}
\]

\[
\approx N(2\pi) + \left[ (n_g - 1) \frac{\ell}{L} + 1 \right] \frac{L}{c} \cdot \Delta \omega + O(\Delta \omega^2) + \ldots,
\]

\[
n_g = 1 + n_1 \cdot \omega_0
\]

where \( n_1 \) is the first-order dispersion slope in the Taylor expansion of medium refractive index \( n(\omega) \) at \( \omega = \omega_0 \), \( N \) is a large integer number, and \( n_g \) is the group index representing the slope of linear dispersion induced by the physical process (for example, the double gain profile) in the medium. As can be seen from eqn.1, it is possible to choose \( n_1 \) (by controlling the dispersion slope) such that \( \phi(\omega) \) becomes independent of frequency with respect to first-order change in frequency \( \Delta \omega \). This happens for a value of \( n_1 = -\frac{1}{\omega_0} \left( \frac{L}{\ell} \right) \), which corresponds to \( n_g = 1 - \frac{L}{\ell} \), a small negative dispersion slope dictated by the ratio \( (L/\ell) \). In a situation where the medium completely fills the entire cavity length, this will correspond to \( n_1 = -\frac{1}{\omega_0} \) or a zero (null) value of \( n_g \), known as the ideal white-light condition. Under this condition, many frequencies around \( \omega_0 \) will resonate simultaneously in the cavity, as the round-trip phase shift becomes independent of first-order change in frequency. Thus, the cavity response gets broadened without sacrificing the cavity buildup and a WLC can be realized. Fig. 1 shows an example of white-light cavity response generated by considering a cavity with internal cavity buildup of \( O(10^3) \) and containing a gain medium with ratio \( (L/\ell) = 10 \). Negative dispersion with a slope corresponding to \( n_g \approx -9 \) is produced by considering narrow resonant gain lines (linewidth \( \sim 1 \) MHz) with frequency separation 7.95 MHz. Broad cavity resonance under the white-light condition is observed without sacrificing the buildup factor shown along the vertical axis.

As discussed earlier, although there is no dephasing in cavity response away from \( \omega_0 \) to first-order in \( \Delta \omega \), dephasing due to higher-orders in \( \Delta \omega \) causes the cavity response to drop. This eventually decides the linewidth of the WLC. One can evaluate the WLC linewidth \( (\gamma') \) in comparison to the empty cavity from the following relation:

\[
\gamma' = \beta \frac{1}{1 + \left[ (n_g - 1) + n_3 \omega_0 (\gamma')^2 \right] \cdot \frac{\ell}{L}}
\]

\[
\beta = \frac{\sin^{-1} \left[ \frac{1 - R}{2\sqrt{R\rho}} \right]}{\sin^{-1} \left[ \frac{1 - R}{2\sqrt{R}} \right]},
\]

\[
n_3 = \frac{1}{6} \cdot \frac{\partial n^3}{\partial \omega} |_{\omega = \omega_0}
\]
where $\gamma$ is the empty cavity linewidth, and $\rho = \exp(-\alpha \ell)$, where $\alpha$ is the overall loss coefficient that causes additional linewidth broadening in the presence of the medium. In deriving the expression, the magnitude of dephasing $\left| n_3 (\Delta \omega)^2 \frac{\ell}{L} \right|$ due to $(\Delta \omega)^2$ is considered small in comparison to the third-order dephasing term $\left| n_3 \omega_o (\Delta \omega)^3 \frac{\ell}{L} \right|$. Note also that dephasing contribution due to second-order dispersion coefficient $n_2$ is ignored, since the characteristic dispersion profile associated with any kind of atomic resonances is anti-symmetric around the resonant frequency $\omega_o$. The linewidth of WLC under perfect white-light condition $n_2 = 1 - \frac{L}{\ell}$ can be estimated from eqn. 2 as

$$\gamma' = 2 \left[ \frac{\beta \gamma}{n^2 \omega_o \frac{L}{\ell}} \right]^{\frac{1}{3}}$$

If the intra-cavity negative dispersion is generated using Raman gain doublets using an atomic medium, the third-order slope $(n_3)$ at the center frequency $\omega_o$ between the gain lines can be approximated as $\frac{n_3}{\gamma^2} = \frac{1}{\omega_o \gamma^2} \frac{L}{\ell}$, where $\Gamma$ corresponds to the frequency separation between the gain lines. In this case, the linewidth of WLC given in eqn. 3 can be simplified to an expression $\gamma' = 2 \left[ \frac{\beta \Gamma}{4 \gamma^2 \Gamma^2} \right]^{\frac{1}{3}}$. Although a small medium length $\ell$ (or large ratio $(L/\ell)$) leads to a large, negative slope needed for white-light condition, the bandwidth of the WLC remains unchanged. These analytic conclusions have also been verified via computer simulations, as described later on. The expression also implies that in order for the white light effect to be evident, the frequency separation $\Gamma$ between the gain peaks should be considered much larger than the empty cavity linewidth $\Delta \omega_{1/2}$. For the example considered in fig.1, if we substitute the values of $\gamma \sim 1$ MHz, $\Gamma = 7.95$ MHz ($\gg \gamma$) and assume $\beta = 1$, the linewidth of the WLC is estimated to be 3.16 MHz. The experimentally observed WLC linewidth (2.9 MHz) is very close to the estimated linewidth, the difference being attributable in part to the fact that our model assumes a single Lorentzian absorption line of width $\Gamma$ for negative dispersion instead of an actual gain doublet. Of course, for applications to the LIGO system, we can simply design the values necessary for the desired bandwidth.

In what follows, we describe an intra-cavity bi-frequency Raman gain experiment that was used to realize WLC. Transparent spectral gain regions and the negative dispersion
associated with gain doublets have been demonstrated in earlier experiments [13-16]. The experiments that demonstrated superluminal pulse propagation relied on a large negative dispersion slope (or negative group index) between closely spaced gain lines. On the other hand, our experiment requires extremely small value of linear negative dispersion slope, typically $O(10^{-15})$ rad$^{-1}$ sec (equivalently, small negative $n_g$) in order to satisfy the white-light condition. This condition has been achieved experimentally under low gain. In order to observe an appreciable white-light effect over a significant bandwidth, frequency separation between gain lines has been set larger than our actual cavity linewidth.

The experimental setup for the white-light cavity is shown in fig. 2a. A 10 cm long Rubidium vapor cell is placed inside a cavity as an intra-cavity gain medium. In order to create the intra-cavity anomalous dispersion, Raman gain doublets are generated in a three-level atomic medium by driving it with dual frequency pumps under the two-photon resonance condition. A 100 cm long ring cavity has been constructed for our study. The cavity consists of four mirrors with two partially transmitting plane mirrors that generate pairs of input and output ports, and two concave mirrors that allow cavity mode-matching for the long cavity path length. The empty cavity finesse is measured to be 100 and the cavity linewidth is measured approximately to be 3 MHz. One of the concave mirrors in the cavity is attached to a piezoelectric transducer (PZT). This is used in frequency-locking the cavity using a feedback control. Damped elastomers with appropriate loads were used in mounting the cavity in order to provide the best possible passive low-frequency vibration isolation. During our experiment, one pair of cavity input and output ports was used to actively lock the cavity length at a desired resonant frequency using a servo control.

The probe and pump beams used in the experiment are obtained from an external cavity frequency-locked CW Ti:Sapphire laser (line width ~ 1 MHz). Figure 2b shows the 5S$_{1/2}$, F=2 and F=3 hyperfine ground states and the Doppler-broadened, unresolved hyperfine 5P$_{3/2}$ excited state manifolds on the D2 line in Rb$^{85}$ that constitute a Λ-type configuration for Raman excitation. An incoherent optical pump from a diode laser is used to create a population inversion between the two coherently coupled ground states. This allows us to obtain efficient gain using relatively weak pump intensities. The optical pump frequency is locked to the 5S$_{1/2}$, F = 2 to 5P$_{3/2}$, $F' = 3$ transition in Rb$^{85}$. All the beams to the system are delivered using optical fibers which provide high-quality, perfectly matched spatial modes for the beams, and robust beam alignments and overlaps inside the optical cavity. The probe beam, which is p-polarized, resonates inside the cavity.
In order to produce negative dispersion, two pump beams of different frequencies are injected into the cavity to generate closely spaced gain lines. These pump beams are derived using two different acousto-optic modulators (AOMs) driven by independent frequency generators. The negative dispersion is observed in the intermediate region between the gain lines. During our experiment, the Raman pumps and optical pump beams, polarized orthogonally to the probe, are all combined with the probe beam inside the cavity in the co- and counter-propagating directions, respectively, using two intra-cavity polarizing beam splitters (PBSs) before and after the cell (fig. 2a). The frequency difference between the probe and the center of the pump lines is initially matched to the ground state splitting in Rb$^{85}$ (3.0357 GHz). Gain at two different probe frequencies is observed by scanning the probe frequency around the center frequency of the pump lines using an AOM set in a double-pass configuration. The incoherent optical pump creates the population inversion between the ground states, and the pump beams drive stimulated Raman excitations to produce gain at distinct probe frequencies.

In the experiment, the optical pump beam does not completely overlap (stays nearly 3 to 4 mm away) on the probe beam while observing the Raman gain. This is because optically pumped Rubidium atoms moving with large average thermal velocity, diffusing quickly through the cross-section of the probe beam to participate in Raman excitation. Raman gain with a large gain coefficient and sub-natural linewidth (~ 1 MHz) has been observed using a medium with modest atomic density [$N \sim O(10^{12})$] and ground-state dephasing by introducing a flipper mirror in the cavity path. The vapor cell is heated to nearly a steady temperature of 60°C using bifilarly wound coils that produce a negligible axial magnetic field. It is also magnetically shielded using μ-metal to reduce the effect of residual magnetic fields.

For optimum gain, the average frequency of the probe and the pump fields is detuned below Doppler resonance where the probe does not get significantly absorbed in the cell even in the absence of pump beams. The cavity is made resonant at the selected probe frequency $\omega_o$ by adjusting its length $L$ simply by tuning the drive voltage to the PZT mirror. The cavity length is then actively locked using a servo signal generated from the cavity output produced by a resonating locking beam sent through the other cavity port (fig.2). A phase sensitive detection was used for active feed-back and servo control to keep the cavity length fixed. The frequency of the lock beam is set at many multiples of cavity free spectral range (FSR) away from the probe frequency $\omega_o$ in order to prevent counter-propagating lock-beam from being absorbed in any Doppler-broadened transitions associated with the medium. This also avoids any undesirable effect on the probe that could be caused by the simultaneous presence of near-resonant locking-beams in the medium.
The cavity linewidth in the absence of gain (pumps turned off) is measured by scanning the probe frequency around the center frequency $\omega_0$. It is found to be approximately 8 MHz, which is nearly three times wider than the empty cavity linewidth. As described earlier, this is due to extra loss introduced by intra-cavity elements and also due to residual medium absorption and scattering loss at the probe frequency. In order to observe the effect of negative dispersion on the cavity linewidth using the probe field, bi-frequency Raman pump beams with frequency separation $\Delta \omega$ centered around the resonant probe frequency $\omega_0$ were turned on, along with the external optical pump beam. Portions of probe frequencies centered between the gain lines experience a negative dispersion. The probe transmission profile (or cavity response) gets modified as the negative dispersion alters the cavity resonance condition seen earlier. In the intermediate region between the gain peaks, the probe might also experience a residual gain due to adjacent gain lines depending on the gain magnitude. The gain at the probe frequencies can be adjusted using the pump intensities in order to control the slope of the negative dispersion. For a particular pump frequency separation, $\Delta \omega$, the gain is adjusted to achieve a negative dispersion slope that corresponds to the white-light condition described earlier.

In our experiment, we measured the dispersion as seen by the probe field under the double-gain condition using a heterodyne technique. This is done by interrupting the cavity resonance by placing a flipper mirror in the probe beam path inside the cavity. For heterodyne measurements, an auxiliary reference wave is produced by frequency shifting a fraction of the probe beam outside the cavity using a 40 MHz AOM. A part of it is combined with the probe that propagates through the cell and the other with a probe that does not propagate through the cell. These two heterodyne rf signals are detected using two fast photodetectors (response time < 10 ns). The phase difference between the two rf signals varies due to dispersion at probe frequencies as it scans around the double gain resonance produced by the bi-frequency pumps. A low noise rf mixer and a low-pass frequency filter were used to demodulate the rf signal from the detectors. The amplitude of the demodulated signal allows us to measure accurately the dispersion in the atomic medium under the gain resonances.

Fig. 3 shows the measured dispersion profiles corresponding to a particular gain separation of 8 MHz under different gain conditions. Very small negative dispersion slopes ($n_1 \sim 3 \times 10^{-16}$ rad$^{-1}$ sec) have been achieved by reducing the gain which is controlled by the pump intensities. It was concluded that gain as low as 3 dB (or smaller) is needed to reach the small, negative values of $n_g$ needed. When such a low gain condition prevails inside the cavity, it also avoids other undesirable effects such as self-
oscillation and bistable behavior that are normally observed in the cavity response in the presence of a stronger gain medium. During our dispersion measurement, the optical power in the probe field outside the cavity was chosen to be $\sim 100 \, \mu W$. This corresponds to a very low transmitted probe intensity at the cell after the input coupler, with a power density of $50 \, \text{mW/cm}^2$. During the closed-cavity gain experiment, the intra-cavity probe intensity is expected to be much higher than the probe intensity chosen during dispersion measurement. Thus, in order to achieve similar negative dispersion conditions predicted from these measurements, the probe power outside the cavity was scaled down by the cavity build up factor in order to see the white-light effect. This is an important step as the gain also depends somewhat on probe intensity if the probe becomes too strong.

The effect of bi-frequency Raman gain induced anomalous dispersion on the cavity resonance is observed by scanning the probe frequency around the cavity resonance ($\omega_o$), while the cavity length is actively locked, as discussed before. Fig. 4 shows a sequence of cavity resonances for different frequency separations between the gain lines. Each time with an increasing frequency separation, the gain was adjusted by controlling the pump intensity to observe a white-light effect in probe transmission. Although broadened cavity response can be clearly seen from the data, the uniformity of the response is considerably degraded with increasing gain separation. This is due to our limitation to peak the gain required to reach the white light condition $n_g = 1 - \frac{L}{\ell}$. However, in future experiments, this constraint can be easily improved using different experimental parameters such as the atomic density, the cell length, and the bandwidths of the AOMs used for generating the pumps.

Fig. 4a shows the cavity linewidth in the absence of gain. Compared to the empty cavity linewidth, this is nearly three times broader, which is likely attributable to additional scattering loss at probe frequency in the Doppler broadened absorption line $\text{F}=2$ of Rb$^{85}$, as mentioned earlier. A comparison also shows that the probe transmissions near white-light conditions are somewhat lower than the peak cavity transmission in the absence of gain. This could possibly be taken to imply that the cavity build up factor is reduced as the bandwidth is increased. However, this is not the case, since the cavity transmission remains nearly the same when the white-light effect is observed with increased gain separations, and therefore increasing bandwidths. We attribute this reduction in peak transmission to possible additional absorption loss in probe transmission caused by optical pumping effects resulting from detuned Raman pumps. This constraints can easily be eliminated in future experiment by using different pumping schemes so that probe will not see any additional absorption between the gain lines. The non-uniform response observed experimentally in cavity transmission is simulated in Fig. 5 by a choosing
dispersion slope slightly away from the white-light condition. It shows the cavity resonance broadening with non-uniformity for an empty cavity linewidth ($\Delta f_{1/2}$) of 6.9 MHz, corresponding to a gain separation $\Gamma = 23$ MHz for a group index value ($n_g = -9.38$) close to the white-light condition. The bandwidth of the WLC cavity is found to be 12 MHz. Fig. 6 shows a comparison of the measured linewidth for white-light response from the experimental data with the estimated linewidth. They agree reasonably well within the measurement error bars.

We have also demonstrated experimentally the reverse of the WLC, using positive dispersion corresponding to slow light. For this case, we have also shown how the system becomes less sensitive to perturbations [17]. Conversely, we have shown how the WLC is more sensitive to perturbations, and how this property may be used to enhance the sensitivity of absolute rotation measurement using a ring laser gyroscope [18]. For relative rotation measurement, we have shown that even the slow-light process can be used to enhance sensitivity.

The WLC described above was demonstrated using light at 780 nm, and rubidium atoms. The idea can be easily extended to most other alkali atoms. Furthermore, as long as there is a nominally two level transition available in some gaseous medium, Zeeman sublevels within these transitions can be used to realize an effective three-level system, which in turn can be used to demonstrate the type of WLC described above. One example is the Zeeman sublevel based EIT we had demonstrated earlier [19]. Another example is the electromagnetically induced transparency observed in Neon gas [20]. Thus, it may easily be possible to find vapor based transitions that would yield a WLC at 1064 nm, as required for the LIGO system. We will perform a systematic search for such transitions. Alternatively, it is possible to realize such a WLC using non-linear wave mixing in photorefractive media. This approach is very versatile, and we will use this approach to demonstrate the WLC for 1064 nm.

In conclusion, we have experimentally demonstrated the effect of anomalous dispersion induced by intracavity gain medium for realizing a white-light cavity. The origin of this effect is understood by including the medium dispersion properties in the cavity transfer function. The technique can be easily perfected to construct white-light cavities with performance criteria meeting the potential requirements for large bandwidth detection of gravity waves in gravitational laser interferometers.

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References


Fig. 1  Cavity response shows white-light effect for $n_g = -9$ obtained using gain-doublets corresponding to $\Gamma = 7.95$ MHz. The cavity buildup (x 2000) with WLC is maintained.
Fig. 2  (a) Schematic of the experimental set-up for the white-light cavity (b) Energy diagram showing detuned Raman excitation in D2 line $^8$Rb to produce bi-frequency gain at probe frequencies.
Fig. 3 Measured dispersion profiles associated with gain doublets for varied gain magnitude and $\Delta f = 7$ MHz. For 3.6 dB gain, first-order dispersion slope $n'$ is estimated to be $-8.7 \times 10^{-16}$ rad$^{-1}$.sec, that corresponds to $n_g = 0.66$. 
Fig. 4  Experimental results showing broadened cavity response in presence of gain doublets with 4 varying gain separation.
Fig. 5  Small deviation ($n_g = -9.38$) from white-light condition shows nonuniform response similar to experimental data.
Fig. 6 Comparison between the estimated white-light cavity linewidth and the actual linewidth measured from experimental data.