5 The MSSM with R-Parity Violation

5.1 Introduction

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5.1.1 Explicit R-parity violation

The Standard Model conserves baryon number $B$ and lepton number $L$ separately at the perturbative level. On the contrast, its minimal supersymmetric extension does allow for the breaking of $B$ and $L$ if one requires `only' gauge invariance and supersymmetry. The case of lepton number violation is easily seen by noting that the Higgs superfield $H_d$ and the lepton superfields $\tilde{L}_i$ have the same gauge and Lorentz quantum numbers and we can redefine them by

$$ W = W_{\text{MSSM}} + W_{R_p}, \quad (5.1) $$

$$ W_{\text{MSSM}} = h_{ij}^E \tilde{L}_i \tilde{H}_d \tilde{E}_j^c + h_{ij}^D \tilde{Q}_i \tilde{H}_d \tilde{D}_j^c + h_{ij}^H \tilde{Q}_i \tilde{H}_u \tilde{H}_d^c \tilde{H}_u - \mu \tilde{H}_d \tilde{H}_u, \quad (5.2) $$

$$ W_{R_p} = \frac{1}{2} \lambda_{ijk} \tilde{L}_i \tilde{L}_j \tilde{E}_k^c + \lambda_{ijk}^c \tilde{L}_i \tilde{Q}_j \tilde{D}_k^c + \frac{1}{2} \lambda_{ijk}'' \tilde{U}_i \tilde{D}_j^c \tilde{D}_k^c + \epsilon_i \tilde{L}_i \tilde{H}_u, \quad (5.3) $$

$i,j,k = 1,2,3$ are generation indices. $\tilde{L}_i$ ($\tilde{Q}_i$) are the lepton (quark) $SU(2)_L$ doublet superfields. $\tilde{E}_j^c$ ($\tilde{D}_j^c$) are the electron (down- and up-quark) $SU(2)_L$ singlet superfields. $\lambda_{ijk}$, $\lambda_{ijk}^c$, and $\lambda_{ijk}''$ are dimensionless Yukawa couplings whereas the $\epsilon_i$ are dimensionful mass parameters. Gauge invariance implies that the first term in $W_{R_p}$ is anti-symmetric in $\{i,j\}$ and the third one is anti-symmetric in $\{j,k\}$.

Equation (5.3) thus contains $9 + 27 + 9 + 3 = 48$ new terms beyond those of the MSSM. Once lepton number is broken, it is obvious from Eqs. (5.2) and (5.3) the MSSM seems to consist of three quark superfields, five $SU(2)$ doublet Higgs superfields and three charged $SU(2)$ singlet Higgs superfields as there are no means to distinguish between lepton and Higgs superfields. From this point of view, the known charged and neutral leptons are higgsinos.

The simultaneous appearance of lepton and baryon number breaking terms leads in general to a phenomenological catastrophe if all involved particles have masses of the order of the electroweak scale: rapid proton decay [1,2]. To avoid this problem a discrete multiplicative symmetry, called R-parity ($R_p$), had been invented [3] which can be written as

$$ R_p = (-1)^{3B + L + 2S}, \quad (5.4) $$

where $S$ is the spin of the corresponding particle. For all superfields of MSSM, the SM field has $R_p = +1$ and its superpartner has $R_p = -1$, e.g. the electron has $R_p = +1$ and the selectron has $R_p = -1$. In this way all terms in Eq. (5.3) are forbidden and one is left with the superpotential given in Eq. (5.2).

Recent neutrino experiments have shown that neutrinos are massive particles which mix among themselves (for a review see e.g. [4]). In contrast to leptons and quarks, neutrinos need not to be Dirac particles but can be Majorana particles. In the latter case the Lagrangian contains a mass term which violates explicitly lepton number by two units. This motivates one to allow the lepton number breaking terms in the superpotential in particular as they automatically imply the existence of massive neutrinos without the need of introducing right-handed neutrinos [5]. The $\lambda''$ terms can still be forbidden by a discrete symmetry which transforms $(\tilde{U}^c, \tilde{D}^c, \tilde{Q})$ into $(-\tilde{U}^c, -\tilde{D}^c, -\tilde{Q})$ while leaving the other fields unchanged. Breaking lepton number has two interesting consequences for the phenomenology of Higgs bosons in supersymmetric theories: (i) the Higgs bosons can mix with the sleptons and (ii) Higgs cascade decays into SUSY particles get altered.

Let us briefly comment on the number of free parameters before discussing the phenomenology in more detail. The last term in Eq. (5.3), $\tilde{L}_i \tilde{H}_u$, mixes the lepton and the Higgs superfields. In supersymmetry $\tilde{L}_i$ and $\tilde{H}_d$ have the same gauge and Lorentz quantum numbers and we can redefine them by
a rotation in \((\hat{H}_d, \hat{L}_i)\). The terms \(\epsilon_i \hat{L}_i \hat{H}_u\) can then be rotated to zero in the superpotential [5]. However, there are still the corresponding terms in the soft supersymmetry breaking Lagrangian

\[ V_{R\, \text{soft}} = B_i \epsilon_i \hat{L}_i \hat{H}_u \quad (5.5) \]

which can only be rotated away if \(B_i = B\) and \(M^2_{H_d} = M^2_{L_i}\) [5]. Such an alignment of the superpotential terms with the soft breaking terms is not stable under the renormalization group equations [6]. Assuming an alignment at the unification scale, the resulting effects are small [6] except for neutrino masses [6–10]. Models containing only bilinear terms do not introduce trilinear terms as can easily be seen from the fact that bilinear terms have dimension mass whereas the trilinear are dimensionless. For this reason we will keep in the following explicitly the bilinear terms in the superpotential.

The presence of the bilinear terms in the soft SUSY breaking potential, Eq. (5.5), implies that not only the usual Higgs bosons get vacuum expectations values (vevs) but also the sneutrinos (for details see [10–12]). As a consequence the neutral Higgs bosons mix with the sneutrinos resulting in five neutral scalar bosons and four neutral pseudoscalar bosons. In addition the charged Higgs boson mixes with the charged sleptons resulting in seven charged states, \(S_i^\pm\) (\(i = 1, \ldots, 7\)). In the following we will simplify the notation by denoting the particles with their MSSM notation indicating their main particle content. The complete set of the corresponding mass matrices is given in ref. [10]. The mixing between sleptons and Higgs bosons leads to additional decay modes for the Higgs bosons [13]:

\[ \phi \rightarrow \nu_{\chi^0_i}, \quad l^\pm \tilde{\chi}^\pm_k, \quad \tilde{\nu} \]

\[ H^+ \rightarrow l^+ \tilde{\chi}^0_i, \quad \nu_{\tilde{\chi}^+_k}, \quad (5.6) \]

where \(\phi\) denotes \(h^0, H^0\) and \(A^0\). Moreover, there is the possibility of associate production of Higgs bosons together with sleptons or slepton-strahlung of t-quarks (in analogy to Higgs-strahlung) as discussed in Section 5.4. Also the sleptons have additional decay modes compared to the MSSM:

\[ \tilde{\nu} \rightarrow q\bar{q}, \quad l^+ l^-, \quad \nu \tilde{\nu} \]

\[ \tilde{l} \rightarrow l^+ \nu, \quad q\tilde{q}', \quad (5.7) \]

where \(\nu\) denotes the leptons. e.g. the sleptons have the same signatures apart from the \(\nu \tilde{\nu}\) channel as the usual Higgs bosons if the R-parity violating decay modes dominate. We want to stress here again, that although we use the MSSM symbols, Higgs bosons and sleptons mix and that the sleptons have to be considered as additional Higgs bosons once lepton number is broken.

How large can the branching ratio for those decay modes be? To answer this question one has to take into account existing constraints on R-parity violating parameters from low energy physics. As most of them are given in terms of trilinear couplings, we will work in the “c-less” basis, e.g. rotate away the bilinear terms in the superpotential Eq. (5.3). Therefore, the trilinear couplings get additional contributions. Assuming, without loss of generality, that the lepton and down type Yukawa couplings are diagonal they are given to leading order in \(\epsilon_i/\mu\) as [14–16]:

\[ \lambda'_{ijk} \rightarrow \lambda'_{ijk} + \delta_{ij} h d_e \frac{\epsilon_i}{\mu} \]

\[ (5.10) \]

and

\[ \lambda_{ijk} \rightarrow \lambda_{ijk} + \delta \lambda_{ijk}, \quad (5.11) \]

\[ \delta \lambda_{121} = h_e \frac{\epsilon_2}{\mu}, \quad \delta \lambda_{122} = h_\mu \frac{\epsilon_1}{\mu}, \quad \delta \lambda_{123} = 0 \]

\[ \delta \lambda_{131} = h_e \frac{\epsilon_3}{\mu}, \quad \delta \lambda_{132} = 0, \quad \delta \lambda_{133} = h_e \frac{\epsilon_1}{\mu} \]

\[ \delta \lambda_{231} = 0, \quad \delta \lambda_{232} = h_\mu \frac{\epsilon_3}{\mu}, \quad \delta \lambda_{233} = h_\mu \frac{\epsilon_2}{\mu} \]
Table 5.1: R-parity violating decays of sfermions via trilinear $\mathcal{O}(R)$ operators $\lambda L_i L_j E^c_k$, $\lambda' L_i Q_j D^c_k$ and $\lambda'' U^c_i D^c_j D^c_k$.

<table>
<thead>
<tr>
<th>Supersymmetric particles</th>
<th>$\lambda$</th>
<th>Couplings</th>
<th>$\lambda'$</th>
<th>$\lambda''$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{\nu}_i L$</td>
<td>$\ell^+_i L \ell^c_R$, $\tilde{\nu}_i L \ell^c_R$</td>
<td>$d_j L d^{c}_{k,R}$</td>
<td>$d_j L d^{c}_{k,R}$</td>
<td>$d_j L d^{c}_{k,R}$</td>
</tr>
<tr>
<td>$\tilde{\ell}_i L$</td>
<td>$\tilde{\ell}^+_i L \ell^c_R$, $\tilde{\ell}^+_i L \ell^c_R$</td>
<td>$\tilde{u}_j L \ell^c_R$, $\tilde{u}_j L \ell^c_R$</td>
<td>$\tilde{u}_j L \ell^c_R$, $\tilde{u}_j L \ell^c_R$</td>
<td>$\tilde{u}_j L \ell^c_R$, $\tilde{u}_j L \ell^c_R$</td>
</tr>
<tr>
<td>$\tilde{\ell}_i R$</td>
<td>$\tilde{\ell}^+_i L \ell^c_R$, $\tilde{\ell}^+_i L \ell^c_R$</td>
<td>$\tilde{\nu}_i L \ell^c_R$, $\tilde{\nu}_i L \ell^c_R$</td>
<td>$\tilde{\nu}_i L \ell^c_R$, $\tilde{\nu}_i L \ell^c_R$</td>
<td>$\tilde{\nu}_i L \ell^c_R$, $\tilde{\nu}_i L \ell^c_R$</td>
</tr>
<tr>
<td>$\tilde{\nu}_j L$</td>
<td>$\tilde{\nu}_j L \ell^c_R$, $\tilde{\nu}_j L \ell^c_R$</td>
<td>$\tilde{\lambda}^+_i L \ell^c_R$, $\tilde{\lambda}^+_i L \ell^c_R$</td>
<td>$\tilde{\lambda}^+_i L \ell^c_R$, $\tilde{\lambda}^+_i L \ell^c_R$</td>
<td>$\tilde{\lambda}^+_i L \ell^c_R$, $\tilde{\lambda}^+_i L \ell^c_R$</td>
</tr>
<tr>
<td>$\tilde{d}_j L$</td>
<td>$\tilde{d}^+_j L \ell^c_R$, $\tilde{d}^+_j L \ell^c_R$</td>
<td>$\tilde{d}^+_j L \ell^c_R$, $\tilde{d}^+_j L \ell^c_R$</td>
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<tr>
<td>$\tilde{d}_k R$</td>
<td>$\tilde{d}^+_k R \ell^c_R$, $\tilde{d}^+_k R \ell^c_R$</td>
<td>$\tilde{d}^+_k R \ell^c_R$, $\tilde{d}^+_k R \ell^c_R$</td>
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<td>$\tilde{d}^+_k R \ell^c_R$, $\tilde{d}^+_k R \ell^c_R$</td>
</tr>
</tbody>
</table>

where we have used the fact that neutrino physics requires $|\epsilon_i/\mu| \ll 1$ [10]. An essential point to notice is that the additional contributions in Eqs. (5.10) and (5.11) follow the hierarchy dictated by the down quark and charged lepton masses of the standard model.

A comprehensive list of bounds on various R-parity violating parameters can be found in [17]. However, there the recent data from neutrino experiments like Super-Kamiokande [18], SNO [19] and KamLAND [20] are not taken into account. These experiments yield strong bounds on trilinear couplings involving the third generation [21,22]. In addition also the sneutrino vevs are constrained by neutrino data [10,21]. Most of the trilinear couplings have a bound of the order $(10^{-4} - 10^{-3}) \times m_i/(100\text{GeV})$ where $m_i$ is the mass of the sfermion in the process under considerations. The cases with stronger limits are: $|\lambda'_{111}| \lesssim O(10^{-4})$ due to neutrinoless double beta decay and $|\lambda'_{333}| \lesssim 5|\lambda'_{111}| \lesssim O(10^{-4})$ due to neutrino oscillation data. Moreover, neutrino oscillation data imply $|\mu| \lesssim 10^{-12}$ where $\nu_i$ are the sneutrino vevs and $\text{det}(M_{\chi})$ is the determinant of the MSSM neutralino mass matrix.

In particular the last constraint implies that in general there is only a small mixing between sleptons and Higgs bosons and usually the R-parity violating decay modes of both, Higgs bosons and sleptons, have only tiny branching ratios of the order $10^{-6}$ and below. One exception is if by chance a Higgs boson is nearly mass degenerate with one of the sleptons which requires quite some fine-tuning. The other exception is if all R-parity conserving decay modes are kinematically forbidden. This can occur if either the sneutrinos or the right sleptons are the lightest supersymmetric particles (LSPs), which will be discussed in detail in Section 5.2. The case of left-slepton LSPs is practically excluded as the sneutrinos are always lighter provided $\tan \beta \geq 1$. In the case of sneutrino LSPs one finds the usual MSSM but misses the ordinary sneutrinos and finds instead additional neutral states behaving nearly like neutral doublet Higgs bosons [16,23]. The two main differences are: (i) The existence of lepton flavour violating decays modes such as $\tilde{\nu}_e \to e^+\tau^-$ which are sizable. (ii) The invisible decay mode into $\tilde{\nu}_\tau$, which turns out to be small with branching ratios in the order of $10^{-4}$. In the case of charged slepton LSPs the situation is reverse: one finds the MSSM sneutrinos but misses the right sleptons and finds instead three additional electrically charged but $SU(2)$ singlet Higgs bosons [24,25]. In both cases one finds that either sneutrinos or charged sleptons have in general couplings to quarks and leptons proportional to the usual Yukawa couplings. The main effect of additional trilinear couplings is to change the SM hierarchy of the couplings enhancing in particular those couplings containing only first and second generation indices.

A further aspect of R-parity violation is that the LSP becomes unstable\footnote{As a side remark we note that it has been shown that a LSP cannot be considered as a cold dark matter candidate in the presence of a single $\mathcal{O}(R)$ coupling with value even as small as $O(10^{-20})$. The only exception is the case of a light gravitino LSP with a mass in the order of 100 eV with a life-time in the order $10^{75}$ Hubble times [27,28].}. This is important for the Higgs sector if the Higgs bosons have sizable decay modes into SUSY particles. For R-parity violating couplings larger than $O(10^{-8} - 10^{-6})$ these decays can be observed in a typical $O(10)$m diameter
Table 5.2: R-parity violating decays of neutralinos and charginos with trilinear $R_p$ operators $\lambda_{L_i L_j E_k^c}$, $\lambda'_{L_i Q_j D_c^k}$ and $\lambda''_{U_i^c D_j^c D_k^c}$ (from [26]).

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</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{\chi}^0$</td>
<td>$\ell^+_i \nu_j d_k$, $\ell^+_i \nu_j L_k$, $\nu_i \ell^+_j d_k$, $\nu_i \ell^+_j L_k$</td>
<td>$\ell^+_i u_j d_k$, $\ell^+_i u_j L_k$, $\nu_i d_j d_k$, $\nu_i d_j L_k$</td>
<td>$\tilde{u}_i d_j d_k$, $\tilde{u}_i d_j L_k$</td>
</tr>
<tr>
<td>$\tilde{\chi}^+$</td>
<td>$\ell^+_i \ell^+_j \ell^+_k$, $\ell^+_i \nu_j E_k^c$, $\nu_i \ell^+_j E_k^c$</td>
<td>$\ell^+_i d_j d_k$, $\ell^+_i d_j L_k$, $\nu_i u_j d_k$, $\nu_i u_j L_k$</td>
<td>$u_i d_j u_k$, $u_i u_j d_k$, $d_i d_j u_k$, $d_i d_j L_k$</td>
</tr>
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</table>

Collider experiment. In the range up to $O(10^{-5} - 10^{-4})$ for those couplings displaced vertices can be observed. The LSP decays are important in those cases where the usual MSSM Higgs bosons have sizable branching ratios in SUSY particles, e.g. decays like $A^0 \rightarrow \tilde{\chi}^0 \tilde{\chi}^0$. In models with conserved R-parity such decays contain large missing momenta as part of their signatures as the LSP, usually the lightest neutralino, escapes detection. In the case of R-parity violation several things change, e.g. all particles can be the LSPs. Tables 5.1 and 5.2 list the R-parity violating final states induced by trilinear couplings of all particles which have tree-level couplings to Higgs bosons. All lepton number violating final states are also induced by sneutrino vevs. The sneutrino vevs induce additional decay modes: $\tilde{\chi}^0 \rightarrow W^\pm l^\mp$, $\tilde{\chi}^0 \rightarrow Z l$, $\tilde{\chi}^0 \rightarrow h^0 \nu$, $\tilde{\nu} \rightarrow \nu \nu$, $\tilde{\nu} \rightarrow t \bar{t}$, and $\tilde{\chi}^+ \rightarrow W^+ \nu$. Several of the R-parity violating decay channels do not have the usual missing energy signal. In other cases it is considerably reduced as the neutrinos carry in average less missing energy compared to neutralinos. R-parity violation implies an enhancement of jet and lepton multiplicities in the final states. For all these reasons decays of Higgs bosons into SUSY particles will look completely different if R-parity is broken compared to the case where it is conserved. Further detailed discussions of R-parity violating decays of SUSY particles can be found in [26, 29] for the case of trilinear R-parity violation and in [23, 24, 28, 30, 31] for bilinear R-parity violation.

5.1.2 Spontaneous R-parity violation

Up to now we have only considered explicit R-parity violation keeping the particle content of the MSSM. In the case that one enlarges the spectrum by gauge singlets one can obtain models where lepton number is fixed via the lepton number assigned as $(MSSM)$ superelds in the presence of the singlet superelds.

The most general superpotential terms involving the Minimal Supersymmetric Standard Model (MSSM) superelds in the presence of the $SU(2) \otimes U(1)$ singlet superelds $(\tilde{\nu}^c_i, \tilde{S}_i, \Phi)$ carrying a conserved lepton number assigned as $(-1, 1, 0)$, respectively, is given as [38]

$$
\mathcal{W} = \varepsilon_{ab} \left( h_{ij} \tilde{Q}_i \tilde{U}_j \tilde{H}_u^b + h_{ij} \tilde{Q}_i \tilde{D}_j \tilde{H}_d^a + h_{ij} \tilde{L}_i \tilde{E}_j \tilde{H}_d^a + h_{ij} \tilde{L}_i \tilde{D}_j \tilde{H}_u^b - \tilde{\mu} \tilde{H}_d^a \tilde{H}_u^b - h_0 \tilde{H}_d^a \tilde{H}_u^b \tilde{\Phi} \right) + h_{ij} \tilde{\Phi} \tilde{\nu}^c_i \tilde{\widehat{S}}_j + M_R \tilde{\nu}^c_i \tilde{\widehat{S}}_j \frac{1}{2} M_\Phi \tilde{\Phi}^2 + \frac{\lambda}{3!} \tilde{\Phi}^3
$$

(5.12)

The first three terms together with the $\tilde{\mu}$ term define the R-parity conserving MSSM, the terms in the second line only involve the $SU(2) \otimes U(1)$ singlet superelds $(\tilde{\nu}^c_i, \tilde{S}_i, \Phi)$, while the remaining terms couple the singlets to the MSSM fields. For completeness we note, that lepton number is fixed via the Dirac-Yukawa $h_\nu$ connecting the right-handed neutrino superelds to the lepton doublet superelds. For simplicity we assume in the discussion below that only one generation of $(\tilde{\nu}^c_i, \tilde{S}_i)$ is present.

The presence of singlets in the model is essential in order to drive the spontaneous violation of R-parity and electroweak symmetries in a phenomenologically consistent way. As in the case of explicit
R-parity violation all sneutrinos obtain a vev beside the Higgs bosons as well as the $\tilde{S}$ field and the singlet field $\Phi$. For completeness we want to note that in the limit, that all sneutrino vevs vanish and all singlets carrying lepton number are very heavy one obtains the NMSSM as an effective theory. The spontaneous breaking of R-parity also entails the spontaneous violation of total lepton number. This implies that one of the neutral CP-odd scalars, which we call majoron $J$ and which is approximately given by the imaginary part of

$$\text{v}_{\text{MAJORON}} = \frac{\text{v}_{\Phi}}{\sqrt{2}} + \frac{\text{v}_{\tilde{S}}}{\sqrt{2}},$$

remains massless, as it is the Nambu-Goldstone boson associated to the breaking of lepton number. $\text{v}_{\Phi}$ and $\text{v}_{\tilde{S}}$ are the vevs of $\Phi^c$ and $\tilde{S}$, respectively and $V = \sqrt{\text{v}_{\Phi}^2 + \text{v}_{\tilde{S}}^2}$. Clearly, the presence of these additional singlets enhances further the number of neutral scalar and pseudoscalar bosons. Explicit formulas for the mass matrices of scalar and pseudoscalar bosons can be found e.g. in [39].

The presence of the singlet fields implies in many respects similar features to the addition of the singlet Higgs in the NMSSM, see Section 4, e.g. the Higgs bosons have reduced couplings to the $Z$-boson:

$$\mathcal{L}_{ZH} = \sum_{i=1}^{8} (\sqrt{2}G_F)^{1/2} M_{Z} \mu_i \eta_i H_i^0.$$

In the basis $(\text{Re}(H_1^0), \text{Re}(H_2^0), \text{Re}(\tilde{\nu}_1), \text{Re}(\tilde{\nu}_2), \text{Re}(S), \text{Re}(\Phi))$, the $\eta_i$ read as:

$$\eta_i = \frac{v_d}{v} R_{i1} S^0 + \frac{v_u}{v} R_{i2} S^0 + \sum_{j=1}^{3} \frac{v_j}{v} R_{ij} S^0 + 2 \frac{v_{ij}}{v} R_{ij+2},$$

where $R_{ij}$ is the $8 \times 8$ mixing matrix of the neutral scalars. As a consequence the production cross section $e^+e^- \to H_1^0 Z$ can be reduced compared to the MSSM implying that one gets weaker bounds from the LEP data. Another feature similar to the NMSSM is that there is an upper bound on the mainly doublet Higgs boson of 150 GeV. Further details are discussed in Section 5.3.

The case of an enlarged gauge symmetry can be obtained for example in left-right symmetric models, e.g. with the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ [37]. Additional details on extra gauge groups can be found in Section 6. The corresponding superpotential is given by:

$$W = h_{\phi \bar{Q}} Q_L^T i\tau_2 \bar{Q}_R^c + h_{\chi \bar{Q}} Q_L^T i\tau_2 \bar{Q}_R^c + h_{\phi L} L_L^T i\tau_2 \bar{L}_R^c + h_{\chi L} L_L^T i\tau_2 \bar{L}_R^c + h_{\Delta L} L_L^T i\tau_2 \bar{L}_R^c + h_{\Delta R} \bar{Q}_R^T i\tau_2 \bar{Q}_R^c + \mu_1 \text{Tr}(i\tau_2 \phi^c i\tau_2 \phi^c) + \mu_2 \text{Tr}(\Delta \delta),$$

where the Higgs sector consists of two triplet and two bi-doublet Higgs superfields with the following $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ quantum numbers:

$$\Delta = \begin{pmatrix} \Delta^- & \Delta^0 & \Delta^+ \\ -\Delta^- & \Delta^0 & -\Delta^+ \end{pmatrix} \sim (1, 3, -2),$$

$$\delta = \begin{pmatrix} \delta^+ & \delta^0 & \delta^- \\ \delta^0 & \delta^0 & -\delta^0 \end{pmatrix} \sim (1, 3, 2),$$

$$\phi = \begin{pmatrix} \phi^0_1 & \phi^+_1 & \phi^0_2 \\ \phi^-_2 & \phi^0_2 & \phi^+_2 \end{pmatrix} \sim (2, 2, 0),$$

$$\bar{\chi} = \begin{pmatrix} \bar{\chi}^0_1 & \bar{\chi}^+_1 & \bar{\chi}^0_2 \\ \bar{\chi}^-_2 & \bar{\chi}^0_2 & \bar{\chi}^+_2 \end{pmatrix} \sim (2, 2, 0).$$

In the fermion sector the ‘right-handed’ matter superfields are combined to $SU(2)_R$ doublets which requires the existence of right-handed neutrinos. The corresponding superfields are denoted by $\bar{Q}_R$ and
for quark and lepton superfield respectively. Also in this case all neutral components of the Higgs fields and all sneutrinos get vevs. However, the majoron now becomes the longitudinal component of the extra $Z'$ gauge boson.

Looking at the decays of the Higgs bosons, one has to distinguish two scenarios: (i) Lepton number is gauged and, thus, the majoron becomes the longitudinal part of an additional neutral gauge boson. (ii) The majoron remains a physical particle in the spectrum. In the first case one has a situation similar to the case of explicit R-parity violation augmented with the possibilities of the NMSSM. There are for example regions in the parameter space where the scalar Higgs, which is mainly a doublet, decays into two pseudoscalar singlet Higgs bosons yielding e.g.

$$H^0_i \rightarrow A^0_i A^0_i \rightarrow b\bar{b}b\bar{b}.$$  (5.18)

In the case of the enlarged gauge group there are additional doubly charged Higgs bosons $H^\pm_i$ which have lepton number violating couplings. In $e^-e^-$ collisions they can be produced according to

$$e^-e^- \rightarrow H^\pm_i$$  (5.19)

and have decays of the type

$$H^\pm_i \rightarrow H^-_j H^-_k$$  (5.20)

$$H^\pm_i \rightarrow l^-_j l^-_k$$  (5.21)

where $l$ denotes $e, \mu$ and $\tau$. Further details on the phenomenology of doubly charged Higgs bosons can be found in Section 13.

The second case, where the majoron is part of the spectrum, leads to additional decay modes of the Higgs bosons. For example, the scalar Higgs bosons can decay according to

$$H^0_i \rightarrow A^0_j J$$  (5.22)

$$H^0_i \rightarrow J J$$  (5.23)

Note, that the later one is completely invisible. It has been shown that there is a sizable region in parameter space with a light scalar Higgs boson which is mainly a doublet and which decays mainly into the invisible mode above [38, 39]. The existence of the majoron leads also to new decay modes of the pseudoscalar Higgs bosons:

$$A^0_i \rightarrow H^0_j J$$  (5.24)

$$A^0_i \rightarrow J J J.$$  (5.25)

The later one is also completely invisible. However, either the production of the decaying pseudoscalar boson or the branching ratio into the invisible state are quite suppressed as discussed in Section 5.3. Therefore, this mode is phenomenologically less important than the decay $H^0_i \rightarrow J J$.

For the decays of supersymmetric particles the same general statements hold as for the Higgs bosons. In case (i) from above, the phenomenology is similar to the case of explicit R-parity breaking. The main difference is the existence of additional singlet neutralinos and/or gauginos which can be produced in the various cascade decays. In the case that all these singlet states turn out to be much heavier than the MSSM states one ends up with the bilinear model of R-parity breaking. In the case that the majoron is present, charginos and neutralinos have additional decay modes:

$$\tilde{\chi}^+_i \rightarrow J \tilde{t}^+$$  (5.26)

$$\tilde{\chi}^0_j \rightarrow J \nu.$$  (5.27)

The latter one is completely invisible. In the case that it dominates one recovers the usual missing energy of the MSSM although R-parity is broken. Further details on the phenomenology of SUSY particles in models with spontaneously broken R-parity can be found e.g. in [40, 41].
5.1.3 Constraints from colliders

A brief summary of the constraints on R-parity violation couplings from low energy effects has been given in Section 5.1.1 and we refer the reader to [26] for a more detailed review. In the following we focus on direct searches at colliders in models with broken R-parity, which have been carried by HERA, LEP and Tevatron collaborations over the past decade. The pair production of supersymmetric particles with the usual R-parity conserving supersymmetric couplings followed by direct or indirect decays involving R-parity violating couplings as well as singly produced supersymmetric particles involving directly the R-parity violating couplings (followed again by direct or indirect decays) have been extensively searched for. No evidence for supersymmetry with R-parity violation have been found at those colliders.

Constraints have been set on the masses of supersymmetric particles produced in pair where it has been assumed that the effect of the R-parity violating couplings is only important in the decays. An example for these constraints is shown in Table 5.3 for pair produced sfermions at LEP from [42–45].

In the case of the lower limits on the mass of $e_R$ and $\nu_e$, the Aleph collaboration assumes $\mu = -200$ GeV and $\tan \beta = 2$. For the lower limits for indirect decays $m_{\tilde{t}_1} - m_{\tilde{\chi}^0} > 10$ GeV is assumed for the $\lambda_{ijk}^n$ couplings. The lower limit on the $\tilde{\ell}_L$ mass (for direct decay) is obtained assuming $BR(\tilde{\ell}_L \rightarrow q\tau) = 1$. The Delphi collaboration takes $\mu = -200$ GeV, $\tan \beta = 1.5$ and $m_{\tilde{f}} - m_{\tilde{\chi}_1^0} \geq 5$ GeV. They also assume the lower mass limit on the lightest neutralino. The $\tilde{t}_1$ and $\tilde{b}_1$ limits from L3 are derived for a squark mixing angle minimizing the cross-section. The Opal collaboration take $m_{\tilde{\chi}_1^0} = 10$ GeV to derive the lower limits on charged sleptons and refer to $\tilde{t}_R$ ($\tilde{\ell}_L$) for the indirect (direct) decays. In case of the sneutrinos (indirect decays) $m_{\tilde{\nu}_i}$ is set to 60 GeV. The limit on $\tilde{\ell}_L$ assumes $i = 3$ (for $i = 1, 2$ this limit rises to 100 GeV). Moreover, for large sneutrino masses an absolute limit of 103 GeV has been set on the chargino mass by the Aleph collaboration [42], irrespective of the R-parity violating coupling. The Delphi collaboration [43] has set a lower limit of 39.5 GeV (103 GeV) for $m_{\tilde{\chi}_1^0}$ ($m_{\tilde{\chi}_1^+}$) for $\lambda_{ijk}$ couplings. For these limits 38.0 GeV and 102.5 GeV, respectively.

Squark pair production and gluino pair production have been considered by CDF and D0. For example, a lower limit on the stop mass of 122 GeV has been set assuming $BR(\tilde{t}_1 \rightarrow b\tau) = 1$ [46]. The CDF collaboration [47] considered the processes $p\bar{p} \rightarrow \tilde{g}\tilde{g} \rightarrow c\bar{c}s\bar{s} \rightarrow c(e^\pm d)c(e^\pm d)$ and $p\bar{p} \rightarrow \tilde{d}\tilde{d} \rightarrow \tilde{\chi}_1^0\tilde{\chi}_1^0 \rightarrow q(c\bar{c}d)q(c\bar{c}d)$ taking only $\lambda_{121}$ to be non-zero. This resulted into the constraint $\sigma \times B > 0.18$ pb for the $c$ search and into lower limits on squark masses i.e. 260 GeV for mass degenerate squarks and 135 GeV for $\tilde{t}_1$ assuming $m_{\tilde{g}} = 200$ GeV and a heavy $\tilde{\chi}_1^0$. The D0 collaboration [48] considered gluino and squark cascade decay till the lightest neutralino and then lightest neutralino decay via $\lambda_{2jk}$ with $j = 1, 2$ and $k = 1, 2, 3$. They obtained a lower limit on squark masses of 240 GeV independent of $m_{\tilde{g}}$ and a lower limit on $m_{\tilde{g}}$ of 224 GeV for all squark masses. In the case of $m_{\tilde{g}} = m_{\tilde{\chi}_1^0}$ a lower limit of 265 GeV has been obtained. In all cases $\tan \beta > 2$, $A_0 = 0$ and $\mu < 0$ has been assumed. The D0 collaboration [49] has also searched for gauginos pair production followed by decays mediated by the $\lambda_{121}$ and $\lambda_{122}$ couplings which allows one to exclude a large region of the parameter space for coupling values of the order of $10^{-4}$.

R-parity violation allows for the possibility of singly produced supersymmetric particles. For example the Delphi collaboration [50] has searched for resonant sneutrino production and decay involving the $\lambda_{121}$ and $\lambda_{131}$ couplings. The obtained constrained on these couplings are in the order of $2.3 \times 10^{-3}$ for $180 \text{ GeV} \leq m_{\tilde{g}} \leq 208$ GeV. The $e^\pm p$ HERA collider is ideally suited to the search for single squark production involving $\lambda_{ij}$ couplings. For example the H1 collaboration [51,52] (see also [53]) has excluded large regions of the planes $(\lambda_{ij1}, m_{\tilde{q}})$ for $j=1,2,3$, e.g. $\lambda_{ij1}^1 < 10^{-2}$ for $m_{\tilde{q}} < 200$ GeV, $|\mu| < 300$ GeV, $70 \text{ GeV} \leq M_2 \leq 350$ and $\tan \beta = 6$. The CDF collaboration [54] has searched for single sneutrino production and direct decays via $\lambda_{ij}$ leading to $e\mu$ (respectively $e\tau$ and $\tau\mu$) final states which resulted into the lower limits $\sigma \times B > 0.14$ pb (respectively 1.2 pb and 1.9 pb). The D0 collaboration [55] has searched for resonant smuon and muon sneutrino single production via $\lambda_{211}$ and has put lower limits of 280 GeV on the corresponding masses.
Table 5.3: Lower limits at the 95% confidence level on the masses of sleptons (unit GeV) from LEP assuming pair production followed by direct or indirect decay involving the R-parity violating couplings $\lambda_{ijk}$, $\lambda'_{ijk}$ and $\lambda''_{ijk}$ from [42–45]. The acronyms A, D, L and O indicate respectively the Aleph, Delphi, L3 and Opal collaborations. Details on the various assumptions are given in the text.

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5.2 The Higgs sector in models with explicitly broken R-parity

Martin Hirsch and Werner Porod

The down-type Higgs field $\tilde{H}_d$ and the left snepton $\tilde{L}_i$ fields of the MSSM carry the same $SU(3) \times SU_2(2) \times U_Y(1)$ quantum numbers. In models with conserved R-parity they can only be distinguished by lepton number. Therefore, in models where lepton number and, thus, R-parity is broken these fields are not distinguishable and the MSSM appears to consist of five Higgs doublets and three electrically charged but $SU_2(2) \times SU_3(1)$ singlet Higgs fields.

The breaking of R-parity can be realized by introducing explicit R-parity breaking terms [5] or by a spontaneous break-down of lepton number [35]. The first class of models can be obtained in mSUGRA scenarios where depending on the choice of discrete symmetries various combinations of R-parity violating parameters are present at the GUT or Planck scale [14]. The latter class of models leads after electroweak symmetry breaking to effective terms, the so-called bilinear terms, which are a sub-class of the terms present in the models with explicit R-parity breaking. These bilinear terms have an interesting feature: They do not introduce trilinear terms when evolved from one scale to another. In contrast, trilinear terms do generate bilinear terms when evolved from one scale to another.

From the point of view of Higgs physics these bilinear terms have the interesting feature that they lead to a mixing between the usual Higgs fields and sleptons, more precisely the charged Higgs boson mixes with the charged sleptons, the real part of the sneutrinos with the neutral scalar Higgs bosons and the imaginary part of the sneutrinos with the pseudoscalar Higgs boson. Therefore we will concentrate on the effect of bilinear R-parity breaking terms and comment on the case of additional tri-linear couplings at the end of this contribution. The model is specified by the following superpotential $W$ and soft SUSY breaking Lagrangian $V_{soft}$:

$$W = W_{MSSM} + \epsilon_i \tilde{L}_i \tilde{H}_u$$

$$V_{soft} = V_{soft,MSSM} + B_i \epsilon_i \tilde{L}_i H_u$$

where $W_{MSSM}$ and $V_{soft,MSSM}$ contain the usual MSSM terms. The $B_i$ induce vevs $v_i$ for the sneutrinos which, however, are not independent quantities. In the following we will trade them against the $\epsilon_i$ as free parameters. In the discussion below we will work in the ‘$\epsilon$-less’ basis presented in Section 5.1.1. In this basis effective trilinear couplings of the form $\lambda_{ijk} = (\epsilon_i/\mu) \cdot h^i_E \delta^{jk}$ and $\lambda'_{ijk} = (\epsilon_i/\mu) \cdot h^i_D \delta^{jk}$ are present, see Eqs. (5.10) and (5.11).

The R-parity violating parameters are constrained due to data from rare decays of sleptons and mesons and other low-energy data. As discussed in Section 5.1.1 the most important ones for this model arise from the observed neutrino data implying that $|\epsilon_i/\mu| \simeq O(10^{-3}) - O(10^{-2})$ and $v/M_2 \simeq 10^{-6}$ where $v = \sqrt{\nu_1^2 + \nu_2^2 + \nu_3^2}$. The smallness of these couplings imply that the mixing between sleptons and Higgs fields is in general small. The largest effects are observable if either the right-sleptons or the sneutrinos are the lightest supersymmetric particles (the left-sleptons are in general heavier than the sneutrinos). In the following we will discuss these two cases. Although lepton number is not defined anymore in this class of models we will nevertheless use the MSSM notation for simplicity.

5.2.1 The charged scalars

In this section we will discuss the case that the right-sleptons are the LSPs. From the experimental point of view they appear to be charged Higgs bosons decaying mainly into sleptons:

$$\tilde{l}_R \rightarrow \nu \nu, \mu \nu, \tau \nu \quad (l = e, \mu, \tau)$$

where $\nu$ denotes the sum over all neutrinos. Decays into quarks are suppressed by the corresponding left-right mixings and only in case of the right stau one can expect a sizable branching ratios for quark
final states if $\tan \beta$ is sufficiently large.

Before discussing the decays in some detail, let us briefly comment on slepton production at future collider experiments. Due to the smallness of the R-parity violating couplings the production of supersymmetric particles is MSSM like. Therefore, at the LHC the direct production of right–sleptons is small, e.g. about 110 (20) fb if $m_\tilde{l} \simeq 100(200)$ GeV \cite{56}. As a result, they will be produced mainly in cascade decays. The relative $\tilde{e}_R$, $\tilde{\mu}_R$ and $\tilde{\tau}_R$ yields will depend on the details of the cascade decays involved. In the cascade decays of squarks and the gluino several neutralinos and charginos will be produced. The gaugino like states will decay into an equal number of $\tilde{e}_R$, $\tilde{\mu}_R$ and $\tilde{\tau}_R$ except for kinematics. In particular the bino-like neutralino is expected to have a large branching ratio into $\tilde{l}_R$ as these are the particles with the biggest hypercharge. In the case of higgsino like states the corresponding branching ratios are proportional to the corresponding Yukawa coupling squared. At a future international linear collider the sleptons can be directly produced in $e^+e^-$ annihilation: $e^+e^- \rightarrow \tilde{l}^-\tilde{l}^+$. Typical cross sections are of the order of a 100 fb (10 fb) for $\tilde{e}$ ($\tilde{\mu}$ and $\tilde{\tau}$).

All three sleptons can decay into all charged leptons as can be seen from Eq. (5.30) and, thus, the question arises if there are any means to distinguish them. It turns out that different generations of sleptons have quite different life times as discussed in detail in Ref. \cite{24}. In Fig. 5.1 we show the charged slepton decay lengths ($\tilde{e}$, $\tilde{\mu}$ and $\tilde{\tau}$, from top to bottom) as a function of the scalar lepton masses performing a scan of the parameter space: $0 \leq M_2 \leq 1.2$ TeV, $0 \leq |\mu| \leq 2.5$ TeV, $0 \leq m_0 \leq 0.5$ TeV, $-3 \leq A_0/m_0, B_0/m_0 \leq 3$ and $2.5 \leq \tan \beta \leq 10$. The R-parity violating parameters are chosen in such a way \cite{10} that the neutrino masses and mixing angles are approximately consistent with the experimental data as described in detail in \cite{24}. As can be seen, all decay lengths are small compared to typical detector sizes, despite the smallness of the neutrino masses. The three generations of sleptons decay with quite different decay lengths and thus it should be possible to separate the different generations experimentally at a future linear collider. Note that the ratio of the decay lengths $L(\tilde{\tau})/L(\tilde{\mu})$ is approximately given by $(h_\mu/h_\tau)^2$ which can easily be understood from Eq. (5.11).

From Eq. (5.11) one expects that ratios of the branching ratios of the 'flavour' violating decay
The MSSM with R-parity violation

modes should be proportional to ratios of $\epsilon_i$ squared:

$$\frac{BR(l_i \rightarrow l_j \sum_r \nu_r)}{BR(l_i \rightarrow l_k \sum_r \nu_r)} \sim \frac{\epsilon_i^2}{\epsilon_k^2}$$

with $l_1 = e$, $l_2 = \mu$, $l_3 = \tau$ and $i \neq j \neq k$. Moreover, this feature should remain valid after taking into account all the mixing effects between SM particles and supersymmetric particles as has been shown in [24] semi-analytically. That this is indeed the case is shown in Fig. 5.2. As can be seen from these figures, the ratio of charged slepton branching ratios are correlated with the ratios of the corresponding BRpV parameters $\epsilon_i$, following very closely the expectation from Eq. (5.31), nearly insensitive to variation of the other parameters. Recall, that all the points were generated through a rather generous scan over the MSSM parameters. Ratios of $\epsilon_i$’s should therefore be very precisely measurable. Moreover, since only two of the three ratios of $\epsilon_i$’s are independent it is possible to derive the following prediction:

$$\frac{BR(\tilde{\tau}_1 \rightarrow e \sum \nu_i)/BR(\tilde{\tau}_1 \rightarrow \mu \sum \nu_i)}{BR(\tilde{\mu}_1 \rightarrow e \sum \nu_i)/BR(\tilde{\mu}_1 \rightarrow \tau \sum \nu_i)} \sim \frac{BR(\tilde{e}_1 \rightarrow \mu \sum \nu_i)}{BR(\tilde{e}_1 \rightarrow \tau \sum \nu_i)}$$

which provides an important cross check of the validity of the bilinear R-parity model. Any significant departure from this equality would be a clear sign that the bilinear model is incomplete. In the parameter range compatible with neutrino data, it turns out that the branching ratios of ‘flavour diagonal’ decay modes hardly vary with the underlying parameters: $BR(\tilde{\tau} \rightarrow \tau \sum \nu_i) \simeq BR(\tilde{\mu} \rightarrow \mu \sum \nu_i) \simeq 0.5$. The variations are of the order of 1%. The selectrons behave differently due to the smallness of lepton Yukawa coupling yielding that $0.96 \lesssim BR(\tilde{e}_R \rightarrow e \sum \nu_i) \lesssim 0.999$.

### 5.2.2 The neutral scalars

In this section we will discuss the scenario where the sneutrinos are the LSPs. The occurrence of sneutrino vevs implies in principle a splitting between real and imaginary parts of the sneutrino in analogy to the neutral Higgs sector (see e.g. [10] for the corresponding mass formulas). However, as a consequence of the smallness of the R-parity violating parameters this splitting is well below the expected accuracy of future collider experiments. Moreover, for the same reason R-parity violating production processes like $e^+ e^- \rightarrow Z \text{Re}(\tilde{\nu})$ have tiny cross sections in the order of $10^{-5}$ fb and the branching ratios of charginos and neutralinos into the real and imaginary parts of the sneutrinos occur with practically the same probability, e.g. $1 - BR(\tilde{\chi} \rightarrow \text{Re}(\tilde{\nu}))/BR(\tilde{\chi} \rightarrow \text{Im}(\tilde{\nu})) \simeq 10^{-5}$. For these reasons we will speak of ‘the’ sneutrino instead of making the distinction between scalar and pseudoscalar particles.
In this scenario the trilinear couplings of the sneutrinos to down quarks and charged leptons follow a hierarchy dictated by the standard model quark and charged lepton masses, see Eqs. (5.10) and (5.11). One expects therefore that the most important final state for sneutrinos is $b\bar{b}$, independent of the sneutrino generation. Electron and muon sneutrinos will decay also to $\tau\bar{\tau}$ final states with a relative ratio of

$$\frac{BR(\tilde{\nu}_e \rightarrow \tau\bar{\tau})}{BR(\tilde{\nu}_e \rightarrow b\bar{b})} \approx \frac{h^2_\tau}{3h^2_b(1 + \Delta_{QCD})}$$

independent of all other parameters. Here $\Delta_{QCD}$ are the QCD radiative corrections. Decays to $\mu\bar{\mu}$ (and non-$b$ jets) final states are suppressed by the corresponding Yukawa couplings squared. From this point of view they behave as a pure down-type Higgs boson of the MSSM. The two main differences are the occurrence of the invisible decay mode $\nu\nu$ and a small decay width implying a finite decay length as discussed below.

Tau sneutrinos, on the other hand, will decay to final states $e\tau$ and $\mu\tau$ with sizable branching ratios

$$\frac{BR(\tilde{\nu}_\tau \rightarrow e\tau)[BR(\tilde{\nu}_\tau \rightarrow \mu\tau)]}{BR(\tilde{\nu}_\tau \rightarrow b\bar{b})} \approx \frac{h^2_\tau}{3h^2_b(1 + \Delta_{QCD})} \frac{\epsilon_1^2}{\epsilon_3^2}$$

The above relation allows one to cross check the consistency of the bilinear scenario with neutrino data, as demonstrated in Fig. 5.3. The current $3\sigma$ allowed range for the solar neutrino mixing angle $\theta_S$ of $0.30 \leq \tan^2 \theta_S \leq 0.59$ fixes $BR(\tilde{\nu}_\tau \rightarrow e\tau)/BR(\tilde{\nu}_\tau \rightarrow \mu\tau)$ to be in the range from about 0.55 to about 1.25, as can be seen in Fig. 5.3.

Non-zero sneutrino vevs induce the decay $\tilde{\nu} \rightarrow \nu\nu$, i.e. by measuring non-zero branching ratios for invisible decays one could establish that sneutrino vevs exist. From the estimate on $\frac{\epsilon_3}{\epsilon_1}$ and $\frac{\epsilon_2}{\epsilon_3}$ discussed above one can estimate that branching ratios of sneutrino decays to invisible states should be of the order $O(10^{-4})$. Figure 5.4a shows the calculated branching ratios for invisible final states, $BR(\tilde{\nu}_i \rightarrow \sum \nu_j \nu_k)$, as a function of the sneutrino mass. The figure shows that the estimate discussed above is correct within an order of magnitude. It also demonstrates that for sneutrinos below $m_{\tilde{\nu}} \leq 500$ GeV one expects $BR(\tilde{\nu}_i \rightarrow \sum \nu_j \nu_k) \geq 10^{-5}$ a few events of the form $e^+e^- \rightarrow \tilde{\nu} \tilde{\nu} \rightarrow b\bar{b}\nu\nu$ are expected per year at a future international linear collider with a center of mass energy of 1 TeV.

To measure absolute values of R-parity violating parameters it would be necessary to measure the decay widths of the sneutrinos. Given the current neutrino data, however, such a measurement seems to be very difficult for the next generation of colliders. Figure 5.4b shows calculated decay lengths, assuming a center of mass energy of $\sqrt{s} = 1$ TeV, versus sneutrino mass. The decay lengths are short compared to sensitivities expected at a future linear collider which are of order 10 $\mu$m [57]. One can turn this argument around to conclude that observing decay lengths much larger than those shown in Fig. 5.4 would rule out explicit R-parity violation as the dominant source of neutrino mass.

![Fig. 5.3: Ratio of branching ratios $BR(\tilde{\nu}_\tau \rightarrow e\tau)/BR(\tilde{\nu}_\tau \rightarrow \mu\tau)$ versus (a) $(\epsilon_1/\epsilon_2)^2$ and b) $\tan^2 \theta_S$. From Ref. [16].](image)
5.2.3 Summary and comments

We have discussed in some detail the Higgs sector in models where R-parity is broken by bilinear terms. In this class of models the Higgs bosons mix with the slepton fields. However, the bounds on the R-parity violating parameters due to requirement of correctly explaining the observed neutrino data imply that large effects occur mainly if one of the following requirements on the SUSY spectrum is fulfilled.

The first possibility is that the right sleptons are the LSPs. In this case their signature is that of three electrically charged but SU$_L$(2) singlet Higgs bosons decaying into all generations of charged leptons. Decays into quarks are in general suppressed. An important property of these charged Higgs bosons is that their life time is quite different and that at least two of them should have a visible decay length at future collider experiments, see e.g. Fig. 5.1.

The second possibility is that the sneutrinos are the LSPs. In this case their signatures are close to those expected for the down-type Higgs boson of the MSSM. The main differences are: (i) the occurrence of the invisible mode $\nu_j$, (ii) small decay widths resulting in decay lengths of the order $\mu m$ and (iii) that one of them has sizable lepton flavour violating decay modes into $e\tau$ and $\mu\tau$.

Let us finally comment on the occurrence of additional trilinear couplings $\lambda_{ijk}$ and $\lambda'_{ijk}$. Their main effects are: (i) The hierarchy of the branching ratios discussed above will be distorted in general. (ii) They can give rise to significantly larger decay widths, in particular if their structure is anti-hierarchical compared to the usual lepton Yukawa couplings. (iii) The invisible decay mode gets tiny. Corresponding scenarios are discussed e.g. in Refs. [16, 25].

5.3 Phenomenology of the neutral Higgs sector in a model with spontaneously broken R-parity

Albert Villanova del Moral

Current neutrino data can be explained in the framework of the Spontaneously Broken R-Parity Model (SBRPM). This model contains a massless Nambu-Goldstone boson associated to spontaneous lepton number violation called majoron which opens additional invisible decay channels for Higgs bosons. We analyze the full neutral Higgs boson sector of the model and demonstrate that there is always a neutral CP-even Higgs boson, whose mass is bounded from above as in the Minimal Supersymmetric Standard Model, which is copiously produced. Moreover we show that its invisible decay mode to two majorons can be dominant in some regions of parameter space. We also study the associated channel where a
neutral CP-odd Higgs boson is produced together with a neutral CP-even one. We show how the lightest CP-odd Higgs boson can have a sizable production cross section and decay to a neutral CP-even Higgs boson and a majoron.

5.3.1 The model

The superpotential of this model is given in Eq. (5.12). Note, that if only trilinear terms are non-zero in the superpotential, this specific realization of the SBRPM would solve the $\mu$-problem in the same way as the Next to Minimal Supersymmetric Standard Model (NMSSM) [58, 59].

Considering only one generation of the superfields $(\tilde{\nu}^c_i, \tilde{S}_i)$ for simplicity, the scalar potential for the electrically neutral fields reads

$$V_{\text{total}} = |h\Phi \tilde{S} + h_0^i \nu_i H_u^0 + M_R \tilde{S}|^2 + |h_0 \Phi H_u^0| + M_\Phi \tilde{S}|^2 + |h\Phi \tilde{c}| + |1 - h_0 \Phi H_d^0 - \mu H_d^0 + h_1^i \nu_i \tilde{c}|^2 + | - h_0 H_u^0 H_d^0 + h\tilde{c} \tilde{S} - \delta^2 + M_\Phi \Phi + \frac{1}{2} \Phi^2|^2$$

$$+ \sum_{i=1}^{3} |h_0 \Phi H_u^0 + A_{h_0} h_0 \Phi H_d^0 + A_{h_i} h_i \nu_i H_u^0 \tilde{c} - B_{\mu} H_u^0 H_d^0|$$

$$- C_\delta \delta^2 \Phi + B_{M_R} M_R \tilde{c} \tilde{S} + \frac{1}{2} B_{M_\Phi} M_\Phi \Phi^2 + \frac{1}{3!} A_\lambda \Phi^3 + h.c.$$}

(5.35)

where $z_\alpha$ denotes any neutral scalar of the model.

As usual, electroweak symmetry is broken by the isodoublet nonzero vacuum expectation values (vevs)

$$\langle H_u^0 \rangle = \frac{v_u}{\sqrt{2}}, \quad \langle H_d^0 \rangle = \frac{v_d}{\sqrt{2}}.$$  (5.36)

R-parity is broken by the lepton-number-carrying isosinglet vevs

$$\langle \tilde{S} \rangle = \frac{v_2}{\sqrt{2}}, \quad \langle \tilde{c} \rangle = \frac{v_R}{\sqrt{2}}.$$  (5.37)

as well as by the (tiny) left sneutrino vevs

$$\langle \tilde{\nu}_{Li} \rangle = \frac{v_1}{\sqrt{2}}.$$  (5.38)

Last but not least, another important vev which is the key ingredient to generate an effective $\mu$-term (and so, solving the $\mu$-problem), is

$$\langle \Phi \rangle = \frac{v_0}{\sqrt{2}}.$$  (5.39)

5.3.2 Neutral Higgs boson masses

The neutral Higgs boson sector of the SBRPM consists of eight CP-even states $H_i^0$, six CP-odd states $A_i^0$ and one massless majoron $J$ (as the electroweak Goldstone boson $G_i^0$ is eaten by the $Z$). The $8 \times 8$ mass matrices for the neutral CP-even and CP-odd Higgs bosons (given in ref. [39]) can be analytically understood in certain limits [60]. First of all, we note that doublet sneutrinos practically do not mix with the rest of the Higgs bosons, as the corresponding entries in the mass matrices are proportional to $h_1^i$ and these parameters are small because of neutrino physics phenomenology. If there were also no mixing
between the \((H_d^0, H_u^0)\) doublet sector, the \(\tilde{\Phi}\) singlet sector and the \((\tilde{S}, \tilde{\nu}^c)\) singlet sector\(^2\), we would have as CP-odd mass eigenstates

\[
m^2_{G^0} = 0, \quad m^2_{A_D^0} = \Omega \left( \frac{v_u}{v_d} + \frac{v_d}{v_u} \right) \tag{5.40}
\]

in the \((H_d^0, H_u^0)\) doublet sector and

\[
m^2_{\tilde{j}} = 0, \quad m^2_{A_S^0} = -\Gamma \left( \frac{v_R}{v_S} + \frac{v_S}{v_R} \right) \tag{5.41}
\]

in the \((\tilde{S}, \tilde{\nu}^c)\) singlet sector, with

\[
\Omega = B\mu - \delta^2 h_0 + \frac{\lambda}{4} h_0 v^2 + \frac{1}{2} h h_0 v_R v_S + \frac{\sqrt{2}}{2} A_h h_0 v_{\Phi} + \frac{\sqrt{2}}{2} h_0 M_{\Phi} v_{\Phi} \tag{5.42}
\]

\[
\Gamma = B_{M,\mu} M_R - \delta^2 h + \frac{1}{4} h \lambda v^2 - \frac{1}{2} h h_0 v_a v_d + \frac{\sqrt{2}}{2} h (A_h + M_{\Phi}) v_{\Phi} . \tag{5.43}
\]

In addition we would have as CP-even mass eigenstates

\[
\begin{align*}
m^2_{H_j^0} &= 2 h^2 \frac{v_R^2 v_S^2}{v_R^2 + v_S^2}, \quad & m^2_{H_S^0} &= -\Gamma \left( \frac{v_R}{v_S} + \frac{v_S}{v_R} \right) - 2 h^2 \frac{v_R^2 v_S^2}{v_R^2 + v_S^2} \tag{5.44}
\end{align*}
\]

in the \((\tilde{S}, \tilde{\nu}^c)\) singlet sector, besides the states \(h_j^0\) and \(H_j^0\) in the \((H_d^0, H_u^0)\) doublet sector, which are analogous to their MSSM counterparts. In Fig. 5.5 a typical scanned Higgs mass spectrum is plotted as a function of the parameter \(\sqrt{\Gamma}\) and we can identify the \(A_S^0\) and \(H_S^0\) states as those which depend on \(\sqrt{\Gamma}\). The various gray-shadings (colors) indicate that the asymptotic states given in Eqs. (5.40), (5.41) and (5.44) constitute more the 50% of the corresponding particle.

Taking into account the phenomenological relation \(v_t \ll v_R, v_S\) we find the following approximation for the majoron

\[
J \simeq \frac{v_S}{V} \text{Im}(\tilde{S}) - \frac{v_R}{V} \text{Im}(\tilde{\nu}^c) \tag{5.45}
\]

where

\[
V^2 = v_S^2 + v_R^2 . \tag{5.46}
\]

\(^2\)This assumption is not strictly valid, as some of their mixings are not negligible, but it is useful to gain some insight on the parameter dependence of the eigenvalues.
5.3.3 Higgs boson production

Neutral Higgs bosons can be produced at an $e^+e^-$ collider via the Bjorken process (or direct production), as shown in Fig. 5.6, on the left. The relevant Lagrangian terms for this production mode are

$$\mathcal{L}_{ZH} = \sum_{i=1}^{8} (\sqrt{2}G_F)^{1/2} M_Z^2 Z_\mu Z^\mu \eta_i H_i^0$$

where $\eta_i$ is the direct production parameter given by

$$\eta_i = \frac{v_d}{v} R^{S_0}_{i1} + \frac{v_u}{v} R^{S_0}_{i2} + \sum_{j=1}^{3} \frac{v_j}{v} R^{S_0}_{ij}$$

We note that if $\eta_i^2$ is nearly one (zero), then $H_i^0$ is mainly a doublet (singlet).

From Fig. 5.7, on the left, we can see that when the direct production parameter $\eta_1^2$ of the lightest CP-even Higgs boson $H_1^0$ is nearly zero, then the direct production parameter $\eta_2^2$ of the next-to-lightest Higgs boson $H_2^0$ approaches one, i.e. $H_2^0$ is largely produced. From Fig. 5.7, on the right, we can see that there is always a state with a mass smaller than about 150 GeV. Combining both plots, we conclude that there is always a light Higgs boson with a large production cross section [60].

Another way of producing neutral Higgs bosons is via the associated production process shown in Fig. 5.6 on the right. The relevant Lagrangian terms for this production mechanism are

$$\mathcal{L}_{ZHA} = \sum_{i,j=1}^{8} (\sqrt{2}G_F)^{1/2} M_Z \zeta_{ij} \left( Z_\mu H_i^0 \tilde{B}_\mu \tilde{F}_j \right)$$

where $\zeta_{ij}$ is the associated production parameter, which is given by

$$\zeta_{ij} = R^{S_0}_{i1} R^{P_0}_{j1} - R^{S_0}_{i2} R^{P_0}_{j2} + \sum_{k=1}^{3} R^{S_0}_{ik} R^{P_0}_{jk}$$

In the MSSM exists a sum rule which relates both the direct and the associated production parameters. In the SBRPM we can construct an analogous but more complicated sum rule taking into account all possible final states [60]. The conclusion is that always at least one state will be produced.

5.3.4 Higgs boson decays

5.3.4.1 CP-even Higgs boson decays

The main decay channels for the lightest (or next-to-lightest) neutral CP-even Higgs bosons $H_{1,2}^0$ are

$$H_{1,2}^0 \to f_i \bar{f}_i \quad \text{if} \quad m_{H_{1,2}^0} > 2m_{f_i}$$

$$H_{1,2}^0 \to JJ$$
where the invisible decay width to majorons is
\[ \Gamma(H_{1,2}^0 \rightarrow JJ) = \frac{g_{H_{1,2}^0JJ}^2}{32\pi m_{H_{1,2}^0}} \] (5.52)
and for the fermionic decay widths all possible final states have been considered. We have taken into account the most important QCD corrections for the quark final states as given in [61].

We define the ratio between the invisible decay width and the visible one as
\[ R_{1,2} = \frac{\Gamma(H_{1,2}^0 \rightarrow JJ)}{\sum_j \Gamma(H_{1,2}^0 \rightarrow f_j f_j)} \] (5.53)
These ratios depend on the couplings \( g_{H_{1,2}^0 JJ} \) which are in general complicated functions of the underlying parameters. However, using Eq. (5.45) one obtains the following couplings to the unrotated doublet Higgs fields \( H_1^0 = \Re(H_d^0) \) and \( H_2^0 = \Re(H_u^0) \):
\[ g'_1 \simeq h h_0 v_u \frac{\nu_{SU}}{V^2} \] (5.54a)
\[ g'_2 \simeq h h_0 v_d \frac{\nu_{SU}}{V^2} - \frac{2v_u}{V^2} \sum_{j=1}^3 c_j^2 \] (5.54b)
Equations (5.54a) and (5.54b) imply that the doublet Higgs bosons can have large branching ratios for the invisible decay mode if the product of the couplings \( h \) and \( h_0 \) is large. Therefore, scenarios exist where neutral Higgs bosons have at the same time a large cross section and a large invisible decay branching ratio [38,39]. This is shown explicitly in Fig. 5.8 where the ratios \( R_i \) for the two lightest Higgs bosons are shown as a function of their direct production parameter \( \eta_i \) for different values of the parameter \( h \) [60].

5.3.4.2 CP-odd Higgs boson decays
The main decay channels for the lightest neutral CP-odd Higgs boson \( A_1^0 \) are:
\[ A_1^0 \rightarrow f_{1\bar{f}_i} \quad \text{if} \quad m_{A_1^0} > 2m_{f_i} \] (5.55a)
\[ A_1^0 \rightarrow H_1^0 J \quad \text{if} \quad m_{A_1^0} > m_{H_1^0} \] (5.55b)
\[ A_1^0 \rightarrow JJ J \] (5.55c)
The width of the CP-odd Higgs boson to a CP-even Higgs boson and a majoron reads

$$\Gamma(A_1^0 \rightarrow H_i^0 J) = \frac{g_{H_i^0 A_1^0 J}^2}{16\pi m_{A_1^0}^2} \left( m_{A_1^0}^2 - m_{H_i^0}^2 \right),$$

(5.56)

and to three majorons

$$\Gamma(A_1^0 \rightarrow JJJ) = \frac{m_{A_1^0} g_{A_1^0 JJJ}^2}{3072\pi^3}.$$  

(5.57)

Contrary to the neutral CP-even case, the corresponding couplings of the majorons to the unrotated doublet Higgs boson ($\exists(H_d^0)$ and $\exists(H_u^{0f})$) appearing in Eqs. (5.56) and (5.57) are zero in first order approximation, using Eq. (5.45) (detailed expressions are given in Ref. [60]). Therefore, the pseudoscalar Higgs boson has to have sizable admixtures of both, doublet and singlet components, if it should be produced at a sizable rate while having at the same time a significant invisible branching ratio. As an example we show in Fig. 5.9 the masses of $A_1^0$ and $H_2^0$, the associated production parameter and the invisible branching ratio of $A_1^0$ as a function of the parameter $\sqrt{\Gamma}$. One sees that in the region where the production cross section is at least 1% of the corresponding MSSM cross section, the branching ratio for the invisible mode varies between 5 and 10% [60].
5.3.5 Conclusions

We have shown that the model with spontaneously broken R-parity contains a light CP-even Higgs boson which is mainly doublet and which has a mass below about 150 GeV (like in the NMSSM). As a new feature we have demonstrated that its invisible decay mode into two majorons can be dominant.

In the case of the CP-even Higgs bosons we have seen that they decay mainly their MSSM counterpart if the doublet component dominates. However, in certain regions of parameter space the singlet component can be large enough to obtain a branching ratio of the invisible mode up to 10%.

5.4 Charged-Higgs-boson and charged-slepton radiation off a top quark at hadron colliders

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In this contribution we study the production of charged scalar particles radiated off a top quark at hadron colliders. The charged particles we consider are charged Higgs bosons and charged sleptons in R-parity-violating models. The remnant of the radiating top quark is the bottom quark in the charged-Higgs-boson case; in the charged-slepton case, it is most likely to be the down quark, but possibly also the strange and the bottom quark. Hereafter we shall refer to this production mechanism as strahlung production.

5.4.1 The charged-Higgs-boson case

Charged Higgs bosons, if detected, would be a clear signal of an extended Higgs sector. They are present in supersymmetric models, which, as is well known, require at least two Higgs doublets of opposite hypercharge. At hadron colliders, charged Higgs bosons can be pair produced in quark-initiated processes, such as the Drell–Yan process mediated by an off-shell photon or Z boson, the b-quark fusion and the W-boson fusion. All these are tree-level processes. Alternatively, they can be pair produced through gluon-fusion processes, which however proceed at the one-loop level. In both cases, the cross sections are not very large, reaching at the LHC ∼ 2–3 fb for $m_{H^\pm} = 400$ GeV and $\tan \beta = 30$ [62]. They can also be singly produced in association with other bosons, such as neutral Higgs bosons, or the W boson. These processes are quark-initiated at the tree level, but they can be gluon-initiated at the quantum level. Their cross sections may reach up to ∼ 100 fb for the same values of $m_{H^\pm}$ and $\tan \beta$ [62].

Strahlung off a third-generation quark, which can be gluon-initiated also at the tree level, can give similarly large or slightly larger cross sections [62–69]. Such a production mechanism can proceed through the $2 \to 2$ elementary process $gb \to tH^-$ and the $2 \to 3$ processes $gg, q\bar{q} \to tH^-b$, which formally give rise to the hadronic processes $p\bar{p}, pp \to tH^-X$ and $p\bar{p}, pp \to tH^-bX$, respectively.

Leading-order predictions for the two production cross sections are given in the two top frames of Fig. 5.10 for the Tevatron and LHC energies, as functions of the charged-Higgs-boson mass, for three different values of $\tan \beta$: $\tan \beta = 2, 10,$ and 50. (For discussions about limits on the $H^\pm$ mass in Two-Higgs-Doublet Models and supersymmetric models, see Refs. [70–75].) The hadronic cross sections $\sigma(p\bar{p}, pp \to tH^-bX)$ are shown by solid lines, $\sigma(p\bar{p}, pp \to tH^-X)$ by dashed lines. The integrations needed to obtain these cross sections are performed by the Monte Carlo integration routine VEGAS [76]. Moreover, the leading-order parton distribution functions CTEQ4L [77] were used, and the renormalization ($\mu_R$) and factorization ($\mu_F$) scales were fixed to the threshold value $m_t + m_{H^\pm}$. A variation of these scales in the interval between $(m_t + m_{H^\pm})/2$ and $2(m_t + m_{H^\pm})$ results in changes up to ±30% in both cross sections. QCD corrections, therefore, may be important, but they have been completed only for the $2 \to 2$ processes [78–81]. Part of these corrections are captured by the QCD correction to the b-quark mass, on which these cross sections depend quite sensitively. A study of their variation for different values of the b-quark mass can be found in Ref. [82]. (Supersymmetric corrections to both decays have also been calculated [83, 84].)

In the kinematical region $m_t > m_{H^\pm} + m_b$ the $2 \to 3$ elementary processes give the largest...
hadronic cross section. This can be well approximated by the much simpler resonant production cross section, given by the on-shell $tt$ production cross section times the branching fraction for the decay $t \rightarrow H^\pm b$. In the region $m_t \sim m_{H^\pm} + m_b$, however, this approximation fails to account for the correct mechanism of production and decay of the charged Higgs boson [85, 86]. When $m_t < m_{H^\pm} + m_b$, the relative size of the two classes of cross sections depends on $\sqrt{s}$ and $m_{H^\pm}$. At the Tevatron centre-of-mass energy, the quark-initiated $2 \rightarrow 3$ processes still have the dominant role up to intermediate values of $m_{H^\pm}$, i.e. up to $m_{H^\pm} \sim 265$ GeV. Both classes of cross sections show the typical behaviour as a function of $\tan \beta$, with a minimum at around $(m_t/m_b)^{1/2}$.

When the charged Higgs boson decays leptonically, $H^- \rightarrow \tau^- \nu_\tau$, the two production mechanisms, which lead to two and one $b$ quark in the final state are independent. (We assume here that it is possible to detect two $b$'s and one $\tau$.) This decay channel is suitable for the discovery of the charged Higgs boson in the region of large and possibly intermediate values of $\tan \beta$ [87], since it is not plagued by QCD background as the $H^- \rightarrow b\bar{b}$ mode [69]. The two production mechanisms can be experimentally distinguished, and studied separately.

When $H^-$ decays hadronically, typically into $b\bar{b}$, the final state to be identified contains at least three $b$'s for the $2 \rightarrow 2$ production mechanism, and at least four for the $2 \rightarrow 3$ one. Since tagging so many $b$'s seems very difficult, even at the LHC, the two production mechanisms result into final states that are indistinguishable. In this case, a sum of the two cross sections is necessary. Care must be taken, however, not to double count the overlapping part, obtained when one of the two initial gluons in the

Fig. 5.10: Cross-sections $\sigma(pp (pp) \rightarrow (t(b)H^-X)$ versus $m_{H^-}$, at the Tevatron and the LHC, for $\tan\beta = 2, 10, 50$, with $m_t = 175$ GeV, $m_b = 3$ GeV. Renormalization and factorization scales are fixed as $\mu_R = \mu_f = m_t + m_{H^-}$. In the two upper frames the solid lines correspond to the $2 \rightarrow 3$ processes, the dashed lines to the $2 \rightarrow 2$ processes. In the two lower frames are shown the cross sections obtained by adding the contributions from the $2 \rightarrow 2$ and $2 \rightarrow 3$ processes, and subtracting overlapping terms.
2 \to 3 \text{ processes produces a } b\bar{b} \text{ pair collinear to the initial } p \text{ or } \bar{p} \text{ [65]. Predictions for the appropriately summed inclusive cross section are shown in lower frames of Fig. 5.10, for both the Tevatron and the LHC. At the LHC, the cross section for } m_{H^\pm} = 400 \text{ GeV and } \tan \beta = 30 \text{ is about } 140 \text{ fb. These cross sections have the same theoretical uncertainty as the individual ones, as well as the same } \tan \beta \text{ dependence.}

5.4.2 The charged-slepton case

As is well known, the component of the charged Higgs boson with hypercharge \(-1/2\) has the same quantum numbers as the three superpartners of the charged leptons, \(\tilde{l}\), except for the lepton number \(L\). In \(R_p\)-violating models, in which \(L\) is violated (by operators with \(L = 1\)), these fields cannot be distinguished. Thus, some of the knowledge acquired by studying the strahlung of \(H^\pm\) off a top-quark line can be applied to investigate a similar production mechanism for charged sleptons.

The relevant operators for this discussion are the superpotential trilinear term \(\lambda'_{ijk} L_i Q_j D^c_k\), with \(i, j, k = 1, 2, 3\). In a basis in which all right-handed quarks and the left-handed down ones are diagonal, the trilinear superpotential operator gives rise to the lagrangian interaction terms:

\[
L \supset \lambda'_{imk} V_{mj} \bar{u}_{Lj} d_{Rk} \tilde{t}^{*}_{L1} - \lambda'_{imk} \bar{d}_{Lm} d_{Rk} \tilde{\nu}^{*}_{L1} + \text{H.c.,}
\]

where \(V_{mj}\) are elements of the CKM matrix. The second operators in this equation give rise to contributions to neutrino masses. Among the first ones, those with couplings such as \(\lambda'_{13k}\), induce the production of single charged sleptons in association with the top quark, in complete analogy with the strahlung production of the charged Higgs boson described before. Couplings like these, with at least one third-generation index, are only very weakly constrained by present experiments, except for those giving indications on the values of the neutrino masses. If we postpone for a moment the discussion of the impact of neutrino physics experiments, values of \(O(1)\) for these couplings, for squark masses of 300 GeV, are still not ruled out by other indirect processes [26].

Also in this case, two classes of elementary processes \(q\bar{q}, gg \to t \tilde{d}_k \tilde{t}_{L1}\) and \(gd_k \to t \tilde{L}_{1k}\), with \(d_k = d, s\) or \(b\) for \(k = 1, 2, 3\) respectively, are induced by the couplings \(\lambda'_{13k}\). Strictly speaking, strahlung off a top-quark line is obtained, more generally, also from couplings \(\lambda'_{imk}\) with \(m \neq 3\). The cross sections from these couplings, however, are suppressed by the factor \(|V_{m3}|^2\) \((m = 1, 2)\), which is smaller than \(10^{-1}\). We therefore neglect this possibility and consider only the contribution from \(\lambda'_{13k}\).
In Fig. 5.11, we show the cross sections for strahlung production of $\tilde{\tau}_L$ (i.e. $i=3$) at Tevatron and LHC energies, for the reference value $\lambda'_{33k} = 0.5$: solid lines denote $\sigma(pp) \rightarrow td_k\tilde{\tau}_L X$, dashed lines denote $\sigma(pp) \rightarrow t\tilde{\tau}_L X$. We assume here that the left-right mixing terms in the slepton mass matrix are small enough to render $\tilde{\tau}_L$ nearly a mass eigenstate, which we indicate simply by $\tilde{\tau}$ in the following. Obviously, the same cross sections are also obtained for the strahlung production of $\tilde{\mu}_L$ and $\tilde{e}_L$, or simply $\tilde{\mu}$ and $\tilde{e}$, when $\lambda'_{23k}$ and $\lambda'_{13k}$ are equal to 0.5. For each of the two sets of cross sections, induced by the $2 \rightarrow 2$ and the $2 \rightarrow 3$ processes, the three lines correspond, from top to bottom, to $k = 1, 2, 3$ in the coupling for $\lambda'_{33k}$, and therefore to $d_k = d, s, b$. We observe that among the hadronic cross sections induced by the $2 \rightarrow 2$ elementary processes, those indicated by the top lines in the two frames of Fig. 5.11 are initiated by a gluon and mainly a valence $d$ quark; those denoted by the central and bottom lines are initiated by a gluon and a sea quark, respectively $s$ and $b$. This is the reason for the larger values of the cross sections induced by $\lambda'_{331}$. Their enhancement with respect to those induced by $\lambda'_{333}$ is $\gtrsim 10$, the enhancement with respect to the cross sections induced by $\lambda'_{332}$ is roughly a factor of 5 at both colliders. Being mainly light-quark- or gluon-initiated, the cross sections induced by the $2 \rightarrow 3$ elementary processes, on the contrary, have all similar sizes for any value of $k$ in the couplings $\lambda'_{33k}$. All three curves are practically indistinguishable in the region $m_t > m_{\tilde{\tau}}$. The two curves corresponding to $\lambda'_{331}$ and $\lambda'_{332}$ are still very similar when $m_s \gtrsim m_t$. In this same region, they deviate from the curve corresponding to $\lambda'_{333}$, only slightly at the Tevatron, but by a factor of 2 at the LHC. This is essentially due to the large logarithms $\alpha_s(m_f) \ln(m_f/m_{d_k})$, originating from the $gg \rightarrow t\bar{d}_k\tilde{\tau}$ diagrams containing a virtual $d_k$ propagator [65], thus enhancing the cross-section for $d_k = s, d$ with respect to that for $d_k = b$. This effect is particularly evident at the LHC, where the $gg$-initiated processes largely dominate over the $q\bar{q}$ ones.

At both colliders, the overall situation for the production cross sections obtained for $k = 3$ is similar to that for the production of the charged Higgs boson: the resonant production, described by the cross section induced by the $2 \rightarrow 3$ processes, well exceeds the production induced by the $2 \rightarrow 2$ process in the region $m_t \gtrsim m_{\tilde{\tau}} + m_b$. The two production mechanisms are of similar size outside this region. The situation is different in the case in which $k = 2$, and much more so when $k = 1$: the cross section induced by the $2 \rightarrow 3$ processes can be neglected with respect to that due to the $2 \rightarrow 2$ process for $m_{\tilde{\tau}} \gtrsim m_t$ (at the precision of our calculation), and starts exceeding that induced by the $2 \rightarrow 2$ process only when $m_{\tilde{\tau}} < m_t$. There is indeed a region at $m_{\tilde{\tau}} \lesssim m_t$ in which the $2 \rightarrow 2$ process gives rise to a cross section still larger than that due to the decay of one of the two pair-produced top quarks, which, as already mentioned, is well described by the cross section induced by the $2 \rightarrow 3$ processes. For $k = 1$, this region is 150 GeV $\lesssim m_{\tilde{\tau}} \lesssim m_t$ at the LHC, and 160 GeV $\lesssim m_{\tilde{\tau}} \lesssim m_t$ at the Tevatron.

To obtain these cross sections, we have assumed that only one of the couplings $\lambda'_{33k}$ is present at a time. There is however no reason why this should be the case. Since $d$ and $s$ jets cannot be distinguished, at least the two sets of cross sections obtained for $k = 1$ and $k = 2$ give rise to the same final states. If the value of 0.5 is allowed for both couplings $\lambda'_{331}$ and $\lambda'_{332}$, for each of the two sets of cross sections, those obtained with these two couplings should be added. (Notice that, in this case, the width of the top quark in the $2 \rightarrow 3$ processes, in the kinematical region $m_t \lesssim m_{\tilde{\tau}} + m_b$, should be calculated accordingly, i.e. by considering the contribution from both couplings.) The case of $\lambda'_{333}$ is a little more complex and whether the corresponding cross sections can or cannot be distinguished from those induced by the couplings $\lambda'_{33k}$ with $k = 1, 2$ depends on the decay modes of $\tilde{\tau}$ and on how many $b$ quarks can be tagged. If it cannot be distinguished, and the same value for the three couplings $\lambda'_{33k}$ is allowed, the overall production cross sections for $\tilde{\tau}$ in the region $m_{\tilde{\tau}} < m_t$ can be considerably larger: three times the values indicated by solid lines in the two frames of Fig. 5.11. In the region $m_{\tilde{\tau}} > m_t$ the overall production cross section, however, still remains that induced by the coupling $\lambda'_{331}$.

This observation brings us back to the aforementioned issue of possible constraints induced on these couplings by neutrino physics. Neutrino masses get contributions induced by these couplings at
the one- and two-loop-level. The expressions for these contributions are \[22, 26, 67, 88\]

\[ m_{\nu_{i'i'}} \sim \frac{3}{8\pi^2} \lambda'_{ikj} \lambda'_{j'k'i'} m_{d_k} m_{d_j} \left( \frac{\mathcal{M}^k_{dLR}}{m_{d_k}} \right)^{kk} , \]  
(5.59)

\[ m_{\nu_{i'i'}} \sim \frac{3g_{\nu}}{2(16\pi^2)^2} \lambda'_{ikk} \lambda'_{i'kk} m_{d_k}^2 \frac{m_{\nu}}{2} \ln \left( \frac{m_{d_{i_1}}^2 m_{d_{i_2}}^2}{m_{d_k}^2} \right) , \]  
(5.60)

where \( j = 1, 2, g_1 \) and \( g_2 \) are the \((U(1) + SU(2))\) gauge couplings, \( m_{\nu} \) is the sneutrino mass, \( m_{\chi_0} \) the \( j \)-th neutralino eigenvalue, \( m_{d_k} \) the mass of the \( k \)-th down quark, \( m_{d_k}(\mathcal{M}^k_{dLR})_{kk} \) the LR mixing term of the \( k \)-th down-squark mass matrix, \( m_{d_k}^2 \) are the two \( k \)-th down-squark eigenvalues, and \( m_{d_k}^2 \) is an average value between these two. Clearly, we have already assumed that intergenerational mixing terms in the squark mass matrix are negligible. Under this assumption, second and third generation indices in the \( R_p \)-violating couplings in the two-loop contribution must be equal, while this is not the case in the one-loop contribution. The importance of the two-loop contribution stem from the fact that the parameter \( \mathcal{M}^k_{dLR} \) can be very small. Even if this is not the case, if we take \( m_{d_k} \sim 3m_{\nu}, m_{\nu} \sim m_{\chi_0}, \mathcal{M}^k_{dLR} \sim m_{d_k} \), and squark masses at 300 GeV, it is evident that the one- and two-loop contributions to neutralino masses differ by only one order of magnitude, when \( k = j \), therefore giving one- and two-loop constraints on the \( \lambda'_{ijk} \) couplings that are numerically very similar. By imposing that the contributions to the neutralino mass in Eqs. (5.59) and (5.60), does not exceed the value of 1 eV, we obtain, up to coefficients of \( \mathcal{O}(1) \):

\[ |(\lambda'_{ijk})_{1\text{-loop}}| \lesssim 10^{-6} \left( \frac{3 \text{ GeV}}{m_{d_k}} \right) \left( \frac{3 \text{ GeV}}{m_{d_j}} \right) \left( \frac{m_{d_k}}{300 \text{ GeV}} \right) \left( \frac{\mathcal{M}^k_{dLR}}{\mathcal{M}^{j\ell}_{dLR}} \right) , \]  
(5.61)

\[ |(\lambda'_{ikk})_{2\text{-loop}}| \lesssim 10^{-5} \left( \frac{3 \text{ GeV}}{m_{d_k}} \right)^2 \left( \frac{m_{\nu}}{100 \text{ GeV}} \right)^2 \left( \frac{100 \text{ GeV}}{|m_{\chi_0}|} \right) \left( \frac{\ln(100)}{\ln(m_{d_k}/m_{d_j})} \right) \]  
(5.62)

The constraints imposed on \( \lambda'_{333} (j = k = 3) \) by the two previous equations are rather severe. Even in the case in which \( |\mathcal{M}^k_{dLR}| \) is practically vanishing, the two-loop constraints say that \( |\lambda'_{333}| \) cannot be larger than \( 3 \times 10^{-3} \). When \( k = 3 \) and \( j \neq 3 \), a constraint on \( \lambda'_{3j3} \), the coupling responsible for the production of a charged slepton in association with a top quark, can only come from Eq. (5.59). Strictly speaking, however, this is a constraint on the product of two different couplings \( \lambda'_{3j3} \) and \( \lambda'_{ij3} \), and there are no a priori reasons why the suppression on the right-hand side has to be inherited only by \( \lambda'_{3j3} \) and does not have to be shared equally between the two couplings, or even be completely borne out by \( \lambda'_{ij3} \). (For \( |\lambda'_{3j3}| \sim |\lambda'_{ij3}| \), when \( j = 2 \) it should be \( |\lambda'_{ij3}| \lesssim 5 \times 10^{-3} \), for \( j = 1 \), \(|\lambda'_{ij3}| \lesssim 3 \times 10^{-2} \). A suppression of \( |\mathcal{M}^j_{dLR}| \) could, however, ease out these upper bounds.) Clearly all these constraints can still be evaded if the contributions to neutralino masses in Eqs. (5.59) and (5.60) are cancelled by other one- and two-loop contributions induced by other \( R_p \)-violating couplings [22, 67]. If this were the case and the value of 0.5 were allowed for both couplings \( \lambda'_{ij3} \) and \( \lambda'_{3j1} \), then the two sets of cross sections in Fig. 5.11 would have to be summed, as mentioned a little earlier. This would have practically no consequences for the cross sections induced by the \( 2 \rightarrow 2 \) processes, but it would double the charged-slepton rate of production at the Tevatron (LHC) in the region \( m_{\tilde{t}} \lesssim 160 \) (150) GeV, where the \( 2 \rightarrow 3 \) processes give the dominant contribution. If no more than 2 \( b \)'s can be detected in the final state obtained after the decay of the charged slepton, and if it could be \( \lambda'_{333} = 0.5 \), also the cross sections generated by this coupling should be combined to the other two.

We focus on the couplings \( \lambda'_{ij3} (j = 1, 2) \) to discuss the signature of the final state obtained after the slepton decay. Its main decay mode is \( \tilde{\ell}_i \rightarrow l_i \chi^0 \). The neutralino, in turn, can decay through the same coupling \( \lambda'_{313} \) into [89]: \( b\nu_l, b\nu_{l'}, t\ell_i, t\tilde{\ell}_i, d\tilde{\ell}_i \). Particularly interesting is the mode \( t\tilde{\ell}_i \); for a mass of \( \tilde{\ell}_i \) larger than 160 GeV at the Tevatron or 150 GeV at the LHC, this gives rise to

\[ t\tilde{\ell}_i \rightarrow t l_i \chi^0 \rightarrow 2t + 2l_i + \text{jet} \rightarrow 2b + 2W + 2l_i + \text{jet} , \]  
(5.63)
with two equal-sign leptons. Notice that these two leptons do not have to be of the same type since two \( \lambda_{i,31} \) couplings, with two different \( i \) indices, may intervene at the level of production and of decay of the charged slepton. If one of the two \( W \)’s decays leptonically, the final state with three leptons, two \( b \)’s jets and missing energy cannot be overlooked. The obvious background for such a final state would be given by the decay products of a \( t\bar{t} \) pair with \( t\bar{t} \to b\bar{b}W^+W^- \), with both \( W \)’s decaying leptonically and one \( b \) quark semileptonically. The identification of two \( b \)’s would then allow us to distinguish the signal from the background without too much loss in the signal, at least in the case in which there are no \( \tau \)’s among the leptons.

REFERENCES


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